Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/120-4.5.1.4-d-tan-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [365]. This is test number [120].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.73 (364)	0.27 (1)
Mathematica	94.52 (345)	5.48 (20)
Maple	90.68 (331)	9.32 (34)
Fricas	71.23 (260)	28.77 (105)
Giac	69.59 (254)	30.41 (111)
Maxima	58.36 (213)	41.64 (152)
Mupad	49.59 (181)	50.41 (184)
Sympy	10.96 (40)	89.04 (325)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

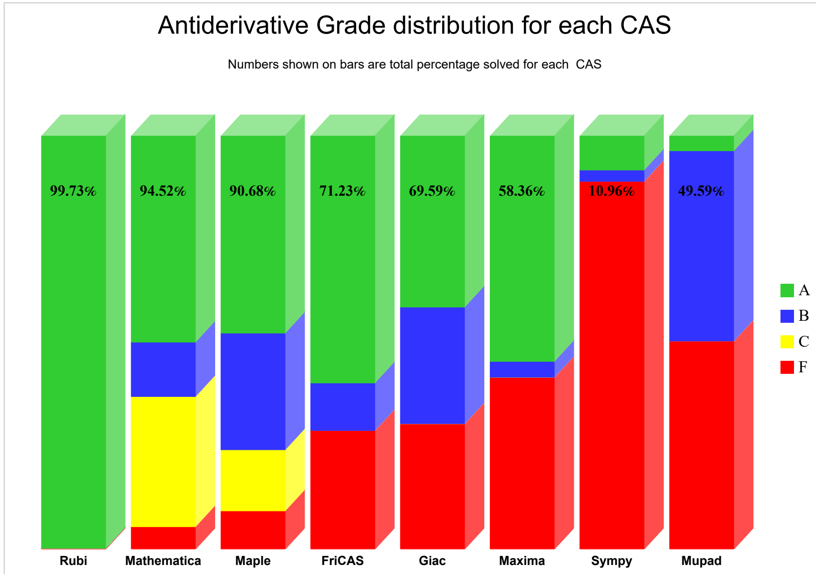
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

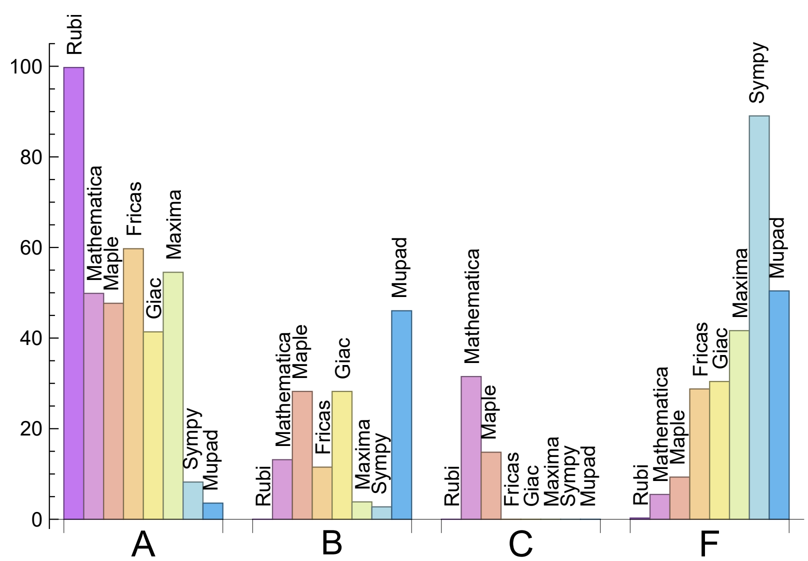
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.73	0.00	0.00	0.27
Fricas	59.73	11.51	0.00	28.77
Maxima	54.52	3.84	0.00	41.64
Mathematica	49.86	13.15	31.51	5.48
Maple	47.67	28.22	14.79	9.32
Giac	41.37	28.22	0.00	30.41
Sympy	8.22	2.74	0.00	89.04
Mupad	N/A	46.03	0.00	50.41

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	20	100.00 %	0.00 %	0.00 %
Maple	34	100.00 %	0.00 %	0.00 %
Fricas	105	40.95 %	59.05 %	0.00 %
Giac	111	96.40 %	0.00 %	3.60 %
Maxima	152	77.63 %	15.79 %	6.58 %
Sympy	325	85.54 %	9.54 %	4.92 %
Mupad	184	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

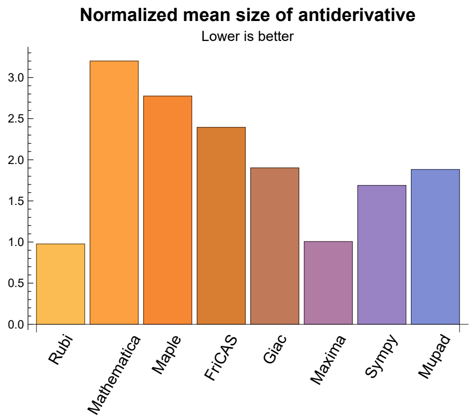
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

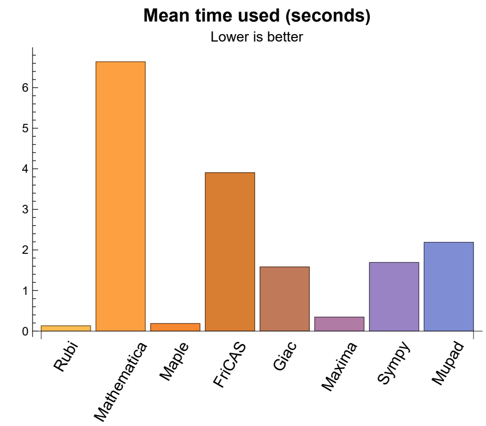
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	169.12	0.98	129.50	1.00
Mathematica	6.64	567.25	3.20	138.00	1.04
Maple	0.19	600.07	2.77	189.00	1.56
Maxima	0.35	110.02	1.01	105.00	0.95
Fricas	3.90	335.31	2.39	178.00	1.58
Sympy	1.69	131.63	1.69	97.00	1.36
Giac	1.58	225.11	1.90	172.50	1.59
Mupad	2.19	236.86	1.88	118.00	1.45

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 143, 144, 145, 146, 156, 157, 158, 169, 170, 177, 178, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 201, 202, 203, 204, 205, 206, 207, 213, 226, 227, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 312, 316, 317, 331, 332, 340, 341, 347, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

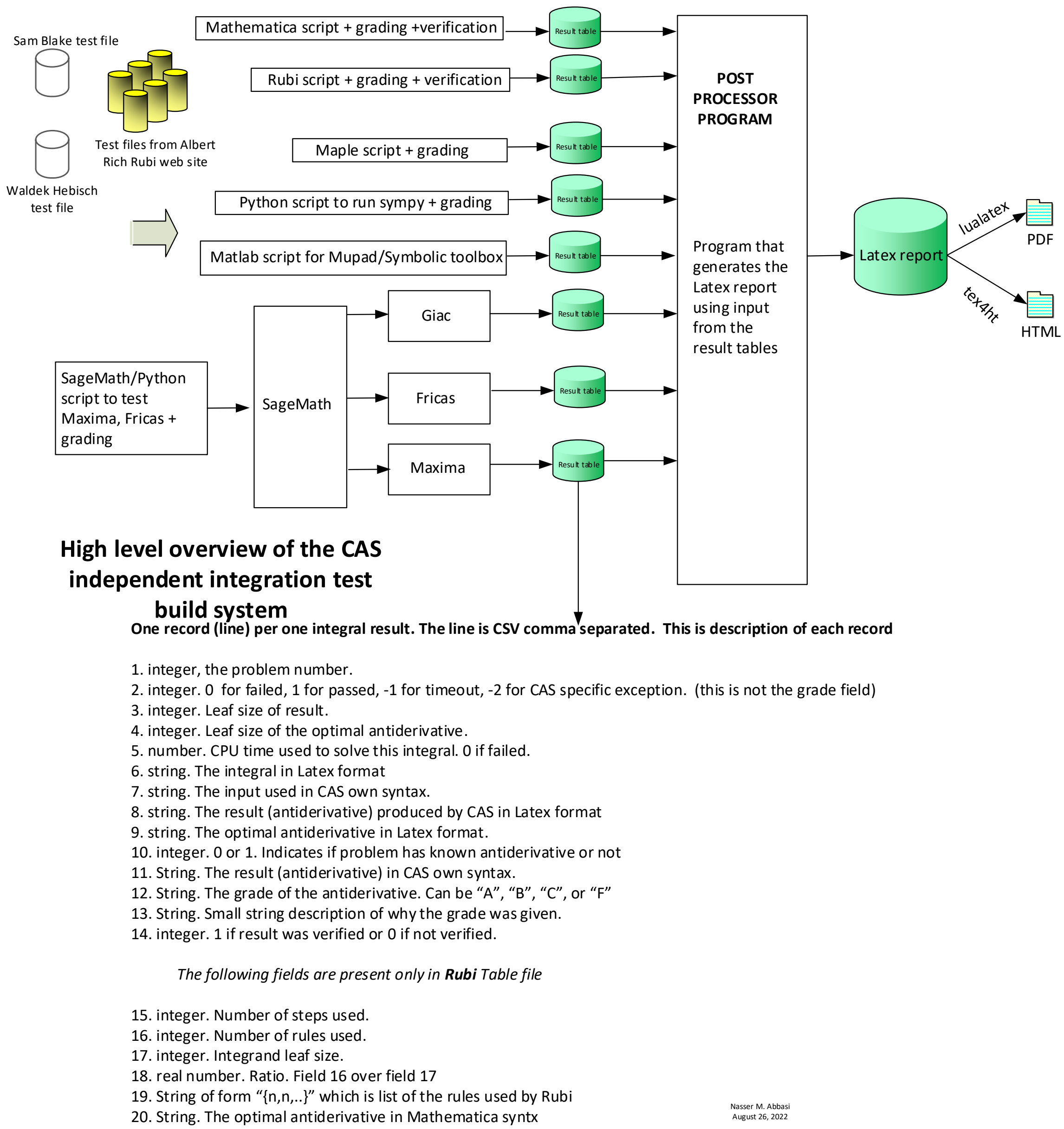
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { }

C grade: { }

F grade: { 347 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 103, 135, 136, 137, 138, 141, 142, 147, 148, 149, 150, 151, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 179, 183, 184, 190, 195, 196, 211, 219, 220, 221, 222, 223, 224, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 291, 292, 296, 297, 301, 302, 303, 304, 309, 310, 311, 318, 319, 324, 325, 326, 333, 335, 336, 340, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 29, 30, 31, 35, 36, 49, 50, 54, 64, 65, 66, 67, 69, 70, 79, 80, 81, 84, 85, 97, 98, 99, 100, 102, 201, 226, 227, 229, 230, 231, 232, 280, 281, 286, 287, 294, 295, 298, 299, 300, 307, 308, 320, 327, 328, 331, 337, 347 }

C grade: { 14, 15, 16, 17, 18, 32, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 139, 140, 143, 144, 145, 146, 152, 157, 158, 169, 170, 174, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 188, 189, 191, 192, 193, 194, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 209, 210, 213, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 267, 268, 269, 270, 293, 305, 306, 312, 313, 314, 315, 316, 317, 321, 322, 323, 329, 330, 332, 334, 338, 339, 341, 342, 343 }

F grade: { 127, 128, 129, 130, 132, 208, 212, 214, 215, 216, 217, 218, 225, 228, 250, 251, 252, 253, 254, 255 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 137, 138, 149, 150, 161, 162, 173, 184, 185, 190, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 324, 328, 333, 337, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 37, 38, 39, 52, 53, 54, 55, 64, 65, 66, 67, 81, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 294, 295, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 341, 342, 343 }

C grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F grade: { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 135, 136, 137, 147, 148, 149, 159, 160, 161, 171, 172, 173, 183, 184, 185, 197, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 299, 300, 301, 302, 303, 304, 305, 318, 319, 320, 326, 327, 328, 335, 336, 337, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 99, 195, 196, 306 }

C grade: { }

F grade: { 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 294, 295, 296, 297, 298, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 162, 165, 166, 167, 169, 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 308, 310, 318, 319, 320, 326, 327, 335, 336, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 9, 13, 17, 18, 28, 46, 150, 151, 152, 156, 163, 164, 168, 173, 174, 175, 185, 186, 187, 188, 196, 197, 198, 199, 200, 207, 266, 293, 298, 305, 306, 307, 309, 311, 321, 322, 328, 329, 330, 337, 338, 339 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 323, 324, 325, 331, 332, 333, 334, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 19, 21, 23, 40, 41, 256, 257, 258, 259, 271, 272, 273, 274, 275, 348, 349, 350, 351, 352, 358, 359, 360, 363, 364, 365 }

B grade: { 20, 22, 37, 38, 39, 60, 75, 90, 91, 290 }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357, 361, 362 }

2.1.7 Giac

A grade: { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 171, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 260, 267, 276, 279, 282, 309, 310, 311, 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 13, 20, 21, 22, 23, 25, 38, 39, 40, 41, 42, 43, 44, 57, 58, 59, 66, 67, 71, 72, 73, 81, 87, 88, 89, 143, 144, 145, 156, 157, 158, 168, 169, 170, 178, 179, 189, 190, 201, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 318, 319, 320, 322, 326, 327, 328, 330, 336 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 321, 323, 324, 325, 329, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.1.8 Mupad

A grade: { 348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 137, 149, 161, 173, 185, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 328, 337 }

C grade: { }

F grade: { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	134	199	116	123	184	293	259
N.S.	1	1.00	0.89	1.32	0.77	0.81	1.22	1.94	1.72
time (sec)	N/A	0.048	0.458	0.163	0.275	2.705	2.337	5.998	5.133

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	159	94	101	148	247	204
N.S.	1	1.00	0.90	1.35	0.80	0.86	1.25	2.09	1.73
time (sec)	N/A	0.044	0.440	0.105	0.275	2.501	1.149	3.508	5.693

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	123	72	79	112	201	151
N.S.	1	1.00	0.94	1.41	0.83	0.91	1.29	2.31	1.74
time (sec)	N/A	0.036	0.314	0.119	0.280	4.539	0.522	1.813	5.734

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	83	50	57	76	155	96
N.S.	1	1.00	0.96	1.46	0.88	1.00	1.33	2.72	1.68
time (sec)	N/A	0.028	0.121	0.082	0.270	3.612	0.217	0.808	1.993

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	34	37	106	40
N.S.	1	1.00	1.00	0.80	1.04	1.36	1.48	4.24	1.60
time (sec)	N/A	0.014	0.024	0.037	0.284	3.810	0.085	0.493	1.172

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	29	26	14	16	0	58	34
N.S.	1	1.00	1.81	1.62	0.88	1.00	0.00	3.62	2.12
time (sec)	N/A	0.015	0.026	0.064	0.272	3.071	0.000	0.433	1.239

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	114	48	42	69	0	103	46
N.S.	1	1.00	2.00	0.84	0.74	1.21	0.00	1.81	0.81
time (sec)	N/A	0.032	0.790	0.118	0.294	2.361	0.000	0.479	1.275

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	127	73	86	150	0	149	88
N.S.	1	1.00	1.34	0.77	0.91	1.58	0.00	1.57	0.93
time (sec)	N/A	0.045	0.554	0.128	0.284	3.033	0.000	0.504	1.178

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	165	96	126	241	0	197	118
N.S.	1	1.00	1.24	0.72	0.95	1.81	0.00	1.48	0.89
time (sec)	N/A	0.061	0.406	0.128	0.275	3.009	0.000	0.543	1.227

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	115	180	164	156	0	174	242
N.S.	1	1.00	0.89	1.40	1.27	1.21	0.00	1.35	1.88
time (sec)	N/A	0.091	1.825	0.112	0.497	2.897	0.000	4.940	2.469

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	95	143	134	134	0	146	188
N.S.	1	1.00	0.93	1.40	1.31	1.31	0.00	1.43	1.84
time (sec)	N/A	0.068	1.297	0.099	0.483	2.840	0.000	2.355	2.383

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	104	102	112	0	118	134
N.S.	1	1.00	1.03	1.42	1.40	1.53	0.00	1.62	1.84
time (sec)	N/A	0.045	0.438	0.099	0.488	3.205	0.000	1.064	1.852

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	67	65	87	0	88	80
N.S.	1	1.00	1.33	1.49	1.44	1.93	0.00	1.96	1.78
time (sec)	N/A	0.024	0.035	0.071	0.495	2.651	0.000	0.624	1.139

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	31	33	0	26	19
N.S.	1	1.00	1.65	1.35	1.19	1.27	0.00	1.00	0.73
time (sec)	N/A	0.017	0.033	0.069	0.513	2.385	0.000	0.465	1.073

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	59	72	0	56	53
N.S.	1	1.00	1.13	1.56	1.07	1.31	0.00	1.02	0.96
time (sec)	N/A	0.035	0.042	0.100	0.479	2.913	0.000	0.494	1.200

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	79	139	0	83	156
N.S.	1	1.00	0.94	1.54	0.94	1.65	0.00	0.99	1.86
time (sec)	N/A	0.058	0.064	0.094	0.502	2.688	0.000	0.494	1.449

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	100	210	0	113	204
N.S.	1	1.00	0.83	1.46	0.90	1.89	0.00	1.02	1.84
time (sec)	N/A	0.085	0.056	0.124	0.470	2.560	0.000	0.507	1.983

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	111	205	119	279	0	140	252
N.S.	1	1.00	0.79	1.46	0.85	1.99	0.00	1.00	1.80
time (sec)	N/A	0.111	0.072	0.161	0.494	3.354	0.000	0.539	3.175

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	140	225	149	156	314	342	308
N.S.	1	1.00	0.73	1.17	0.78	0.81	1.64	1.78	1.60
time (sec)	N/A	0.068	0.499	0.175	0.278	4.484	3.240	6.402	5.142

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	185	110	117	252	292	249
N.S.	1	1.00	0.83	1.40	0.83	0.89	1.91	2.21	1.89
time (sec)	N/A	0.056	0.292	0.128	0.273	3.216	1.734	3.663	4.819

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	125	149	97	104	189	242	192
N.S.	1	1.00	1.04	1.24	0.81	0.87	1.58	2.02	1.60
time (sec)	N/A	0.052	0.391	0.109	0.263	3.049	0.790	1.930	5.052

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	83	109	58	65	126	192	133
N.S.	1	1.00	1.28	1.68	0.89	1.00	1.94	2.95	2.05
time (sec)	N/A	0.040	0.183	0.091	0.278	2.771	0.345	0.902	3.870

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	34	43	52	60	142	76
N.S.	1	1.00	1.06	0.71	0.90	1.08	1.25	2.96	1.58
time (sec)	N/A	0.025	0.104	0.056	0.268	3.288	0.147	0.535	1.214

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	28	31	35	0	64	36
N.S.	1	1.00	0.83	0.80	0.89	1.00	0.00	1.83	1.03
time (sec)	N/A	0.027	0.041	0.096	0.274	4.268	0.000	0.485	1.255

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	39	34	48	0	111	50
N.S.	1	1.00	1.40	0.98	0.85	1.20	0.00	2.78	1.25
time (sec)	N/A	0.035	0.070	0.096	0.272	4.553	0.000	0.489	1.229

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	63	72	122	0	138	62
N.S.	1	1.00	1.01	0.74	0.85	1.44	0.00	1.62	0.73
time (sec)	N/A	0.049	0.287	0.112	0.274	3.732	0.000	0.548	1.260

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	114	86	109	191	0	186	113
N.S.	1	1.00	0.90	0.68	0.86	1.50	0.00	1.46	0.89
time (sec)	N/A	0.063	0.246	0.125	0.263	3.056	0.000	0.572	1.269

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	146	111	165	322	0	238	149
N.S.	1	1.00	0.86	0.66	0.98	1.91	0.00	1.41	0.88
time (sec)	N/A	0.081	0.351	0.149	0.280	3.547	0.000	0.608	1.589

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	337	169	151	165	0	180	234
N.S.	1	1.00	2.09	1.05	0.94	1.02	0.00	1.12	1.45
time (sec)	N/A	0.136	1.474	0.119	0.473	2.910	0.000	2.659	2.648

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	558	130	119	139	0	148	174
N.S.	1	1.00	4.69	1.09	1.00	1.17	0.00	1.24	1.46
time (sec)	N/A	0.100	5.687	0.092	0.488	2.855	0.000	1.240	1.953

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	773	93	83	111	0	99	101
N.S.	1	1.00	10.74	1.29	1.15	1.54	0.00	1.38	1.40
time (sec)	N/A	0.081	6.333	0.092	0.473	2.374	0.000	0.735	1.208

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	50	48	42	0	31	24
N.S.	1	1.00	1.31	1.43	1.37	1.20	0.00	0.89	0.69
time (sec)	N/A	0.053	0.043	0.082	0.475	2.159	0.000	0.459	1.078

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	112	112	77	81	0	50	39
N.S.	1	1.00	1.62	1.62	1.12	1.17	0.00	0.72	0.57
time (sec)	N/A	0.081	0.276	0.082	0.480	2.523	0.000	0.481	1.096

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	194	155	97	118	0	80	78
N.S.	1	1.00	1.81	1.45	0.91	1.10	0.00	0.75	0.73
time (sec)	N/A	0.091	0.835	0.092	0.478	3.593	0.000	0.510	1.259

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	312	188	117	173	0	112	182
N.S.	1	1.00	2.24	1.35	0.84	1.24	0.00	0.81	1.31
time (sec)	N/A	0.105	1.133	0.116	0.482	3.702	0.000	0.540	1.755

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	428	231	137	274	0	145	230
N.S.	1	1.00	2.39	1.29	0.77	1.53	0.00	0.81	1.28
time (sec)	N/A	0.121	1.927	0.135	0.477	2.917	0.000	0.582	2.778

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	214	386	162	169	439	367	337
N.S.	1	1.00	1.02	1.84	0.77	0.80	2.09	1.75	1.60
time (sec)	N/A	0.072	0.699	0.197	0.263	3.580	4.609	6.189	5.263

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	110	318	110	117	350	317	278
N.S.	1	1.00	0.80	2.32	0.80	0.85	2.55	2.31	2.03
time (sec)	N/A	0.056	0.381	0.148	0.265	3.273	2.402	3.968	5.169

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	140	254	110	117	255	267	221
N.S.	1	1.00	1.01	1.84	0.80	0.85	1.85	1.93	1.60
time (sec)	N/A	0.055	0.407	0.122	0.284	2.224	1.214	2.133	5.483

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	186	84	91	165	217	162
N.S.	1	1.00	0.93	1.88	0.85	0.92	1.67	2.19	1.64
time (sec)	N/A	0.046	0.294	0.101	0.267	2.930	0.533	1.064	5.546

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	44	58	65	76	167	105
N.S.	1	1.00	0.97	0.67	0.88	0.98	1.15	2.53	1.59
time (sec)	N/A	0.028	0.168	0.070	0.274	3.266	0.215	0.588	1.940

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	36	43	61	0	145	86
N.S.	1	1.00	0.75	0.75	0.90	1.27	0.00	3.02	1.79
time (sec)	N/A	0.033	0.092	0.101	0.267	3.326	0.000	0.506	1.201

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	39	34	50	0	109	48
N.S.	1	1.00	1.15	0.98	0.85	1.25	0.00	2.72	1.20
time (sec)	N/A	0.036	0.142	0.111	0.264	2.369	0.000	0.515	1.234

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	72	52	59	82	0	138	78
N.S.	1	1.00	1.18	0.85	0.97	1.34	0.00	2.26	1.28
time (sec)	N/A	0.043	0.188	0.122	0.266	2.473	0.000	0.546	1.220

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	102	74	96	178	0	165	94
N.S.	1	1.00	0.95	0.69	0.90	1.66	0.00	1.54	0.88
time (sec)	N/A	0.055	0.692	0.130	0.270	2.192	0.000	0.567	1.291

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	130	99	142	272	0	213	130
N.S.	1	1.00	0.87	0.66	0.95	1.83	0.00	1.43	0.87
time (sec)	N/A	0.071	0.367	0.148	0.268	3.012	0.000	0.615	1.204

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	363	290	262	178	0	196	263
N.S.	1	1.00	1.53	1.22	1.11	0.75	0.00	0.83	1.11
time (sec)	N/A	0.229	2.125	0.135	0.497	2.610	0.000	2.682	2.429

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	303	223	210	152	0	164	203
N.S.	1	1.00	1.79	1.32	1.24	0.90	0.00	0.97	1.20
time (sec)	N/A	0.170	1.173	0.110	0.499	3.079	0.000	1.367	2.439

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	230	157	147	113	0	132	146
N.S.	1	1.00	2.35	1.60	1.50	1.15	0.00	1.35	1.49
time (sec)	N/A	0.119	0.877	0.086	0.488	3.118	0.000	0.817	1.909

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	109	79	85	84	0	66	35
N.S.	1	1.00	2.22	1.61	1.73	1.71	0.00	1.35	0.71
time (sec)	N/A	0.072	0.249	0.082	0.491	3.158	0.000	0.519	1.200

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	112	125	90	82	0	50	39
N.S.	1	1.00	1.62	1.81	1.30	1.19	0.00	0.72	0.57
time (sec)	N/A	0.098	0.253	0.098	0.483	2.695	0.000	0.534	1.198

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	112	232	122	118	0	66	62
N.S.	1	1.00	1.05	2.17	1.14	1.10	0.00	0.62	0.58
time (sec)	N/A	0.117	0.699	0.110	0.483	3.452	0.000	0.526	1.446

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	252	293	152	160	0	96	91
N.S.	1	1.00	1.79	2.08	1.08	1.13	0.00	0.68	0.65
time (sec)	N/A	0.133	0.994	0.112	0.497	3.140	0.000	0.568	1.700

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	370	364	182	235	0	128	206
N.S.	1	1.00	2.07	2.03	1.02	1.31	0.00	0.72	1.15
time (sec)	N/A	0.153	1.359	0.132	0.504	2.977	0.000	0.605	1.944

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	268	425	212	314	0	161	254
N.S.	1	1.00	1.26	2.00	1.00	1.47	0.00	0.76	1.19
time (sec)	N/A	0.171	6.194	0.174	0.490	3.108	0.000	0.660	3.032

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	137	82	90	95	0	245	208
N.S.	1	1.00	1.01	0.61	0.67	0.70	0.00	1.81	1.54
time (sec)	N/A	0.057	0.525	0.154	0.290	4.925	0.000	5.548	5.142

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	103	65	70	75	0	201	153
N.S.	1	1.00	1.06	0.67	0.72	0.77	0.00	2.07	1.58
time (sec)	N/A	0.051	0.257	0.113	0.273	3.764	0.000	3.358	6.021

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	44	50	55	0	157	99
N.S.	1	1.00	0.98	0.67	0.76	0.83	0.00	2.38	1.50
time (sec)	N/A	0.042	0.191	0.100	0.268	3.789	0.000	1.643	2.193

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	25	28	33	0	111	44
N.S.	1	1.00	0.75	0.89	1.00	1.18	0.00	3.96	1.57
time (sec)	N/A	0.034	0.067	0.065	0.266	2.912	0.000	0.739	1.224

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	27	17	19	41	31	21
N.S.	1	1.00	1.12	1.59	1.00	1.12	2.41	1.82	1.24
time (sec)	N/A	0.019	0.020	0.033	0.264	2.723	2.136	0.457	1.211

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	67	43	47	60	0	86	49
N.S.	1	1.00	1.10	0.70	0.77	0.98	0.00	1.41	0.80
time (sec)	N/A	0.040	0.123	0.106	0.275	2.598	0.000	0.443	1.252

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	67	91	139	0	157	76
N.S.	1	1.00	1.04	0.65	0.88	1.35	0.00	1.52	0.74
time (sec)	N/A	0.058	0.605	0.125	0.266	2.536	0.000	0.476	1.343

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	135	91	130	217	0	211	132
N.S.	1	1.00	0.93	0.63	0.90	1.50	0.00	1.46	0.91
time (sec)	N/A	0.071	0.552	0.137	0.276	3.102	0.000	0.517	1.313

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	301	230	329	127	0	149	193
N.S.	1	1.00	2.87	2.19	3.13	1.21	0.00	1.42	1.84
time (sec)	N/A	0.102	0.870	0.129	0.484	3.948	0.000	4.411	2.524

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	893	170	247	107	0	123	139
N.S.	1	1.00	11.45	2.18	3.17	1.37	0.00	1.58	1.78
time (sec)	N/A	0.078	6.469	0.111	0.476	5.207	0.000	2.086	1.997

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	241	110	163	86	0	96	83
N.S.	1	1.00	4.92	2.24	3.33	1.76	0.00	1.96	1.69
time (sec)	N/A	0.056	0.951	0.091	0.483	3.633	0.000	1.005	1.287

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	60	50	78	35	0	50	25
N.S.	1	1.00	2.86	2.38	3.71	1.67	0.00	2.38	1.19
time (sec)	N/A	0.036	0.102	0.067	0.492	3.674	0.000	0.595	1.108

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	100	59	93	64	0	66	65
N.S.	1	1.00	1.64	0.97	1.52	1.05	0.00	1.08	1.07
time (sec)	N/A	0.070	0.875	0.091	0.482	3.095	0.000	0.498	1.293

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	254	85	137	134	0	98	158
N.S.	1	1.00	2.89	0.97	1.56	1.52	0.00	1.11	1.80
time (sec)	N/A	0.094	0.862	0.115	0.482	2.202	0.000	0.478	1.434

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	359	111	177	198	0	127	206
N.S.	1	1.00	3.07	0.95	1.51	1.69	0.00	1.09	1.76
time (sec)	N/A	0.122	1.139	0.113	0.470	3.463	0.000	0.518	2.036

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	125	74	80	85	0	223	193
N.S.	1	1.00	1.04	0.62	0.67	0.71	0.00	1.86	1.61
time (sec)	N/A	0.054	0.517	0.148	0.267	2.635	0.000	5.850	5.181

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	83	45	50	55	0	180	135
N.S.	1	1.00	1.28	0.69	0.77	0.85	0.00	2.77	2.08
time (sec)	N/A	0.042	0.199	0.111	0.263	4.514	0.000	3.688	3.869

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	34	40	45	0	136	77
N.S.	1	1.00	1.06	0.71	0.83	0.94	0.00	2.83	1.60
time (sec)	N/A	0.037	0.118	0.087	0.281	3.821	0.000	1.816	1.350

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	31	31	0	33	22
N.S.	1	1.00	0.91	0.85	0.94	0.94	0.00	1.00	0.67
time (sec)	N/A	0.034	0.068	0.092	0.267	3.096	0.000	0.766	1.388

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	56	37	35	43	177	57	35
N.S.	1	1.00	1.56	1.03	0.97	1.19	4.92	1.58	0.97
time (sec)	N/A	0.028	0.141	0.042	0.267	2.892	11.710	0.513	1.130

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	55	74	106	0	117	62
N.S.	1	1.00	1.02	0.68	0.91	1.31	0.00	1.44	0.77
time (sec)	N/A	0.046	0.188	0.118	0.277	2.561	0.000	0.454	1.263

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	79	110	162	0	186	89
N.S.	1	1.00	0.98	0.64	0.89	1.32	0.00	1.51	0.72
time (sec)	N/A	0.062	0.371	0.132	0.265	2.383	0.000	0.536	1.366

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	154	103	167	283	0	236	151
N.S.	1	1.00	0.93	0.62	1.01	1.72	0.00	1.43	0.92
time (sec)	N/A	0.081	0.825	0.138	0.274	2.277	0.000	0.557	1.245

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	495	200	301	117	0	136	179
N.S.	1	1.00	4.16	1.68	2.53	0.98	0.00	1.14	1.50
time (sec)	N/A	0.143	5.880	0.126	0.484	3.025	0.000	4.825	2.258

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	767	140	196	97	0	99	111
N.S.	1	1.00	10.65	1.94	2.72	1.35	0.00	1.38	1.54
time (sec)	N/A	0.113	6.330	0.107	0.475	3.193	0.000	2.320	1.389

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	177	80	123	66	0	79	61
N.S.	1	1.00	5.21	2.35	3.62	1.94	0.00	2.32	1.79
time (sec)	N/A	0.051	0.562	0.086	0.491	3.830	0.000	1.134	1.192

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	35	42	31	49	42	0	29	22
N.S.	1	1.06	1.27	0.94	1.48	1.27	0.00	0.88	0.67
time (sec)	N/A	0.085	0.022	0.096	0.471	2.993	0.000	0.634	1.096

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	149	72	113	106	0	84	78
N.S.	1	1.00	1.39	0.67	1.06	0.99	0.00	0.79	0.73
time (sec)	N/A	0.130	1.396	0.111	0.485	2.699	0.000	0.490	1.395

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	314	98	157	154	0	114	182
N.S.	1	1.00	2.26	0.71	1.13	1.11	0.00	0.82	1.31
time (sec)	N/A	0.148	1.017	0.119	0.477	3.050	0.000	0.525	1.645

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	802	124	197	250	0	144	230
N.S.	1	1.00	4.48	0.69	1.10	1.40	0.00	0.80	1.28
time (sec)	N/A	0.158	6.602	0.129	0.490	2.451	0.000	0.532	2.507

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	140	85	90	95	0	246	225
N.S.	1	1.00	1.02	0.62	0.66	0.69	0.00	1.80	1.64
time (sec)	N/A	0.058	0.311	0.171	0.274	2.607	0.000	18.012	5.287

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	62	70	75	0	202	167
N.S.	1	1.00	0.94	0.63	0.71	0.76	0.00	2.04	1.69
time (sec)	N/A	0.048	0.357	0.125	0.266	2.527	0.000	5.886	5.916

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	64	45	50	55	0	158	109
N.S.	1	1.00	0.98	0.69	0.77	0.85	0.00	2.43	1.68
time (sec)	N/A	0.040	0.184	0.105	0.268	3.667	0.000	3.721	2.033

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	33	45	53	0	112	72
N.S.	1	1.00	0.78	0.72	0.98	1.15	0.00	2.43	1.57
time (sec)	N/A	0.039	0.117	0.092	0.264	3.735	0.000	1.809	1.269

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	40	36	42	457	56	36
N.S.	1	1.00	0.94	1.14	1.03	1.20	13.06	1.60	1.03
time (sec)	N/A	0.039	0.068	0.106	0.270	3.011	11.445	0.896	1.171

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	77	49	60	76	411	87	48
N.S.	1	1.00	1.38	0.88	1.07	1.36	7.34	1.55	0.86
time (sec)	N/A	0.031	0.189	0.054	0.276	2.632	11.129	0.580	1.154

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	67	98	151	0	143	75
N.S.	1	1.00	0.96	0.66	0.97	1.50	0.00	1.42	0.74
time (sec)	N/A	0.051	0.331	0.135	0.264	2.412	0.000	0.538	1.245

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	140	91	146	240	0	212	102
N.S.	1	1.00	0.98	0.64	1.02	1.68	0.00	1.48	0.71
time (sec)	N/A	0.070	0.635	0.145	0.273	3.225	0.000	0.569	1.399

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	169	115	188	317	0	261	170
N.S.	1	1.00	0.91	0.62	1.02	1.71	0.00	1.41	0.92
time (sec)	N/A	0.089	1.184	0.151	0.268	2.570	0.000	0.647	1.328

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	362	290	429	147	0	175	265
N.S.	1	1.00	1.53	1.22	1.81	0.62	0.00	0.74	1.12
time (sec)	N/A	0.274	1.332	0.181	0.487	3.374	0.000	20.478	2.564

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	303	230	343	127	0	149	208
N.S.	1	1.00	1.79	1.36	2.03	0.75	0.00	0.88	1.23
time (sec)	N/A	0.202	0.868	0.145	0.503	3.001	0.000	12.017	2.355

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	230	170	257	97	0	123	148
N.S.	1	1.00	2.32	1.72	2.60	0.98	0.00	1.24	1.49
time (sec)	N/A	0.147	0.806	0.117	0.482	3.635	0.000	4.902	1.934

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	241	110	171	87	0	97	92
N.S.	1	1.00	3.65	1.67	2.59	1.32	0.00	1.47	1.39
time (sec)	N/A	0.066	0.997	0.101	0.497	3.935	0.000	2.462	1.309

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	117	61	98	83	0	63	37
N.S.	1	1.04	2.54	1.33	2.13	1.80	0.00	1.37	0.80
time (sec)	N/A	0.104	0.279	0.095	0.490	3.390	0.000	1.174	1.161

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	71	125	45	72	80	0	50	35
N.S.	1	1.18	2.08	0.75	1.20	1.33	0.00	0.83	0.58
time (sec)	N/A	0.129	0.401	0.095	0.498	2.947	0.000	0.722	1.161

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	252	85	133	142	0	99	91
N.S.	1	1.00	1.76	0.59	0.93	0.99	0.00	0.69	0.64
time (sec)	N/A	0.175	1.267	0.108	0.490	2.823	0.000	0.550	1.703

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	366	111	177	216	0	131	205
N.S.	1	1.00	2.07	0.63	1.00	1.22	0.00	0.74	1.16
time (sec)	N/A	0.194	1.192	0.125	0.492	2.890	0.000	0.623	2.075

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	394	137	218	282	0	160	254
N.S.	1	1.00	1.83	0.64	1.01	1.31	0.00	0.74	1.18
time (sec)	N/A	0.208	3.801	0.142	0.494	3.072	0.000	0.630	3.237

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	186	1519	122	0	0	0	-1
N.S.	1	1.00	0.60	4.90	0.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	2.212	2.362	0.233	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	214	698	121	0	0	0	-1
N.S.	1	1.00	0.76	2.48	0.43	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	12.176	0.233	0.238	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	182	1429	111	0	0	0	-1
N.S.	1	1.00	0.67	5.25	0.41	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	1.478	0.247	0.233	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	220	289	111	0	0	0	-1
N.S.	1	1.00	0.90	1.18	0.45	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	1.767	0.248	0.232	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	196	1414	121	0	0	0	-1
N.S.	1	1.00	0.64	4.64	0.40	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	2.588	0.235	0.239	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	200	658	122	0	0	0	-1
N.S.	1	1.00	0.71	2.33	0.43	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	11.619	0.217	0.234	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	254	1451	134	0	0	0	-1
N.S.	1	1.00	0.73	4.19	0.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	2.574	0.255	0.240	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	117	1542	140	0	0	0	-1
N.S.	1	1.00	0.32	4.21	0.38	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	18.221	0.274	0.233	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	257	731	139	0	0	0	-1
N.S.	1	1.00	0.77	2.18	0.41	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	53.797	0.249	0.229	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	106	1504	129	0	0	0	-1
N.S.	1	1.00	0.34	4.87	0.42	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	11.374	0.251	0.249	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	220	663	128	0	0	0	-1
N.S.	1	1.00	0.79	2.38	0.46	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	32.563	0.300	0.240	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	238	1416	138	0	0	0	-1
N.S.	1	1.00	0.77	4.57	0.45	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	7.365	0.248	0.235	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	224	660	139	0	0	0	-1
N.S.	1	1.00	0.71	2.09	0.44	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	35.177	0.238	0.240	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	2820	1453	151	0	0	0	-1
N.S.	1	1.00	7.62	3.93	0.41	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	14.322	0.342	0.239	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	332	744	0	0	0	0	-1
N.S.	1	1.00	1.01	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	48.735	0.251	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	129	1529	0	0	0	0	-1
N.S.	1	1.00	0.40	4.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	22.160	0.236	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	271	708	0	0	0	0	-1
N.S.	1	1.00	0.92	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	79.691	0.244	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	105	1443	0	0	0	0	-1
N.S.	1	1.00	0.37	5.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	15.046	0.241	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	1211	325	0	0	0	0	-1
N.S.	1	1.00	4.71	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	22.828	0.238	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	2715	359	0	0	0	0	-1
N.S.	1	1.00	8.62	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	7.745	0.229	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	1253	1289	0	0	0	0	-1
N.S.	1	1.00	4.32	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	19.144	0.246	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	180	2149	0	0	0	0	-1
N.S.	1	1.00	0.50	5.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	24.578	0.237	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	1299	1926	0	0	0	0	-1
N.S.	1	1.00	3.96	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	19.220	0.241	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	0	1542	0	0	0	0	-1
N.S.	1	1.00	0.00	4.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	12.807	0.280	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	0	731	0	0	0	0	-1
N.S.	1	1.00	0.00	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	104.515	0.250	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	0	1504	0	0	0	0	-1
N.S.	1	1.00	0.00	4.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	4.373	0.293	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	0	663	0	0	0	0	-1
N.S.	1	1.00	0.00	2.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	44.028	0.263	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	812	367	0	0	0	0	-1
N.S.	1	1.00	2.62	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	6.748	0.240	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	0	1287	0	0	0	0	-1
N.S.	1	1.00	0.00	4.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	76.832	0.241	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	2792	2153	0	0	0	0	-1
N.S.	1	1.00	7.69	5.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	8.370	0.262	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	1281	1926	0	0	0	0	-1
N.S.	1	1.00	3.51	5.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	18.914	0.262	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	102	359	145	299	0	193	-1
N.S.	1	1.00	0.69	2.44	0.99	2.03	0.00	1.31	-0.01
time (sec)	N/A	0.084	0.575	0.402	0.511	4.516	0.000	2.056	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	221	107	259	0	152	-1
N.S.	1	1.00	0.81	2.23	1.08	2.62	0.00	1.54	-0.01
time (sec)	N/A	0.060	0.167	0.170	0.509	3.196	0.000	1.082	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	42	67	184	0	75	47
N.S.	1	1.00	1.18	0.82	1.31	3.61	0.00	1.47	0.92
time (sec)	N/A	0.033	0.048	0.052	0.500	3.980	0.000	0.817	1.481

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	98	0	242	0	88	-1
N.S.	1	1.00	0.99	1.34	0.00	3.32	0.00	1.21	-0.01
time (sec)	N/A	0.052	0.057	0.142	0.000	3.761	0.000	0.786	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	87	267	0	426	0	140	-1
N.S.	1	1.00	0.66	2.04	0.00	3.25	0.00	1.07	-0.01
time (sec)	N/A	0.085	0.297	0.197	0.000	3.274	0.000	0.845	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	102	407	0	529	0	201	-1
N.S.	1	1.00	0.53	2.11	0.00	2.74	0.00	1.04	-0.01
time (sec)	N/A	0.117	0.317	0.276	0.000	3.748	0.000	0.849	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	134	566	0	371	0	284	-1
N.S.	1	1.00	0.60	2.55	0.00	1.67	0.00	1.28	-0.00
time (sec)	N/A	0.076	7.244	0.261	0.000	3.695	0.000	2.672	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	110	317	0	331	0	246	-1
N.S.	1	1.00	0.69	1.98	0.00	2.07	0.00	1.54	-0.01
time (sec)	N/A	0.064	5.804	0.185	0.000	3.790	0.000	1.577	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	226	210	0	283	0	208	-1
N.S.	1	1.00	2.35	2.19	0.00	2.95	0.00	2.17	-0.01
time (sec)	N/A	0.051	4.100	0.148	0.000	2.176	0.000	1.154	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	5502	189	0	422	0	236	-1
N.S.	1	1.00	50.48	1.73	0.00	3.87	0.00	2.17	-0.01
time (sec)	N/A	0.070	24.261	0.177	0.000	3.597	0.000	1.073	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	5552	381	0	547	0	365	-1
N.S.	1	1.00	28.33	1.94	0.00	2.79	0.00	1.86	-0.01
time (sec)	N/A	0.140	24.109	0.223	0.000	3.914	0.000	1.125	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	5594	573	0	646	0	476	-1
N.S.	1	1.00	19.98	2.05	0.00	2.31	0.00	1.70	-0.00
time (sec)	N/A	0.185	24.047	0.184	0.000	2.269	0.000	1.222	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	112	429	162	334	0	218	-1
N.S.	1	1.00	0.66	2.54	0.96	1.98	0.00	1.29	-0.01
time (sec)	N/A	0.096	0.515	0.184	0.493	3.582	0.000	2.691	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	92	291	124	290	0	173	-1
N.S.	1	1.00	0.76	2.40	1.02	2.40	0.00	1.43	-0.01
time (sec)	N/A	0.072	0.257	0.165	0.503	2.555	0.000	1.562	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	57	86	238	0	122	67
N.S.	1	1.00	0.96	0.78	1.18	3.26	0.00	1.67	0.92
time (sec)	N/A	0.042	0.145	0.041	0.496	2.441	0.000	1.036	1.460

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	101	0	243	0	89	-1
N.S.	1	1.00	0.99	1.38	0.00	3.33	0.00	1.22	-0.01
time (sec)	N/A	0.057	0.061	0.194	0.000	2.328	0.000	0.907	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	258	0	378	0	138	-1
N.S.	1	1.00	0.91	2.37	0.00	3.47	0.00	1.27	-0.01
time (sec)	N/A	0.078	0.337	0.178	0.000	2.818	0.000	0.927	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	104	502	0	589	0	211	-1
N.S.	1	1.00	0.61	2.94	0.00	3.44	0.00	1.23	-0.01
time (sec)	N/A	0.103	0.311	0.218	0.000	3.630	0.000	0.988	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	147	656	0	415	0	369	-1
N.S.	1	1.00	0.57	2.54	0.00	1.61	0.00	1.43	-0.00
time (sec)	N/A	0.088	8.406	0.254	0.000	4.128	0.000	3.516	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	123	407	0	371	0	310	-1
N.S.	1	1.00	0.63	2.10	0.00	1.91	0.00	1.60	-0.01
time (sec)	N/A	0.074	6.496	0.180	0.000	3.299	0.000	2.121	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	97	300	0	321	0	224	-1
N.S.	1	1.00	0.76	2.34	0.00	2.51	0.00	1.75	-0.01
time (sec)	N/A	0.064	5.560	0.144	0.000	2.378	0.000	1.536	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	102	115	0	264	0	197	-1
N.S.	1	1.00	1.59	1.80	0.00	4.12	0.00	3.08	-0.02
time (sec)	N/A	0.049	0.389	0.155	0.000	2.634	0.000	1.202	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-2)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	5542	372	0	531	0	369	-1
N.S.	1	1.00	38.49	2.58	0.00	3.69	0.00	2.56	-0.01
time (sec)	N/A	0.101	24.063	0.192	0.000	3.140	0.000	1.558	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-2)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	5582	720	0	708	0	519	-1
N.S.	1	1.00	24.70	3.19	0.00	3.13	0.00	2.30	-0.00
time (sec)	N/A	0.156	24.011	0.263	0.000	4.205	0.000	2.372	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	156	500	181	386	0	244	-1
N.S.	1	1.00	0.81	2.59	0.94	2.00	0.00	1.26	-0.01
time (sec)	N/A	0.108	0.744	0.187	0.474	3.932	0.000	2.682	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	102	362	143	334	0	198	-1
N.S.	1	1.00	0.70	2.50	0.99	2.30	0.00	1.37	-0.01
time (sec)	N/A	0.081	0.610	0.161	0.516	4.130	0.000	1.636	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	74	105	282	0	148	92
N.S.	1	1.00	0.85	0.76	1.08	2.91	0.00	1.53	0.95
time (sec)	N/A	0.048	0.222	0.042	0.476	2.668	0.000	1.167	1.823

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	124	0	300	0	112	-1
N.S.	1	1.00	0.87	1.31	0.00	3.16	0.00	1.18	-0.01
time (sec)	N/A	0.068	0.099	0.129	0.000	2.379	0.000	0.959	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	115	248	0	398	0	140	-1
N.S.	1	1.00	1.08	2.34	0.00	3.75	0.00	1.32	-0.01
time (sec)	N/A	0.075	0.276	0.174	0.000	2.578	0.000	0.944	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	138	376	0	503	0	177	-1
N.S.	1	1.00	0.94	2.56	0.00	3.42	0.00	1.20	-0.01
time (sec)	N/A	0.092	1.307	0.200	0.000	2.639	0.000	1.066	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	173	747	0	477	0	397	-1
N.S.	1	1.00	0.60	2.58	0.00	1.64	0.00	1.37	-0.00
time (sec)	N/A	0.092	9.937	0.272	0.000	3.291	0.000	3.126	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	149	498	0	425	0	339	-1
N.S.	1	1.00	0.67	2.22	0.00	1.90	0.00	1.51	-0.00
time (sec)	N/A	0.080	7.402	0.178	0.000	2.883	0.000	2.321	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	125	391	0	374	0	281	-1
N.S.	1	1.00	0.78	2.44	0.00	2.34	0.00	1.76	-0.01
time (sec)	N/A	0.068	5.802	0.148	0.000	2.571	0.000	1.913	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	124	192	0	270	0	192	-1
N.S.	1	1.00	1.88	2.91	0.00	4.09	0.00	2.91	-0.02
time (sec)	N/A	0.052	0.777	0.134	0.000	3.517	0.000	1.381	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	214	0	355	0	311	-1
N.S.	1	1.00	0.84	2.23	0.00	3.70	0.00	3.24	-0.01
time (sec)	N/A	0.057	0.232	0.177	0.000	4.167	0.000	2.381	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	5562	542	0	650	0	481	-1
N.S.	1	1.00	31.60	3.08	0.00	3.69	0.00	2.73	-0.01
time (sec)	N/A	0.125	24.092	0.251	0.000	3.644	0.000	3.909	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	88	293	129	285	0	167	-1
N.S.	1	1.00	0.70	2.33	1.02	2.26	0.00	1.33	-0.01
time (sec)	N/A	0.079	0.203	0.166	0.504	3.257	0.000	2.108	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	155	91	241	0	126	-1
N.S.	1	1.00	0.85	1.99	1.17	3.09	0.00	1.62	-0.01
time (sec)	N/A	0.056	0.094	0.165	0.487	2.872	0.000	1.192	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	44	26	49	137	0	48	27
N.S.	1	1.00	1.42	0.84	1.58	4.42	0.00	1.55	0.87
time (sec)	N/A	0.029	0.046	0.058	0.481	3.083	0.000	0.884	1.415

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	57	259	0	384	0	111	-1
N.S.	1	1.00	0.62	2.82	0.00	4.17	0.00	1.21	-0.01
time (sec)	N/A	0.066	0.069	0.165	0.000	2.550	0.000	0.867	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	90	504	0	546	0	174	-1
N.S.	1	1.00	0.59	3.32	0.00	3.59	0.00	1.14	-0.01
time (sec)	N/A	0.099	0.213	0.227	0.000	3.409	0.000	0.963	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	102	746	0	705	0	241	-1
N.S.	1	1.00	0.48	3.49	0.00	3.29	0.00	1.13	-0.00
time (sec)	N/A	0.130	0.288	0.318	0.000	2.799	0.000	1.002	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	467	480	0	355	0	266	-1
N.S.	1	1.00	2.47	2.54	0.00	1.88	0.00	1.41	-0.01
time (sec)	N/A	0.072	18.991	0.187	0.000	2.320	0.000	2.810	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	238	231	0	311	0	228	-1
N.S.	1	1.00	1.90	1.85	0.00	2.49	0.00	1.82	-0.01
time (sec)	N/A	0.058	2.877	0.172	0.000	3.909	0.000	1.650	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	119	116	0	235	0	187	-1
N.S.	1	1.00	1.89	1.84	0.00	3.73	0.00	2.97	-0.02
time (sec)	N/A	0.043	0.772	0.129	0.000	3.423	0.000	1.241	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	5534	374	0	503	0	269	-1
N.S.	1	1.00	33.54	2.27	0.00	3.05	0.00	1.63	-0.01
time (sec)	N/A	0.098	24.303	0.191	0.000	3.628	0.000	1.154	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	5574	722	0	666	0	385	-1
N.S.	1	1.00	22.21	2.88	0.00	2.65	0.00	1.53	-0.00
time (sec)	N/A	0.155	24.085	0.245	0.000	2.416	0.000	1.232	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	5618	1068	0	823	0	500	-1
N.S.	1	1.00	16.77	3.19	0.00	2.46	0.00	1.49	-0.00
time (sec)	N/A	0.209	23.996	0.204	0.000	3.179	0.000	1.317	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	79	224	110	261	0	154	-1
N.S.	1	1.00	0.79	2.24	1.10	2.61	0.00	1.54	-0.01
time (sec)	N/A	0.072	0.172	0.161	0.489	4.081	0.000	2.431	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	81	71	191	0	85	-1
N.S.	1	1.00	1.04	1.50	1.31	3.54	0.00	1.57	-0.02
time (sec)	N/A	0.055	0.078	0.144	0.490	3.395	0.000	1.411	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	45	70	244	0	87	50
N.S.	1	1.00	0.70	0.83	1.30	4.52	0.00	1.61	0.93
time (sec)	N/A	0.037	0.030	0.036	0.486	2.926	0.000	1.067	1.586

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	60	376	0	485	0	164	-1
N.S.	1	1.00	0.50	3.13	0.00	4.04	0.00	1.37	-0.01
time (sec)	N/A	0.078	0.074	0.169	0.000	3.027	0.000	1.021	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	90	514	0	592	0	253	-1
N.S.	1	1.00	0.51	2.92	0.00	3.36	0.00	1.44	-0.01
time (sec)	N/A	0.111	0.192	0.227	0.000	3.049	0.000	1.204	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	99	866	0	837	0	322	-1
N.S.	1	1.00	0.42	3.64	0.00	3.52	0.00	1.35	-0.00
time (sec)	N/A	0.150	0.307	0.195	0.000	3.190	0.000	1.106	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	248	391	0	343	0	296	-1
N.S.	1	1.00	1.58	2.49	0.00	2.18	0.00	1.89	-0.01
time (sec)	N/A	0.071	2.678	0.172	0.000	2.204	0.000	3.614	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	162	142	0	295	0	230	-1
N.S.	1	1.00	1.71	1.49	0.00	3.11	0.00	2.42	-0.01
time (sec)	N/A	0.058	4.375	0.161	0.000	2.838	0.000	2.357	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	4739	142	0	295	0	72	-1
N.S.	1	1.00	55.75	1.67	0.00	3.47	0.00	0.85	-0.01
time (sec)	N/A	0.060	23.939	0.120	0.000	2.715	0.000	1.262	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	5578	542	0	603	0	143	-1
N.S.	1	1.00	25.94	2.52	0.00	2.80	0.00	0.67	-0.00
time (sec)	N/A	0.132	24.032	0.197	0.000	3.281	0.000	1.145	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	5620	732	0	712	0	261	-1
N.S.	1	1.00	18.55	2.42	0.00	2.35	0.00	0.86	-0.00
time (sec)	N/A	0.191	24.013	0.269	0.000	3.602	0.000	1.244	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	5662	1240	0	955	0	379	-1
N.S.	1	1.00	14.63	3.20	0.00	2.47	0.00	0.98	-0.00
time (sec)	N/A	0.243	24.163	0.217	0.000	3.698	0.000	1.415	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	155	163	241	0	126	-1
N.S.	1	1.00	0.88	1.99	2.09	3.09	0.00	1.62	-0.01
time (sec)	N/A	0.064	0.146	0.158	0.490	3.841	0.000	2.762	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	154	125	245	0	90	-1
N.S.	1	1.00	0.93	2.85	2.31	4.54	0.00	1.67	-0.02
time (sec)	N/A	0.056	0.058	0.133	0.506	3.132	0.000	1.769	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	40	62	87	321	0	116	69
N.S.	1	1.00	0.51	0.79	1.12	4.12	0.00	1.49	0.88
time (sec)	N/A	0.046	0.059	0.036	0.500	3.564	0.000	1.298	1.907

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	60	496	0	573	0	206	-1
N.S.	1	1.00	0.42	3.44	0.00	3.98	0.00	1.43	-0.01
time (sec)	N/A	0.092	0.086	0.167	0.000	2.989	0.000	1.033	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	744	0	748	0	295	-1
N.S.	1	1.00	0.45	3.72	0.00	3.74	0.00	1.48	-0.00
time (sec)	N/A	0.126	0.222	0.238	0.000	3.319	0.000	1.177	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	99	986	0	905	0	364	-1
N.S.	1	1.00	0.38	3.76	0.00	3.45	0.00	1.39	-0.00
time (sec)	N/A	0.165	0.330	0.226	0.000	3.210	0.000	1.346	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	447	302	0	323	0	257	-1
N.S.	1	1.00	3.52	2.38	0.00	2.54	0.00	2.02	-0.01
time (sec)	N/A	0.063	6.073	0.172	0.000	2.673	0.000	4.177	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	5491	327	0	414	0	66	-1
N.S.	1	1.00	48.59	2.89	0.00	3.66	0.00	0.58	-0.01
time (sec)	N/A	0.071	23.717	0.161	0.000	3.287	0.000	2.074	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	5521	370	0	492	0	47	-1
N.S.	1	1.00	43.47	2.91	0.00	3.87	0.00	0.37	-0.01
time (sec)	N/A	0.077	23.973	0.116	0.000	2.504	0.000	1.557	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	5604	714	0	691	0	177	-1
N.S.	1	1.00	21.15	2.69	0.00	2.61	0.00	0.67	-0.00
time (sec)	N/A	0.159	24.066	0.208	0.000	4.003	0.000	1.179	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	5646	1066	0	868	0	296	-1
N.S.	1	1.00	15.90	3.00	0.00	2.45	0.00	0.83	-0.00
time (sec)	N/A	0.226	24.242	0.299	0.000	3.704	0.000	1.484	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	5688	1412	0	1023	0	412	-1
N.S.	1	1.00	12.96	3.22	0.00	2.33	0.00	0.94	-0.00
time (sec)	N/A	0.282	24.131	0.270	0.000	4.292	0.000	1.636	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	227	5584	724	0	733	0	109	-1
N.S.	1	1.28	31.55	4.09	0.00	4.14	0.00	0.62	-0.01
time (sec)	N/A	0.139	24.183	0.954	0.000	3.463	0.000	1.375	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	1.273	0.174	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	391	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	6.257	0.166	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	358	0	0	0	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	3.373	0.150	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	105	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.760	0.115	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.566	0.127	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	329	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	6.247	0.143	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	10.781	0.151	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	20.038	0.160	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	53.236	0.164	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	62.303	0.171	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	48.994	0.156	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	87	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.481	0.126	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	72	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.179	0.108	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	49	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.058	0.112	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.049	0.079	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.054	0.104	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.265	0.116	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	1.404	0.098	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	910	0	0	0	0	0	-1
N.S.	1	1.00	8.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	9.745	0.079	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	893	0	0	0	0	0	-1
N.S.	1	1.00	8.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	3.907	0.102	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	1.756	0.112	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	2072	0	0	0	0	0	-1
N.S.	1	1.00	18.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	19.769	0.182	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	238	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	3.086	0.165	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	229	0	0	0	0	0	-1
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	2.422	0.157	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	2164	0	0	0	0	0	-1
N.S.	1	1.00	19.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	17.858	0.149	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	185	658	126	0	0	0	-1
N.S.	1	1.00	0.58	2.06	0.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	14.069	1.214	0.250	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	191	1414	125	0	0	0	-1
N.S.	1	1.00	0.55	4.09	0.36	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	1.214	0.270	0.239	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	169	289	115	0	0	0	-1
N.S.	1	1.00	0.62	1.05	0.42	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	11.916	0.290	0.236	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	189	1433	115	0	0	0	-1
N.S.	1	1.00	0.63	4.79	0.38	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	1.705	0.618	0.243	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	224	698	125	0	0	0	-1
N.S.	1	1.00	0.70	2.18	0.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	12.655	0.263	0.236	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	93	660	143	0	0	0	-1
N.S.	1	1.00	0.26	1.85	0.40	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	32.478	0.289	0.254	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	220	1416	142	0	0	0	-1
N.S.	1	1.00	0.64	4.13	0.41	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	15.264	0.292	0.246	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	118	663	132	0	0	0	-1
N.S.	1	1.00	0.38	2.13	0.42	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	32.012	0.286	0.249	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	221	1504	133	0	0	0	-1
N.S.	1	1.00	0.65	4.44	0.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	23.937	0.287	0.238	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	127	731	143	0	0	0	-1
N.S.	1	1.00	0.34	1.95	0.38	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	38.605	0.280	0.238	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	316	2149	0	0	0	0	-1
N.S.	1	1.00	0.78	5.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	14.388	0.271	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	135	1289	0	0	0	0	-1
N.S.	1	1.00	0.42	3.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.236	31.857	0.411	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	249	359	0	0	0	0	-1
N.S.	1	1.00	0.72	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	13.872	0.253	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	112	325	0	0	0	0	-1
N.S.	1	1.00	0.39	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	57.989	0.268	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	194	1443	0	0	0	0	-1
N.S.	1	1.00	0.60	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	70.858	0.267	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	130	708	0	0	0	0	-1
N.S.	1	1.00	0.39	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	43.963	0.260	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	261	1529	0	0	0	0	-1
N.S.	1	1.00	0.70	4.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	28.175	0.273	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	0	2153	0	0	0	0	-1
N.S.	1	1.00	0.00	5.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	14.122	0.289	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	0	1287	0	0	0	0	-1
N.S.	1	1.00	0.00	3.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	70.661	0.342	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	0	367	0	0	0	0	-1
N.S.	1	1.00	0.00	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	5.366	0.264	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	665	0	0	0	0	-1
N.S.	1	1.00	0.00	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	65.271	0.289	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	0	1504	0	0	0	0	-1
N.S.	1	1.00	0.00	4.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	57.273	0.279	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	0	731	0	0	0	0	-1
N.S.	1	1.00	0.00	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	63.452	0.282	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	159	94	101	148	317	221
N.S.	1	1.00	0.95	1.43	0.85	0.91	1.33	2.86	1.99
time (sec)	N/A	0.079	0.498	0.129	0.307	3.610	1.233	3.703	5.240

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	123	72	79	112	248	162
N.S.	1	1.00	0.98	1.46	0.86	0.94	1.33	2.95	1.93
time (sec)	N/A	0.064	0.217	0.115	0.289	3.332	0.549	1.807	5.846

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	83	50	57	76	179	102
N.S.	1	1.00	1.00	1.51	0.91	1.04	1.38	3.25	1.85
time (sec)	N/A	0.038	0.141	0.092	0.282	3.410	0.224	0.809	2.228

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	26	34	37	107	40
N.S.	1	1.00	1.00	0.92	1.04	1.36	1.48	4.28	1.60
time (sec)	N/A	0.019	0.021	0.034	0.283	3.659	0.098	0.475	1.302

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	33	34	38	0	61	51
N.S.	1	1.00	1.40	0.77	0.79	0.88	0.00	1.42	1.19
time (sec)	N/A	0.039	0.044	0.082	0.297	3.298	0.000	0.438	1.306

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	114	75	62	99	0	170	86
N.S.	1	1.00	1.58	1.04	0.86	1.38	0.00	2.36	1.19
time (sec)	N/A	0.064	1.400	0.118	0.278	2.802	0.000	0.500	1.356

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	166	111	99	168	0	266	128
N.S.	1	1.00	1.63	1.09	0.97	1.65	0.00	2.61	1.25
time (sec)	N/A	0.091	0.362	0.135	0.277	3.663	0.000	0.527	1.344

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	216	151	133	237	0	358	170
N.S.	1	1.00	1.66	1.16	1.02	1.82	0.00	2.75	1.31
time (sec)	N/A	0.116	0.639	0.131	0.286	3.847	0.000	0.546	1.503

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	103	143	134	134	0	228	331
N.S.	1	1.00	1.01	1.40	1.31	1.31	0.00	2.24	3.25
time (sec)	N/A	0.069	1.139	0.095	0.491	3.104	0.000	2.353	2.510

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	79	104	102	112	0	172	267
N.S.	1	1.00	1.08	1.42	1.40	1.53	0.00	2.36	3.66
time (sec)	N/A	0.045	0.645	0.098	0.493	3.341	0.000	1.073	2.234

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	67	65	87	0	115	96
N.S.	1	1.00	1.33	1.49	1.44	1.93	0.00	2.56	2.13
time (sec)	N/A	0.025	0.031	0.067	0.485	3.972	0.000	0.617	1.391

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	31	33	0	52	48
N.S.	1	1.00	1.65	1.35	1.19	1.27	0.00	2.00	1.85
time (sec)	N/A	0.017	0.028	0.070	0.490	2.923	0.000	0.452	1.305

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	59	87	0	112	90
N.S.	1	1.00	1.13	1.56	1.07	1.58	0.00	2.04	1.64
time (sec)	N/A	0.037	0.036	0.094	0.485	2.174	0.000	0.478	1.561

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	79	130	0	170	132
N.S.	1	1.00	0.94	1.54	0.94	1.55	0.00	2.02	1.57
time (sec)	N/A	0.058	0.051	0.092	0.480	3.858	0.000	0.488	1.386

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	100	179	0	225	174
N.S.	1	1.00	0.83	1.46	0.90	1.61	0.00	2.03	1.57
time (sec)	N/A	0.082	0.053	0.128	0.484	4.034	0.000	0.523	1.617

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	217	173	224	174	181	314	489	344
N.S.	1	1.17	0.94	1.21	0.94	0.98	1.70	2.64	1.86
time (sec)	N/A	0.095	0.459	0.194	0.273	3.474	3.280	6.353	5.071

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	169	138	184	139	146	252	415	280
N.S.	1	1.13	0.93	1.23	0.93	0.98	1.69	2.79	1.88
time (sec)	N/A	0.076	0.362	0.147	0.272	4.031	1.642	3.951	5.173

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	131	105	148	108	115	189	341	215
N.S.	1	1.14	0.91	1.29	0.94	1.00	1.64	2.97	1.87
time (sec)	N/A	0.064	0.281	0.135	0.270	5.050	0.812	1.924	4.968

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	108	75	82	126	267	151
N.S.	1	1.00	0.85	1.24	0.86	0.94	1.45	3.07	1.74
time (sec)	N/A	0.048	0.463	0.095	0.275	3.561	0.333	0.917	3.733

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	40	42	51	60	191	81
N.S.	1	1.00	0.89	0.85	0.89	1.09	1.28	4.06	1.72
time (sec)	N/A	0.023	0.063	0.054	0.277	3.183	0.135	0.558	1.533

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	48	62	68	0	101	96
N.S.	1	1.00	0.87	0.79	1.02	1.11	0.00	1.66	1.57
time (sec)	N/A	0.068	0.115	0.109	0.271	2.424	0.000	0.457	1.429

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	92	72	113	0	209	98
N.S.	1	1.00	0.89	1.00	0.78	1.23	0.00	2.27	1.07
time (sec)	N/A	0.097	0.491	0.118	0.273	4.308	0.000	0.523	1.364

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	148	136	122	203	0	360	164
N.S.	1	1.00	1.17	1.08	0.97	1.61	0.00	2.86	1.30
time (sec)	N/A	0.110	3.039	0.121	0.286	3.228	0.000	0.560	1.445

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	293	168	150	184	0	282	403
N.S.	1	1.00	1.87	1.07	0.96	1.17	0.00	1.80	2.57
time (sec)	N/A	0.144	1.423	0.115	0.522	3.062	0.000	2.516	2.645

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	355	129	118	151	0	220	332
N.S.	1	1.00	3.06	1.11	1.02	1.30	0.00	1.90	2.86
time (sec)	N/A	0.103	0.965	0.094	0.496	2.994	0.000	1.225	2.563

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	201	92	82	115	0	158	227
N.S.	1	1.00	2.87	1.31	1.17	1.64	0.00	2.26	3.24
time (sec)	N/A	0.078	1.237	0.096	0.494	4.073	0.000	0.729	1.669

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	49	47	44	0	80	58
N.S.	1	1.00	0.81	1.02	0.98	0.92	0.00	1.67	1.21
time (sec)	N/A	0.053	0.407	0.079	0.498	4.636	0.000	0.444	1.431

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	122	111	76	102	0	176	118
N.S.	1	1.00	1.44	1.31	0.89	1.20	0.00	2.07	1.39
time (sec)	N/A	0.079	0.505	0.086	0.481	3.858	0.000	0.514	1.460

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	198	154	96	152	0	273	191
N.S.	1	1.00	1.62	1.26	0.79	1.25	0.00	2.24	1.57
time (sec)	N/A	0.092	0.617	0.102	0.481	2.738	0.000	0.567	1.497

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	257	187	116	206	0	366	258
N.S.	1	1.00	1.68	1.22	0.76	1.35	0.00	2.39	1.69
time (sec)	N/A	0.107	0.869	0.137	0.492	3.184	0.000	0.635	1.497

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	520	273	268	293	0	1768	631
N.S.	1	1.00	2.08	1.09	1.07	1.17	0.00	7.07	2.52
time (sec)	N/A	0.138	6.241	0.197	0.311	4.105	0.000	5.269	2.969

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	371	184	183	205	0	1052	395
N.S.	1	1.00	2.18	1.08	1.08	1.21	0.00	6.19	2.32
time (sec)	N/A	0.100	6.191	0.134	0.272	2.769	0.000	3.319	2.403

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	115	110	129	0	560	227
N.S.	1	1.00	1.00	1.06	1.02	1.19	0.00	5.19	2.10
time (sec)	N/A	0.068	0.386	0.131	0.279	2.885	0.000	1.950	1.881

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	57	57	69	0	289	115
N.S.	1	1.00	0.88	0.97	0.97	1.17	0.00	4.90	1.95
time (sec)	N/A	0.051	0.136	0.087	0.267	3.105	0.000	0.826	1.538

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	19	33	19	19	82	114	71
N.S.	1	1.00	0.54	0.94	0.54	0.54	2.34	3.26	2.03
time (sec)	N/A	0.022	0.043	0.036	0.266	3.339	3.390	0.671	1.468

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	70	75	68	75	0	257	93
N.S.	1	1.00	0.74	0.80	0.72	0.80	0.00	2.73	0.99
time (sec)	N/A	0.073	0.114	0.117	0.283	3.529	0.000	0.746	1.749

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	141	126	144	263	0	403	174
N.S.	1	1.00	0.90	0.80	0.92	1.68	0.00	2.57	1.11
time (sec)	N/A	0.132	1.070	0.177	0.280	3.190	0.000	0.631	1.845

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	625	193	289	576	0	649	290
N.S.	1	1.00	2.67	0.82	1.24	2.46	0.00	2.77	1.24
time (sec)	N/A	0.222	6.253	0.213	0.287	4.022	0.000	0.788	2.266

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	271	907	411	0	603	0	746	2500
N.S.	1	1.37	4.58	2.08	0.00	3.05	0.00	3.77	12.63
time (sec)	N/A	0.267	6.184	0.240	0.000	3.369	0.000	2.325	4.129

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	287	234	0	444	0	476	2500
N.S.	1	1.00	2.28	1.86	0.00	3.52	0.00	3.78	19.84
time (sec)	N/A	0.239	2.218	0.201	0.000	2.410	0.000	1.060	3.292

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	115	108	0	253	0	140	121
N.S.	1	1.00	1.51	1.42	0.00	3.33	0.00	1.84	1.59
time (sec)	N/A	0.132	0.159	0.133	0.000	4.497	0.000	0.624	1.625

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	135	147	115	0	362	0	582	1002
N.S.	1	1.27	1.39	1.08	0.00	3.42	0.00	5.49	9.45
time (sec)	N/A	0.167	0.455	0.147	0.000	3.111	0.000	0.515	3.940

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	256	416	184	0	742	0	1073	2500
N.S.	1	1.45	2.35	1.04	0.00	4.19	0.00	6.06	14.12
time (sec)	N/A	0.275	6.196	0.200	0.000	6.956	0.000	0.608	11.109

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	528	287	321	423	0	1696	760
N.S.	1	1.00	2.07	1.13	1.26	1.66	0.00	6.65	2.98
time (sec)	N/A	0.153	6.295	0.197	0.291	2.980	0.000	6.040	4.769

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	383	200	227	312	0	1023	505
N.S.	1	1.00	2.14	1.12	1.27	1.74	0.00	5.72	2.82
time (sec)	N/A	0.106	6.218	0.147	0.285	4.444	0.000	3.638	3.090

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	187	127	149	219	0	568	286
N.S.	1	1.00	1.55	1.05	1.23	1.81	0.00	4.69	2.36
time (sec)	N/A	0.077	0.612	0.106	0.287	2.467	0.000	1.778	2.127

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	72	74	102	0	313	124
N.S.	1	1.00	0.84	0.97	1.00	1.38	0.00	4.23	1.68
time (sec)	N/A	0.059	0.275	0.093	0.285	1.031	0.000	0.813	1.651

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	41	46	0	238	257
N.S.	1	1.00	1.00	0.91	0.76	0.85	0.00	4.41	4.76
time (sec)	N/A	0.030	0.044	0.050	0.282	1.472	0.000	0.515	1.484

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	189	114	142	234	0	303	160
N.S.	1	1.00	1.37	0.83	1.03	1.70	0.00	2.20	1.16
time (sec)	N/A	0.104	0.374	0.161	0.300	3.336	0.000	0.509	1.978

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	351	164	303	693	0	656	313
N.S.	1	1.00	1.78	0.83	1.54	3.52	0.00	3.33	1.59
time (sec)	N/A	0.174	2.299	0.198	0.289	3.619	0.000	0.540	2.484

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	473	230	558	1378	0	795	471
N.S.	1	1.00	1.70	0.83	2.01	4.96	0.00	2.86	1.69
time (sec)	N/A	0.284	3.146	0.270	0.310	4.105	0.000	0.590	2.967

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	283	865	365	0	843	0	411	2500
N.S.	1	1.42	4.32	1.82	0.00	4.22	0.00	2.06	12.50
time (sec)	N/A	0.313	6.263	0.253	0.000	2.583	0.000	2.310	4.816

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	327	219	0	584	0	294	2500
N.S.	1	1.00	2.18	1.46	0.00	3.89	0.00	1.96	16.67
time (sec)	N/A	0.220	1.700	0.178	0.000	2.986	0.000	1.132	3.468

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	115	0	383	0	144	551
N.S.	1	1.00	0.94	1.35	0.00	4.51	0.00	1.69	6.48
time (sec)	N/A	0.107	0.286	0.131	0.000	1.903	0.000	0.648	2.145

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	209	184	0	705	0	332	2500
N.S.	1	1.00	0.92	0.81	0.00	3.11	0.00	1.46	11.01
time (sec)	N/A	0.306	1.824	0.216	0.000	1.591	0.000	0.531	6.458

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	303	260	0	1481	0	487	2500
N.S.	1	1.00	0.84	0.72	0.00	4.11	0.00	1.35	6.94
time (sec)	N/A	0.422	2.566	0.256	0.000	1.355	0.000	0.558	6.972

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	1846	3803	0	0	0	0	-1
N.S.	1	1.00	2.43	5.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	56.624	0.513	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	740	740	202	1830	0	0	0	0	-1
N.S.	1	1.00	0.27	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	17.198	0.238	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	224	874	0	0	0	0	-1
N.S.	1	1.00	0.54	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	15.802	0.235	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	246	2352	0	0	0	0	-1
N.S.	1	1.00	0.58	5.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	33.216	0.231	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	863	863	1571	6530	0	0	0	0	-1
N.S.	1	1.00	1.82	7.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	47.569	0.230	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	836	836	2169	16410	0	0	0	0	-1
N.S.	1	1.00	2.59	19.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	54.290	0.287	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	254	3268	191	425	0	966	-1
N.S.	1	1.00	1.50	19.34	1.13	2.51	0.00	5.72	-0.01
time (sec)	N/A	0.117	6.298	0.842	0.491	3.933	0.000	1.971	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	194	2342	108	311	0	539	-1
N.S.	1	1.00	1.94	23.42	1.08	3.11	0.00	5.39	-0.01
time (sec)	N/A	0.077	6.202	0.324	0.508	4.277	0.000	0.890	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	137	42	67	192	0	185	47
N.S.	1	1.00	2.69	0.82	1.31	3.76	0.00	3.63	0.92
time (sec)	N/A	0.033	0.253	0.074	0.488	3.566	0.000	0.576	1.861

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	224	575	0	2132	0	0	-1
N.S.	1	1.00	2.11	5.42	0.00	20.11	0.00	0.00	-0.01
time (sec)	N/A	0.108	3.565	0.232	0.000	5.166	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	331	2844	0	3523	0	514	-1
N.S.	1	1.00	1.54	13.23	0.00	16.39	0.00	2.39	-0.00
time (sec)	N/A	0.209	18.239	0.204	0.000	8.145	0.000	0.995	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	692	1109	0	0	0	0	-1
N.S.	1	1.00	2.01	3.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	17.539	0.257	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	215	0	0	0	0	-1
N.S.	1	1.00	1.21	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.264	0.182	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	154	628	0	0	0	0	-1
N.S.	1	1.00	0.63	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	3.689	0.353	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	248	4997	175	381	0	699	-1
N.S.	1	1.00	1.68	33.76	1.18	2.57	0.00	4.72	-0.01
time (sec)	N/A	0.095	6.336	0.412	0.485	4.077	0.000	2.115	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	194	3003	92	273	0	265	-1
N.S.	1	1.00	2.46	38.01	1.16	3.46	0.00	3.35	-0.01
time (sec)	N/A	0.067	1.166	0.267	0.492	3.552	0.000	0.961	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	108	26	49	145	0	102	27
N.S.	1	1.00	3.48	0.84	1.58	4.68	0.00	3.29	0.87
time (sec)	N/A	0.030	0.216	0.055	0.500	2.257	0.000	0.649	1.634

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	251	691	0	2420	0	0	-1
N.S.	1	1.00	2.37	6.52	0.00	22.83	0.00	0.00	-0.01
time (sec)	N/A	0.094	6.068	0.219	0.000	5.936	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	329	4203	0	4336	0	546	-1
N.S.	1	1.00	1.27	16.17	0.00	16.68	0.00	2.10	-0.00
time (sec)	N/A	0.193	2.191	0.213	0.000	52.931	0.000	1.095	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	610	835	1779	0	0	0	0	-1
N.S.	1	1.51	2.07	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.530	17.105	0.366	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	2752	823	0	0	0	0	-1
N.S.	1	1.00	8.88	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	18.301	0.231	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	178	0	0	0	0	-1
N.S.	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.225	0.260	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	1198	1408	0	0	0	0	-1
N.S.	1	1.00	3.32	3.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	18.359	0.260	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	263	6612	194	467	0	0	-1
N.S.	1	1.00	1.78	44.68	1.31	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.117	6.428	0.788	0.519	6.549	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	167	2830	110	317	0	258	-1
N.S.	1	1.00	1.90	32.16	1.25	3.60	0.00	2.93	-0.01
time (sec)	N/A	0.083	0.991	0.254	0.496	4.576	0.000	1.222	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	128	45	70	260	0	0	50
N.S.	1	1.00	2.37	0.83	1.30	4.81	0.00	0.00	0.93
time (sec)	N/A	0.037	0.399	0.033	0.473	3.820	0.000	0.000	1.955

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	322	2766	0	3924	0	0	-1
N.S.	1	1.00	2.27	19.48	0.00	27.63	0.00	0.00	-0.01
time (sec)	N/A	0.142	1.580	0.179	0.000	46.017	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	316	381	10977	0	8098	0	0	-1
N.S.	1	1.34	1.61	46.51	0.00	34.31	0.00	0.00	-0.00
time (sec)	N/A	0.291	3.857	0.257	0.000	116.621	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	530	907	859	1545	0	0	0	0	-1
N.S.	1	1.71	1.62	2.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.886	16.821	0.315	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	5162	633	0	0	0	0	-1
N.S.	1	1.00	15.01	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	23.715	0.214	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0	-1
N.S.	1	1.00	3.60	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	6.154	0.220	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	664	663	2239	0	0	0	0	-1
N.S.	1	1.48	1.48	4.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.652	13.682	0.213	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	3.658	0.187	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	178	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	1.308	0.152	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.723	0.111	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	0	786	0	0	0	0	0	-1
N.S.	1	0.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.034	4.918	0.128	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	14.789	0.171	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.700	0.159	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	5.032	0.162	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	6.025	0.158	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	3.515	0.158	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	298	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	3.257	0.115	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	118	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.425	0.109	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.448	0.095	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	163	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	1.572	0.106	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	256	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	7.090	0.112	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	6.975	0.102	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	3.677	0.080	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	3.614	0.104	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	5.820	0.106	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	5.353	0.175	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	6.778	0.184	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	6.354	0.165	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	7.553	0.152	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [111] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	3	2	1.00	19	0.105
3	A	3	2	1.00	19	0.105
4	A	3	2	1.00	19	0.105
5	A	3	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	2	1.00	19	0.105
8	A	3	2	1.00	19	0.105
9	A	3	2	1.00	19	0.105
10	A	6	2	1.00	19	0.105
11	A	5	2	1.00	19	0.105
12	A	4	2	1.00	19	0.105
13	A	3	2	1.00	19	0.105
14	A	2	2	1.00	19	0.105
15	A	3	2	1.00	19	0.105
16	A	4	2	1.00	19	0.105
17	A	5	2	1.00	19	0.105
18	A	6	2	1.00	19	0.105
19	A	3	2	1.00	21	0.095
20	A	3	2	1.00	21	0.095
21	A	3	2	1.00	21	0.095
22	A	3	2	1.00	21	0.095
23	A	3	2	1.00	19	0.105
24	A	3	2	1.00	19	0.105
25	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	21	0.095
27	A	3	2	1.00	21	0.095
28	A	3	2	1.00	21	0.095
29	A	12	7	1.00	21	0.333
30	A	10	7	1.00	21	0.333
31	A	8	7	1.00	21	0.333
32	A	8	5	1.00	21	0.238
33	A	9	6	1.00	21	0.286
34	A	11	7	1.00	21	0.333
35	A	12	7	1.00	21	0.333
36	A	13	7	1.00	21	0.333
37	A	3	2	1.00	21	0.095
38	A	3	2	1.00	21	0.095
39	A	3	2	1.00	21	0.095
40	A	3	2	1.00	21	0.095
41	A	3	2	1.00	19	0.105
42	A	3	2	1.00	19	0.105
43	A	3	2	1.00	21	0.095
44	A	3	2	1.00	21	0.095
45	A	3	2	1.00	21	0.095
46	A	3	2	1.00	21	0.095
47	A	17	8	1.00	21	0.381
48	A	14	8	1.00	21	0.381
49	A	11	8	1.00	21	0.381
50	A	11	8	1.00	21	0.381
51	A	11	6	1.00	21	0.286
52	A	14	8	1.00	21	0.381
53	A	15	8	1.00	21	0.381
54	A	16	8	1.00	21	0.381
55	A	17	8	1.00	21	0.381
56	A	3	2	1.00	21	0.095
57	A	3	2	1.00	21	0.095
58	A	3	2	1.00	21	0.095
59	A	3	2	1.00	21	0.095
60	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	19	0.105
62	A	3	2	1.00	21	0.095
63	A	3	2	1.00	21	0.095
64	A	6	3	1.00	21	0.143
65	A	5	3	1.00	21	0.143
66	A	4	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	5	3	1.00	21	0.143
70	A	6	3	1.00	21	0.143
71	A	3	2	1.00	21	0.095
72	A	3	2	1.00	21	0.095
73	A	3	2	1.00	21	0.095
74	A	3	2	1.00	21	0.095
75	A	3	2	1.00	19	0.105
76	A	3	2	1.00	19	0.105
77	A	3	2	1.00	21	0.095
78	A	3	2	1.00	21	0.095
79	A	11	8	1.00	21	0.381
80	A	9	8	1.00	21	0.381
81	A	5	5	1.00	21	0.238
82	A	9	6	1.06	21	0.286
83	A	12	8	1.00	21	0.381
84	A	13	8	1.00	21	0.381
85	A	14	8	1.00	21	0.381
86	A	3	2	1.00	21	0.095
87	A	3	2	1.00	21	0.095
88	A	3	2	1.00	21	0.095
89	A	3	2	1.00	21	0.095
90	A	3	2	1.00	21	0.095
91	A	3	2	1.00	19	0.105
92	A	3	2	1.00	19	0.105
93	A	3	2	1.00	21	0.095
94	A	3	2	1.00	21	0.095
95	A	18	9	1.00	21	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	15	9	1.00	21	0.429
97	A	12	9	1.00	21	0.429
98	A	6	6	1.00	21	0.286
99	A	12	9	1.04	21	0.429
100	A	12	7	1.18	21	0.333
101	A	16	9	1.00	21	0.429
102	A	17	9	1.00	21	0.429
103	A	18	9	1.00	21	0.429
104	A	17	14	1.00	23	0.609
105	A	16	13	1.00	23	0.565
106	A	16	13	1.00	23	0.565
107	A	15	12	1.00	23	0.522
108	A	17	14	1.00	23	0.609
109	A	16	13	1.00	23	0.565
110	A	18	14	1.00	23	0.609
111	A	21	17	1.00	25	0.680
112	A	20	16	1.00	25	0.640
113	A	19	15	1.00	25	0.600
114	A	18	14	1.00	25	0.560
115	A	20	16	1.00	25	0.640
116	A	20	16	1.00	25	0.640
117	A	22	17	1.00	25	0.680
118	A	18	14	1.00	25	0.560
119	A	18	15	1.00	25	0.600
120	A	17	14	1.00	25	0.560
121	A	17	14	1.00	25	0.560
122	A	16	13	1.00	25	0.520
123	A	18	15	1.00	25	0.600
124	A	17	14	1.00	25	0.560
125	A	19	15	1.00	25	0.600
126	A	18	14	1.00	25	0.560
127	A	22	18	1.00	25	0.720
128	A	21	17	1.00	25	0.680
129	A	20	16	1.00	25	0.640
130	A	19	15	1.00	25	0.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	21	17	1.00	25	0.680
132	A	21	17	1.00	25	0.680
133	A	23	18	1.00	25	0.720
134	A	23	17	1.00	25	0.680
135	A	8	5	1.00	23	0.217
136	A	6	5	1.00	23	0.217
137	A	4	4	1.00	21	0.190
138	A	6	4	1.00	21	0.190
139	A	8	6	1.00	23	0.261
140	A	10	7	1.00	23	0.304
141	A	4	3	1.00	23	0.130
142	A	4	3	1.00	23	0.130
143	A	4	4	1.00	23	0.174
144	A	5	4	1.00	23	0.174
145	A	7	5	1.00	23	0.217
146	A	9	6	1.00	23	0.261
147	A	9	5	1.00	23	0.217
148	A	7	5	1.00	23	0.217
149	A	5	4	1.00	21	0.190
150	A	6	4	1.00	21	0.190
151	A	7	5	1.00	23	0.217
152	A	9	7	1.00	23	0.304
153	A	4	3	1.00	23	0.130
154	A	4	3	1.00	23	0.130
155	A	4	3	1.00	23	0.130
156	A	3	3	1.00	23	0.130
157	A	6	5	1.00	23	0.217
158	A	8	5	1.00	23	0.217
159	A	10	5	1.00	23	0.217
160	A	8	5	1.00	23	0.217
161	A	6	4	1.00	21	0.190
162	A	7	5	1.00	21	0.238
163	A	7	5	1.00	23	0.217
164	A	8	6	1.00	23	0.261
165	A	4	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.00	23	0.130
167	A	4	3	1.00	23	0.130
168	A	3	3	1.00	23	0.130
169	A	4	3	1.00	23	0.130
170	A	7	5	1.00	23	0.217
171	A	7	5	1.00	23	0.217
172	A	5	5	1.00	23	0.217
173	A	3	3	1.00	21	0.143
174	A	7	5	1.00	21	0.238
175	A	9	6	1.00	23	0.261
176	A	11	7	1.00	23	0.304
177	A	4	3	1.00	23	0.130
178	A	5	4	1.00	23	0.174
179	A	3	3	1.00	23	0.130
180	A	6	5	1.00	23	0.217
181	A	8	6	1.00	23	0.261
182	A	10	6	1.00	23	0.261
183	A	6	5	1.00	23	0.217
184	A	4	4	1.00	23	0.174
185	A	4	4	1.00	21	0.190
186	A	8	6	1.00	21	0.286
187	A	10	6	1.00	23	0.261
188	A	12	7	1.00	23	0.304
189	A	5	4	1.00	23	0.174
190	A	4	3	1.00	23	0.130
191	A	4	3	1.00	23	0.130
192	A	7	6	1.00	23	0.261
193	A	9	6	1.00	23	0.261
194	A	11	6	1.00	23	0.261
195	A	5	4	1.00	23	0.174
196	A	4	4	1.00	23	0.174
197	A	5	4	1.00	21	0.190
198	A	9	6	1.00	21	0.286
199	A	11	6	1.00	23	0.261
200	A	13	7	1.00	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	23	0.130
202	A	5	4	1.00	23	0.174
203	A	5	4	1.00	23	0.174
204	A	8	6	1.00	23	0.261
205	A	10	6	1.00	23	0.261
206	A	12	6	1.00	23	0.261
207	A	7	5	1.28	23	0.217
208	A	1	1	1.00	23	0.043
209	A	8	6	1.00	23	0.261
210	A	7	6	1.00	23	0.261
211	A	4	4	1.00	21	0.190
212	A	5	5	1.00	23	0.217
213	A	8	7	1.00	23	0.304
214	A	9	7	1.00	23	0.304
215	A	1	1	1.00	25	0.040
216	A	1	1	1.00	25	0.040
217	A	1	1	1.00	25	0.040
218	A	1	1	1.00	25	0.040
219	A	4	3	1.00	21	0.143
220	A	4	3	1.00	21	0.143
221	A	3	3	1.00	21	0.143
222	A	2	2	1.00	19	0.105
223	A	4	4	1.00	19	0.210
224	A	5	5	1.00	21	0.238
225	A	1	1	1.00	21	0.048
226	A	1	1	1.00	21	0.048
227	A	1	1	1.00	21	0.048
228	A	1	1	1.00	21	0.048
229	A	1	1	1.00	23	0.043
230	A	1	1	1.00	23	0.043
231	A	1	1	1.00	23	0.043
232	A	1	1	1.00	23	0.043
233	A	17	14	1.00	23	0.609
234	A	18	15	1.00	23	0.652
235	A	16	13	1.00	23	0.565

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	17	14	1.00	23	0.609
237	A	17	14	1.00	23	0.609
238	A	21	17	1.00	25	0.680
239	A	21	17	1.00	25	0.680
240	A	19	15	1.00	25	0.600
241	A	20	16	1.00	25	0.640
242	A	21	17	1.00	25	0.680
243	A	20	16	1.00	25	0.640
244	A	18	15	1.00	25	0.600
245	A	19	16	1.00	25	0.640
246	A	17	14	1.00	25	0.560
247	A	18	15	1.00	25	0.600
248	A	18	15	1.00	25	0.600
249	A	19	16	1.00	25	0.640
250	A	24	19	1.00	25	0.760
251	A	22	18	1.00	25	0.720
252	A	22	18	1.00	25	0.720
253	A	20	16	1.00	25	0.640
254	A	21	17	1.00	25	0.680
255	A	22	18	1.00	25	0.720
256	A	7	5	1.00	19	0.263
257	A	6	5	1.00	19	0.263
258	A	5	5	1.00	19	0.263
259	A	4	4	1.00	17	0.235
260	A	5	4	1.00	17	0.235
261	A	6	5	1.00	19	0.263
262	A	7	5	1.00	19	0.263
263	A	8	5	1.00	19	0.263
264	A	5	2	1.00	19	0.105
265	A	4	2	1.00	19	0.105
266	A	3	2	1.00	19	0.105
267	A	2	2	1.00	19	0.105
268	A	3	2	1.00	19	0.105
269	A	4	2	1.00	19	0.105
270	A	5	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	2	1.17	21	0.095
272	A	3	2	1.13	21	0.095
273	A	3	2	1.14	21	0.095
274	A	3	2	1.00	21	0.095
275	A	3	2	1.00	19	0.105
276	A	3	2	1.00	19	0.105
277	A	4	3	1.00	21	0.143
278	A	5	4	1.00	21	0.190
279	A	12	7	1.00	21	0.333
280	A	10	7	1.00	21	0.333
281	A	8	7	1.00	21	0.333
282	A	8	5	1.00	21	0.238
283	A	9	6	1.00	21	0.286
284	A	11	7	1.00	21	0.333
285	A	12	7	1.00	21	0.333
286	A	3	2	1.00	21	0.095
287	A	3	2	1.00	21	0.095
288	A	3	2	1.00	21	0.095
289	A	3	2	1.00	21	0.095
290	A	4	4	1.00	19	0.210
291	A	3	2	1.00	19	0.105
292	A	3	2	1.00	21	0.095
293	A	3	2	1.00	21	0.095
294	A	15	8	1.37	21	0.381
295	A	6	6	1.00	21	0.286
296	A	7	7	1.00	21	0.333
297	A	9	8	1.27	21	0.381
298	A	15	8	1.45	21	0.381
299	A	3	2	1.00	21	0.095
300	A	3	2	1.00	21	0.095
301	A	3	2	1.00	21	0.095
302	A	3	2	1.00	21	0.095
303	A	3	2	1.00	19	0.105
304	A	3	2	1.00	19	0.105
305	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	2	1.00	21	0.095
307	A	16	10	1.42	21	0.476
308	A	6	6	1.00	21	0.286
309	A	6	6	1.00	21	0.286
310	A	11	7	1.00	21	0.333
311	A	15	8	1.00	21	0.381
312	A	38	22	1.00	25	0.880
313	A	35	19	1.00	25	0.760
314	A	21	16	1.00	25	0.640
315	A	19	14	1.00	25	0.560
316	A	39	23	1.00	25	0.920
317	A	36	20	1.00	25	0.800
318	A	5	4	1.00	23	0.174
319	A	5	4	1.00	23	0.174
320	A	4	4	1.00	21	0.190
321	A	7	5	1.00	21	0.238
322	A	13	9	1.00	23	0.391
323	A	7	7	1.00	23	0.304
324	A	1	1	1.00	14	0.071
325	A	5	4	1.00	23	0.174
326	A	5	4	1.00	23	0.174
327	A	5	4	1.00	23	0.174
328	A	3	3	1.00	21	0.143
329	A	7	5	1.00	21	0.238
330	A	11	6	1.00	23	0.261
331	A	11	8	1.51	23	0.348
332	A	6	6	1.00	23	0.261
333	A	1	1	1.00	14	0.071
334	A	9	8	1.00	23	0.348
335	A	5	4	1.00	23	0.174
336	A	5	4	1.00	23	0.174
337	A	4	4	1.00	21	0.190
338	A	7	4	1.00	21	0.190
339	A	11	5	1.34	23	0.217
340	A	17	11	1.71	23	0.478

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	7	7	1.00	23	0.304
342	A	6	6	1.00	14	0.429
343	A	14	11	1.48	23	0.478
344	A	8	6	1.00	23	0.261
345	A	7	6	1.00	23	0.261
346	A	4	4	1.00	21	0.190
347	F	0	0	N/A	0.	N/A
348	A	0	0	0.00	0	0.000
349	A	0	0	0.00	0	0.000
350	A	0	0	0.00	0	0.000
351	A	0	0	0.00	0	0.000
352	A	0	0	0.00	0	0.000
353	A	5	4	1.00	21	0.190
354	A	4	4	1.00	21	0.190
355	A	2	2	1.00	19	0.105
356	A	8	5	1.00	19	0.263
357	A	10	5	1.00	21	0.238
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	0	0	0.00	0	0.000
365	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx$

Optimal. Leaf size=151

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{2a\sec^2(c+dx)}{d}-\frac{4a\sec^3(c+dx)}{3d}+\frac{3a\sec^4(c+dx)}{2d}+\frac{6a\sec^5(c+dx)}{5d}-2$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-2*a*sec(d*x+c)^2/d-4/3*a*sec(d*x+c)^3/d+
3/2*a*sec(d*x+c)^4/d+6/5*a*sec(d*x+c)^5/d-2/3*a*sec(d*x+c)^6/d-4/7*a*sec(d*
x+c)^7/d+1/8*a*sec(d*x+c)^8/d+1/9*a*sec(d*x+c)^9/d
```

Rubi [A]

time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{a\sec^9(c+dx)}{9d}+\frac{a\sec^8(c+dx)}{8d}-\frac{4a\sec^7(c+dx)}{7d}-\frac{2a\sec^6(c+dx)}{3d}+\frac{6a\sec^5(c+dx)}{5d}+\frac{3a\sec^4(c+dx)}{2d}-\frac{4a\sec^3(c+dx)}{3d}-\frac{2a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^2)/d -
(4*a*Sec[c + d*x]^3)/(3*d) + (3*a*Sec[c + d*x]^4)/(2*d) + (6*a*Sec[c + d*x]
^5)/(5*d) - (2*a*Sec[c + d*x]^6)/(3*d) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Se
c[c + d*x]^8)/(8*d) + (a*Sec[c + d*x]^9)/(9*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^5}{x^{10}}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}} + \frac{a^9}{x^9} - \frac{4a^9}{x^8} - \frac{4a^9}{x^7} + \frac{6a^9}{x^6} + \frac{6a^9}{x^5} - \frac{4a^9}{x^4} - \frac{4a^9}{x^3} + \frac{a^9}{x^2} + \frac{a^9}{x}\right.\right.}{a^8 d}$$

$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} - \frac{2a\sec^2(c+dx)}{d} - \frac{4a\sec^3(c}{3d}$$

Mathematica [A]
time = 0.46, size = 134, normalized size = 0.89

$$\frac{a\sec(c+dx)}{d} - \frac{4a\sec^3(c+dx)}{3d} + \frac{6a\sec^5(c+dx)}{5d} - \frac{4a\sec^7(c+dx)}{7d} + \frac{a\sec^9(c+dx)}{9d} - \frac{a(24\log(\cos(c+dx)) + 12\tan^2(c+dx) - 6\tan^4(c+dx) + 4\tan^6(c+dx) - 3\tan^8(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] (a*Sec[c + d*x])/d - (4*a*Sec[c + d*x]^3)/(3*d) + (6*a*Sec[c + d*x]^5)/(5*d
) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Sec[c + d*x]^9)/(9*d) - (a*(24*Log[Cos[
c + d*x]] + 12*Tan[c + d*x]^2 - 6*Tan[c + d*x]^4 + 4*Tan[c + d*x]^6 - 3*Tan
[c + d*x]^8))/(24*d)
```

Maple [A]
time = 0.16, size = 199, normalized size = 1.32

method	result
derivativedivides	$\frac{a\left(\frac{\sin^{10}(dx+c)}{9\cos(dx+c)^9} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^7} + \frac{\sin^{10}(dx+c)}{105\cos(dx+c)^5} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^3} + \frac{\sin^{10}(dx+c)}{9\cos(dx+c)} + \frac{\left(\frac{128}{35} + \sin^8(dx+c) + \frac{8\left(\sin^6(dx+c)\right)}{7} + \frac{48\left(\sin^4\right.}{3}\right.}{9}\right.}{d}$
default	$\frac{a\left(\frac{\sin^{10}(dx+c)}{9\cos(dx+c)^9} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^7} + \frac{\sin^{10}(dx+c)}{105\cos(dx+c)^5} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^3} + \frac{\sin^{10}(dx+c)}{9\cos(dx+c)} + \frac{\left(\frac{128}{35} + \sin^8(dx+c) + \frac{8\left(\sin^6(dx+c)\right)}{7} + \frac{48\left(\sin^4\right.}{3}\right.}{9}\right.}{d}$
risch	$iax + \frac{2iac}{d} + \frac{2a\left(315\,e^{17i(dx+c)} - 1260\,e^{16i(dx+c)} + 840\,e^{15i(dx+c)} - 5040\,e^{14i(dx+c)} + 4788\,e^{13i(dx+c)} - 14280\,e^{12i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/9*sin(d*x+c)^10/cos(d*x+c)^9-1/63*sin(d*x+c)^10/cos(d*x+c)^7+1/10
5*sin(d*x+c)^10/cos(d*x+c)^5-1/63*sin(d*x+c)^10/cos(d*x+c)^3+1/9*sin(d*x+c)
```

```
^10/cos(d*x+c)+1/9*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4
+64/35*sin(d*x+c)^2)*cos(d*x+c))+a*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*t
an(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 116, normalized size = 0.77

$$-\frac{2520\,a\log\left(\cos\left(dx+c\right)\right)-\frac{2520\,a\cos\left(dx+c\right)^{8}-5040\,a\cos\left(dx+c\right)^{7}-3360\,a\cos\left(dx+c\right)^{6}+3780\,a\cos\left(dx+c\right)^{5}+3024\,a\cos\left(dx+c\right)^{4}-1680\,a\cos\left(dx+c\right)^{3}-1440\,a\cos\left(dx+c\right)^{2}+315\,a\cos\left(dx+c\right)+280\,a}{\cos\left(dx+c\right)^{9}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a*log(cos(d*x + c)) - (2520*a*cos(d*x + c)^8 - 5040*a*cos(d*x
 + c)^7 - 3360*a*cos(d*x + c)^6 + 3780*a*cos(d*x + c)^5 + 3024*a*cos(d*x +
c)^4 - 1680*a*cos(d*x + c)^3 - 1440*a*cos(d*x + c)^2 + 315*a*cos(d*x + c) +
 280*a)/cos(d*x + c)^9)/d
```

Fricas [A]
time = 2.70, size = 123, normalized size = 0.81

$$-\frac{2520\,a\cos\left(dx+c\right)^{9}\log\left(-\cos\left(dx+c\right)\right)-2520\,a\cos\left(dx+c\right)^{8}+5040\,a\cos\left(dx+c\right)^{7}+3360\,a\cos\left(dx+c\right)^{6}-3780\,a\cos\left(dx+c\right)^{5}-3024\,a\cos\left(dx+c\right)^{4}+1680\,a\cos\left(dx+c\right)^{3}+1440\,a\cos\left(dx+c\right)^{2}-315\,a\cos\left(dx+c\right)-280\,a}{2520\,d\cos\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a*cos(d*x + c)^9*log(-cos(d*x + c)) - 2520*a*cos(d*x + c)^8 +
 5040*a*cos(d*x + c)^7 + 3360*a*cos(d*x + c)^6 - 3780*a*cos(d*x + c)^5 - 30
24*a*cos(d*x + c)^4 + 1680*a*cos(d*x + c)^3 + 1440*a*cos(d*x + c)^2 - 315*a
*cos(d*x + c) - 280*a)/(d*cos(d*x + c)^9)
```

Sympy [A]
time = 2.34, size = 184, normalized size = 1.22

$$\begin{cases} \frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{8}\left(c+dx\right)\sec\left(c+dx\right)}{9d}+\frac{a\tan^{8}\left(c+dx\right)}{8d}-\frac{8a\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)}{63d}-\frac{a\tan^{6}\left(c+dx\right)}{6d}+\frac{16a\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{105d}+\frac{a\tan^{4}\left(c+dx\right)}{4d}-\frac{64a\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{315d}-\frac{a\tan^{2}\left(c+dx\right)}{2d}+\frac{128a\sec\left(c+dx\right)}{315d} & \text{for}\: d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)\tan^{9}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**8*sec(c + d*x
)/(9*d) + a*tan(c + d*x)**8/(8*d) - 8*a*tan(c + d*x)**6*sec(c + d*x)/(63*d)
 - a*tan(c + d*x)**6/(6*d) + 16*a*tan(c + d*x)**4*sec(c + d*x)/(105*d) + a*
tan(c + d*x)**4/(4*d) - 64*a*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a*tan(c
 + d*x)**2/(2*d) + 128*a*sec(c + d*x)/(315*d), Ne(d, 0)), (x*(a*sec(c) + a)
*tan(c)**9, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(137) = 274.

time = 6.00, size = 293, normalized size = 1.94

$$\frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9177\,a+\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{953988\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{1594782\,a(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1336734\,a(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{781956\,a(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{302004\,a(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{69201\,a(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}+\frac{7129\,a(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^9}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a + 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^9)/d
```

Mupad [B]

time = 5.13, size = 259, normalized size = 1.72

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-18\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+\frac{218\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3}-174\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{1382\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{5}-\frac{2114\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{15}+\frac{1654\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{35}-\frac{326\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{35}+\frac{256\,a}{315}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((256*a)/315 - (326*a*tan(c/2 + (d*x)
/2)^2)/35 + (1654*a*tan(c/2 + (d*x)/2)^4)/35 - (2114*a*tan(c/2 + (d*x)/2)^6
)/15 + (1382*a*tan(c/2 + (d*x)/2)^8)/5 - 174*a*tan(c/2 + (d*x)/2)^10 + (218
*a*tan(c/2 + (d*x)/2)^12)/3 - 18*a*tan(c/2 + (d*x)/2)^14 + 2*a*tan(c/2 + (d
*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/
2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*
tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16
+ tan(c/2 + (d*x)/2)^18 - 1))
```

3.2 $\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal. Leaf size=118

$$\frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{d}-\frac{3a\sec^4(c+dx)}{4d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec}{}$$

```
[Out] a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+3/2*a*sec(d*x+c)^2/d+a*sec(d*x+c)^3/d-3/4
*a*sec(d*x+c)^4/d-3/5*a*sec(d*x+c)^5/d+1/6*a*sec(d*x+c)^6/d+1/7*a*sec(d*x+c
)^7/d
```

Rubi [A]
time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{a\sec^7(c+dx)}{7d}+\frac{a\sec^6(c+dx)}{6d}-\frac{3a\sec^5(c+dx)}{5d}-\frac{3a\sec^4(c+dx)}{4d}+\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (3*a*Sec[c + d*x]^2)/(2*d) +
 (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^4)/(4*d) - (3*a*Sec[c + d*x]^5)/(
5*d) + (a*Sec[c + d*x]^6)/(6*d) + (a*Sec[c + d*x]^7)/(7*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^4}{x^8}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{a^7}{x^7}-\frac{3a^7}{x^6}-\frac{3a^7}{x^5}+\frac{3a^7}{x^4}+\frac{3a^7}{x^3}-\frac{a^7}{x^2}-\frac{a^7}{x}\right)dx, x, \cos(\right.}{a^6 d}$$
$$= \frac{a\log(\cos(c+dx))}{d} - \frac{a\sec(c+dx)}{d} + \frac{3a\sec^2(c+dx)}{2d} + \frac{a\sec^3(c+}{d}$$

Mathematica [A]
time = 0.44, size = 106, normalized size = 0.90

$$-\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec^7(c+dx)}{7d}+\frac{a(12\log(\cos(c+dx))+6\tan^2(c+dx)-3\tan^4(c+dx)+2\tan^6(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^5)/(5*d) +
 (a*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A]
time = 0.10, size = 159, normalized size = 1.35

method	result
derivativedivides	$\frac{a\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}\right.}{d}$
default	$\frac{a\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2a\left(105\,e^{13i(dx+c)}-315\,e^{12i(dx+c)}+210\,e^{11i(dx+c)}-945\,e^{10i(dx+c)}+903\,e^{9i(dx+c)}-1820\,e^{8i(dx+c)}+\right.}{105d(e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/7*sin(d*x+c)^8/cos(d*x+c)^7-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35*s
in(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)^6
+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a*(1/6*tan(d*x+c)^6-1/4*tan
(d*x+c)^4+1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 0.80

$$\frac{420\,a\log\left(\cos\left(dx+c\right)\right)-\frac{420\,a\cos\left(dx+c\right)^{6}-630\,a\cos\left(dx+c\right)^{5}-420\,a\cos\left(dx+c\right)^{4}+315\,a\cos\left(dx+c\right)^{3}+252\,a\cos\left(dx+c\right)^{2}-70\,a\cos\left(dx+c\right)-60\,a}{\cos\left(dx+c\right)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*a*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*a)/cos(d*x + c)^7)/d
```

Fricas [A]
time = 2.50, size = 101, normalized size = 0.86

$$\frac{420\,a\cos\left(dx+c\right)^{7}\log\left(-\cos\left(dx+c\right)\right)-420\,a\cos\left(dx+c\right)^{6}+630\,a\cos\left(dx+c\right)^{5}+420\,a\cos\left(dx+c\right)^{4}-315\,a\cos\left(dx+c\right)^{3}-252\,a\cos\left(dx+c\right)^{2}+70\,a\cos\left(dx+c\right)+60\,a}{420\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*a*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*a*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*a*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*a)/(d*cos(d*x + c)^7)
```

Sympy [A]
time = 1.15, size = 148, normalized size = 1.25

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)}{7d}+\frac{a\tan^{6}\left(c+dx\right)}{6d}-\frac{6a\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{35d}-\frac{a\tan^{4}\left(c+dx\right)}{4d}+\frac{8a\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{a\tan^{2}\left(c+dx\right)}{2d}-\frac{16a\sec\left(c+dx\right)}{35d} & \text{for}\: d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)\tan^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6*sec(c + d*
x)/(7*d) + a*tan(c + d*x)**6/(6*d) - 6*a*tan(c + d*x)**4*sec(c + d*x)/(35*d
) - a*tan(c + d*x)**4/(4*d) + 8*a*tan(c + d*x)**2*sec(c + d*x)/(35*d) + a*t
an(c + d*x)**2/(2*d) - 16*a*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
a)*tan(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(108) = 216.
time = 3.51, size = 247, normalized size = 2.09

$$-\frac{420\,a\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)+\frac{1473\,a+\frac{11151\,a\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{36813\,a\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{69475\,a\left(\cos\left(dx+c\right)-1\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{56035\,a\left(\cos\left(dx+c\right)-1\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+\frac{28749\,a\left(\cos\left(dx+c\right)-1\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}+\frac{8463\,a\left(\cos\left(dx+c\right)-1\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}+\frac{1089\,a\left(\cos\left(dx+c\right)-1\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}}{\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")

[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a + 11151*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d

Mupad [B]
time = 5.69, size = 204, normalized size = 1.73

$$\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-\frac{224\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{166\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}-\frac{42\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{32\,a}{35}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x)),x)

[Out] ((32*a)/35 - (42*a*tan(c/2 + (d*x)/2)^2)/5 + (166*a*tan(c/2 + (d*x)/2)^4)/5 - (224*a*tan(c/2 + (d*x)/2)^6)/3 + (128*a*tan(c/2 + (d*x)/2)^8)/3 - 14*a*tan(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d

3.3 $\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\sec^2(c+dx)}{d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec^4(c+dx)}{4d}+\frac{a\sec^5(c+dx)}{5d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-a*sec(d*x+c)^2/d-2/3*a*sec(d*x+c)^3/d+1/
4*a*sec(d*x+c)^4/d+1/5*a*sec(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{a\sec^5(c+dx)}{5d}+\frac{a\sec^4(c+dx)}{4d}-\frac{2a\sec^3(c+dx)}{3d}-\frac{a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (a*Sec[c + d*x]^2)/d - (2
*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^5)/(5
*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^3}{x^6}\,dx,x,\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^5}{x^5}-\frac{2a^5}{x^4}-\frac{2a^5}{x^3}+\frac{a^5}{x^2}+\frac{a^5}{x}\right)dx,x,\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\sec^2(c+dx)}{d}-\frac{2a\sec^3(c}{3d}$$

Mathematica [A]
time = 0.31, size = 82, normalized size = 0.94

$$\frac{a\sec(c+dx)}{d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.12, size = 123, normalized size = 1.41

method	result
derivativedivides	$\frac{a\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx-\right.}{2}\right.}{d}$
default	$\frac{a\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx-\right.}{2}\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2a\left(15\,e^{9i(dx+c)}-30\,e^{8i(dx+c)}+20\,e^{7i(dx+c)}-60\,e^{6i(dx+c)}+58\,e^{5i(dx+c)}-60\,e^{4i(dx+c)}+20\,e^{3i(dx+c)}-3\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/5*sin(d*x+c)^6/cos(d*x+c)^5-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*si
n(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a
*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.83

$$-\frac{60\,a\log\left(\cos\left(dx+c\right)\right)-\frac{60\,a\cos\left(dx+c\right)^{4}-60\,a\cos\left(dx+c\right)^{3}-40\,a\cos\left(dx+c\right)^{2}+15\,a\cos\left(dx+c\right)+12\,a}{\cos\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*a*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*a*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*a)/cos(d*x + c)^5)/d
```

Fricas [A]
time = 4.54, size = 79, normalized size = 0.91

$$-\frac{60\,a\cos\left(dx+c\right)^{5}\log\left(-\cos\left(dx+c\right)\right)-60\,a\cos\left(dx+c\right)^{4}+60\,a\cos\left(dx+c\right)^{3}+40\,a\cos\left(dx+c\right)^{2}-15\,a\cos\left(dx+c\right)-12\,a}{60\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*a*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*a)/(d*cos(d*x
 + c)^5)
```

Sympy [A]
time = 0.52, size = 112, normalized size = 1.29

$$\begin{cases}\frac{a\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{5d}+\frac{a\tan^{4}\left(c+dx\right)}{4d}-\frac{4a\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{15d}-\frac{a\tan^{2}\left(c+dx\right)}{2d}+\frac{8a\sec\left(c+dx\right)}{15d} & \text{for}\: d\neq 0\\x\left(a\sec\left(c\right)+a\right)\tan^{5}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4*sec(c + d*x
)/(5*d) + a*tan(c + d*x)**4/(4*d) - 4*a*tan(c + d*x)**2*sec(c + d*x)/(15*d)
 - a*tan(c + d*x)**2/(2*d) + 8*a*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(
c) + a)*tan(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(81) = 162.
time = 1.81, size = 201, normalized size = 2.31

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{201\,a+\frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1970\,a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{805\,a(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{137\,a(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a + 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

Mupad [B]
time = 5.73, size = 151, normalized size = 1.74

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{22\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{16\,a}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*a)/15 - (22*a*tan(c/2 + (d*x)/2)
^2)/3 + (62*a*tan(c/2 + (d*x)/2)^4)/3 - 10*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan
(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 1
0*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1
))
```

3.4 $\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=57

$$\frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}$$

```
[Out] a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+1/2*a*sec(d*x+c)^2/d+1/3*a*sec(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3964, 76}

$$\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^2)/(2*d) + (
a*Sec[c + d*x]^3)/(3*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-ax)(a+ax)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{a^2d}\\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{a^3}{x^3}-\frac{a^3}{x^2}-\frac{a^3}{x}\right)\,dx,x,\cos(c+dx)\right)}{a^2d}\\ &= \frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.96

$$-\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.08, size = 83, normalized size = 1.46

method	result	size
derivativedivides	$\frac{a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	83
default	$\frac{a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	83
risch	$-iax-\frac{2iac}{d}-\frac{2a\left(3\,\mathrm{e}^{5i(dx+c)}-3\,\mathrm{e}^{4i(dx+c)}+2\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+si
n(d*x+c)^2)*cos(d*x+c))+a*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.88

$$\frac{6\,a\log\left(\cos\left(dx+c\right)\right)-\frac{6\,a\cos(dx+c)^2-3\,a\cos(dx+c)-2\,a}{\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*a)/
cos(d*x + c)^3)/d
```

Fricas [A]
time = 3.61, size = 57, normalized size = 1.00

$$\frac{6\,a\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-6\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+2\,a}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*a*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*a)/(d*cos(d*x + c)^3)
```

Sympy [A]
time = 0.22, size = 76, normalized size = 1.33

$$\begin{cases} -\frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a\tan^2\left(c+dx\right)}{2d}-\frac{2a\sec\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2*sec(c + d*
x)/(3*d) + a*tan(c + d*x)**2/(2*d) - 2*a*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a*sec(c) + a)*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(53) = 106.
time = 0.81, size = 155, normalized size = 2.72

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{19\,a+\frac{69\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (19*a + 69*a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 11*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1)^3)/d
```

Mupad [B]
time = 1.99, size = 96, normalized size = 1.68

$$\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\,a}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] ((4*a)/3 - 6*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c
/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*
a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.5 $\int (a+a\sec(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=25

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3964, 45}

$$\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\tan(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{a+ax}{x^2}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{a}{x}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.80

method	result	size
derivativedivides	$\frac{a(\sec(dx+c)+\ln(\sec(dx+c)))}{d}$	20
default	$\frac{a(\sec(dx+c)+\ln(\sec(dx+c)))}{d}$	20
risch	$iax+\frac{2iac}{d}+\frac{2a\,e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-\frac{a\ln(e^{2i(dx+c)}+1)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*a*(sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.04

$$-\frac{a\log\left(\cos\left(dx+c\right)\right)-\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
[Out] -(a*log(cos(d*x + c)) - a/cos(d*x + c))/d
```

Fricas [A]
time = 3.81, size = 34, normalized size = 1.36

$$-\frac{a\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-a}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - a)/(d*cos(d*x + c))
```

Sympy [A]
time = 0.09, size = 37, normalized size = 1.48

$$\begin{cases} \frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\sec\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a\sec\left(c\right)+a)\tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*sec(c + d*x)/d, Ne(d, 0)),
(x*(a*sec(c) + a)*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(25) = 50.
time = 0.49, size = 106, normalized size = 4.24

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3\,a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B]
time = 1.17, size = 40, normalized size = 1.60

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.6 $\int \cot(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$\frac{a\log(1-\cos(c+dx))}{d}$$

```
[Out] a*ln(1-cos(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3964, 31}

$$\frac{a\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{a^2\text{Subst}\left(\int \frac{1}{a-ax}\,dx, x, \cos(c+dx)\right)}{d}\\ &= \frac{a\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.81

$$\frac{2a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2]]))/d
```

Maple [A]
time = 0.06, size = 26, normalized size = 1.62

method	result	size
derivativedivides	$-\frac{a(-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c)))}{d}$	26
default	$-\frac{a(-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c)))}{d}$	26
risch	$-iax-\frac{2iac}{d}+\frac{2a\ln(e^{i(dx+c)}-1)}{d}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(-ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$\frac{a\log\left(\cos\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] a*log(cos(d*x + c) - 1)/d
```

Fricas [A]
time = 3.07, size = 16, normalized size = 1.00

$$\frac{a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] a*log(-1/2*cos(d*x + c) + 1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.
time = 0.43, size = 58, normalized size = 3.62

$$\frac{a \log \left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a \log \left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 1.24, size = 34, normalized size = 2.12

$$\frac{a \left(2 \ln \left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - \ln \left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

3.7 $\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=57

$$-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(1+\cos(c+dx))}{4d}$$

```
[Out] -1/2*a/d/(1-cos(d*x+c))-3/4*a*ln(1-cos(d*x+c))/d-1/4*a*ln(1+cos(d*x+c))/d
```

Rubi [A]

time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*a/(d*(1 - Cos[c + d*x])) - (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Log[
1 + Cos[c + d*x]])/(4*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{a^4\text{Subst}\left(\int \frac{x^2}{(a-ax)^2(a+ax)}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^4\text{Subst}\left(\int \left(\frac{1}{2a^3(-1+x)^2}+\frac{3}{4a^3(-1+x)}+\frac{1}{4a^3(1+x)}\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(1+\cos(c+dx))}{4d}\end{aligned}$$

Mathematica [A]
time = 0.79, size = 114, normalized size = 2.00

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.12, size = 48, normalized size = 0.84

method	result	size
derivativedivides	$\frac{a\left(-\frac{\ln(1+\sec(dx+c))}{4}-\frac{1}{2(-1+\sec(dx+c))}-\frac{3\ln(-1+\sec(dx+c))}{4}+\ln(\sec(dx+c))\right)}{d}$	48
default	$\frac{a\left(-\frac{\ln(1+\sec(dx+c))}{4}-\frac{1}{2(-1+\sec(dx+c))}-\frac{3\ln(-1+\sec(dx+c))}{4}+\ln(\sec(dx+c))\right)}{d}$	48
risch	$iax+\frac{2iac}{d}+\frac{a\,e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}-\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(-1/4*ln(1+sec(d*x+c))-1/2/(-1+sec(d*x+c))-3/4*ln(-1+sec(d*x+c))+ln(s
ec(d*x+c)))
```

Maxima [A]
time = 0.29, size = 42, normalized size = 0.74

$$-\frac{a\log\left(\cos\left(dx+c\right)+1\right)+3\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,a}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 2*a/(cos(d*x +
c) - 1))/d
```

Fricas [A]
time = 2.36, size = 69, normalized size = 1.21

$$-\frac{(a\cos\left(dx+c\right)-a)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\,(a\cos\left(dx+c\right)-a)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,a}{4\,(d\cos\left(dx+c\right)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) - 2*a)/(d*cos(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cot^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int \cot^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**3, x)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(49) = 98.
time = 0.48, size = 103, normalized size = 1.81

$$-\frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 3*a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B]
time = 1.28, size = 46, normalized size = 0.81

$$-\frac{a\left(\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(6*log(tan(c/2 + (d*x)/2)) - 4*log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2))/(4*d)
```

3.8 $\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=95

$$-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(1+\cos(c+dx))}+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(1+\cos(c+dx))}{16d}$$

```
[Out] -1/8*a/d/(1-cos(d*x+c))^2+3/4*a/d/(1-cos(d*x+c))+1/8*a/d/(1+cos(d*x+c))+11/
16*a*ln(1-cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(\cos(c+dx)+1)}-\frac{a}{8d(1-\cos(c+dx))^2}+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*a/(d*(1 - Cos[c + d*x])^2) + (3*a)/(4*d*(1 - Cos[c + d*x])) + a/(8*d*(
1 + Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) + (5*a*Log[1 + Cos
[c + d*x]])/(16*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^6\text{Subst}\left(\int \frac{x^4}{(a-ax)^3(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{4a^5(-1+x)^3} - \frac{3}{4a^5(-1+x)^2} - \frac{11}{16a^5(-1+x)} + \frac{1}{8a^5(1+x)^2} - \frac{1}{\ldots}\right.\right.}{d}$$

$$= -\frac{a}{8d(1-\cos(c+dx))^2} + \frac{3a}{4d(1-\cos(c+dx))} + \frac{a}{8d(1+\cos(c+da\ldots}$$

Mathematica [A]
time = 0.55, size = 127, normalized size = 1.34

$$\frac{a\left(32\cot^2(c+dx) - 16\cot^4(c+dx) + 10\csc^2\left(\frac{1}{2}(c+dx)\right) - \csc^4\left(\frac{1}{2}(c+dx)\right) - 24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 64\log(\cos(c+dx)) + 24\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 64\log(\tan(c+dx)) - 10\sec^2\left(\frac{1}{2}(c+dx)\right) + \sec^4\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(32*Cot[c + d*x]^2 - 16*Cot[c + d*x]^4 + 10*Csc[(c + d*x)/2]^2 - Csc[(c
+ d*x)/2]^4 - 24*Log[Cos[(c + d*x)/2]] + 64*Log[Cos[c + d*x]] + 24*Log[Sin[
(c + d*x)/2]] + 64*Log[Tan[c + d*x]] - 10*Sec[(c + d*x)/2]^2 + Sec[(c + d*x
)/2]^4))/(64*d)
```

Maple [A]
time = 0.13, size = 73, normalized size = 0.77

method	result
derivativedivides	$-\frac{a\left(\frac{1}{8+8\sec(dx+c)}-\frac{5\ln(1+\sec(dx+c))}{16}+\frac{1}{8(-1+\sec(dx+c))^2}-\frac{1}{2(-1+\sec(dx+c))}-\frac{11\ln(-1+\sec(dx+c))}{16}+\ln(\sec(dx+c))\right)}{d}$
default	$-\frac{a\left(\frac{1}{8+8\sec(dx+c)}-\frac{5\ln(1+\sec(dx+c))}{16}+\frac{1}{8(-1+\sec(dx+c))^2}-\frac{1}{2(-1+\sec(dx+c))}-\frac{11\ln(-1+\sec(dx+c))}{16}+\ln(\sec(dx+c))\right)}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{a\left(5\,e^{5i(dx+c)}+6\,e^{4i(dx+c)}-14\,e^{3i(dx+c)}+6\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{4d\left(e^{i(dx+c)}-1\right)^4\left(e^{i(dx+c)}+1\right)^2}+\frac{11a\ln\left(e^{i(dx+c)}-1\right)}{8d}+\frac{5a\ln\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(1/8/(1+sec(d*x+c))-5/16*ln(1+sec(d*x+c))+1/8/(-1+sec(d*x+c))^2-1/2/
(-1+sec(d*x+c))-11/16*ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.91

$$\frac{5\,a\log\left(\cos\left(dx+c\right)+1\right)+11\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(5\,a\cos(dx+c)^2+3\,a\cos(dx+c)-6\,a\right)}{\cos(dx+c)^3-\cos(dx+c)^2-\cos(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 2*(5*a*cos(d
*x + c)^2 + 3*a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3 - cos(d*x + c)^2 - cos(
d*x + c) + 1))/d
```

Fricas [A]
time = 3.03, size = 150, normalized size = 1.58

$$-\frac{10\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)-5\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-11\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,a}{16\,\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*a*cos(d*x + c)^2 + 6*a*cos(d*x + c) - 5*(a*cos(d*x + c)^3 - a*cos
(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) - 11*(a*cos(d
*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x + c) +
1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^{5}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\cot^{5}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**5, x)
)
```

Giac [A]
time = 0.50, size = 149, normalized size = 1.57

$$\frac{22\,a\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)^{2}}{(\cos(dx+c)-1)^{2}}-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

Mupad [B]
time = 1.18, size = 88, normalized size = 0.93

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a}{4}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}\right)}{8\,d}+\frac{11\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (co
t(c/2 + (d*x)/2)^4*(a/4 - (5*a*tan(c/2 + (d*x)/2)^2)/2))/(8*d) + (11*a*log(
tan(c/2 + (d*x)/2)))/(8*d)
```

3.9 $\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=133

$$-\frac{a}{24d(1-\cos(c+dx))^3}+\frac{9a}{32d(1-\cos(c+dx))^2}-\frac{15a}{16d(1-\cos(c+dx))}+\frac{a}{32d(1+\cos(c+dx))^2}-\frac{a}{4d(1+\cos}$$

```
[Out] -1/24*a/d/(1-cos(d*x+c))^3+9/32*a/d/(1-cos(d*x+c))^2-15/16*a/d/(1-cos(d*x+c
))+1/32*a/d/(1+cos(d*x+c))^2-1/4*a/d/(1+cos(d*x+c))-21/32*a*ln(1-cos(d*x+c)
)/d-11/32*a*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$-\frac{15a}{16d(1-\cos(c+dx))}-\frac{a}{4d(\cos(c+dx)+1)}+\frac{9a}{32d(1-\cos(c+dx))^2}+\frac{a}{32d(\cos(c+dx)+1)^2}-\frac{a}{24d(1-\cos(c+dx))^3}-\frac{21a\log(1-\cos(c+dx))}{32d}-\frac{11a\log(\cos(c+dx)+1)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/24*a/(d*(1 - Cos[c + d*x])^3) + (9*a)/(32*d*(1 - Cos[c + d*x])^2) - (15*
a)/(16*d*(1 - Cos[c + d*x])) + a/(32*d*(1 + Cos[c + d*x])^2) - a/(4*d*(1 +
Cos[c + d*x])) - (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (11*a*Log[1 + Cos[c
+ d*x]])/(32*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^8\text{Subst}\left(\int \frac{x^6}{(a-ax)^4(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^8\text{Subst}\left(\int \left(\frac{1}{8a^7(-1+x)^4} + \frac{9}{16a^7(-1+x)^3} + \frac{15}{16a^7(-1+x)^2} + \frac{21}{32a^7(-1+x)} + \right.\right.}{d}$$
$$= -\frac{a}{24d(1-\cos(c+dx))^3} + \frac{9a}{32d(1-\cos(c+dx))^2} - \frac{15a}{16d(1-\cos(c}$$

Mathematica [A]

time = 0.41, size = 165, normalized size = 1.24

$$-\frac{a\left(192\cot^2(c+dx)-96\cot^4(c+dx)+64\cot^6(c+dx)+66\csc^2\left(\frac{1}{2}(c+dx)\right)-12\csc^4\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)-120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+384\log(\cos(c+dx))+120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+384\log(\tan(c+dx))-66\sec^2\left(\frac{1}{2}(c+dx)\right)+12\sec^4\left(\frac{1}{2}(c+dx)\right)-\sec^6\left(\frac{1}{2}(c+dx)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/384*(a*(192*Cot[c + d*x]^2 - 96*Cot[c + d*x]^4 + 64*Cot[c + d*x]^6 + 66*
Csc[(c + d*x)/2]^2 - 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 - 120*Log[C
os[(c + d*x)/2]] + 384*Log[Cos[c + d*x]] + 120*Log[Sin[(c + d*x)/2]] + 384*
Log[Tan[c + d*x]] - 66*Sec[(c + d*x)/2]^2 + 12*Sec[(c + d*x)/2]^4 - Sec[(c
+ d*x)/2]^6))/d
```

Maple [A]

time = 0.13, size = 96, normalized size = 0.72

method	result
derivativedivides	$\frac{a\left(\frac{1}{32(1+\sec(dx+c))^2}+\frac{3}{16(1+\sec(dx+c))}-\frac{11\ln(1+\sec(dx+c))}{32}-\frac{1}{24(-1+\sec(dx+c))^3}+\frac{5}{32(-1+\sec(dx+c))^2}-\frac{1}{2(-1+\sec(dx+c))}-\right.}{d}$
default	$\frac{a\left(\frac{1}{32(1+\sec(dx+c))^2}+\frac{3}{16(1+\sec(dx+c))}-\frac{11\ln(1+\sec(dx+c))}{32}-\frac{1}{24(-1+\sec(dx+c))^3}+\frac{5}{32(-1+\sec(dx+c))^2}-\frac{1}{2(-1+\sec(dx+c))}-\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{a\left(33\,e^{9i(dx+c)}+78\,e^{8i(dx+c)}-184\,e^{7i(dx+c)}+2\,e^{6i(dx+c)}+270\,e^{5i(dx+c)}+2\,e^{4i(dx+c)}-184\,e^{3i(dx+c)}+\right.}{24d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(1/32/(1+sec(d*x+c))^2+3/16/(1+sec(d*x+c))-11/32*ln(1+sec(d*x+c))-1/2
4/(-1+sec(d*x+c))^3+5/32/(-1+sec(d*x+c))^2-1/2/(-1+sec(d*x+c))-21/32*ln(-1+
sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]

time = 0.27, size = 126, normalized size = 0.95

$$-\frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a\cos\left(dx+c\right)^4+39\,a\cos\left(dx+c\right)^3-79\,a\cos\left(dx+c\right)^2-29\,a\cos\left(dx+c\right)+44\,a\right)}{\cos\left(dx+c\right)^5-\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 2*(33*a*co
s(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 79*a*cos(d*x + c)^2 - 29*a*cos(d*x + c
) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x +
 c)^2 + cos(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(113) = 226$.
time = 3.01, size = 241, normalized size = 1.81

$$\frac{66\,a\cos\left(dx+c\right)^{4}+78\,a\cos\left(dx+c\right)^{3}-158\,a\cos\left(dx+c\right)^{2}-58\,a\cos\left(dx+c\right)-33\,\left(a\cos\left(dx+c\right)^{5}-a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{3}+2\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-63\,\left(a\cos\left(dx+c\right)^{5}-a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{3}+2\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+88\,a}{96\,\left(d\cos\left(dx+c\right)^{5}-d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{3}+2\,d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*a*cos(d*x + c)^4 + 78*a*cos(d*x + c)^3 - 158*a*cos(d*x + c)^2 - 58
*a*cos(d*x + c) - 33*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c
)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2)
- 63*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*
x + c)^2 + a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(
d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d
*cos(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^{7}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\cot^{7}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**7, x)
)
```

Giac [A]
time = 0.54, size = 197, normalized size = 1.48

$$-\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a+\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{462\,a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}\right)(\cos(dx+c)+1)^{3}}{(\cos(dx+c)-1)^{3}}-\frac{42\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a + 21*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(
cos(d*x + c) - 1)^3 - 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

Mupad [B]
time = 1.23, size = 118, normalized size = 0.89

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{a}{6}\right)}{32\,d}-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}-\frac{21\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^6*(a/6 - (7*a*tan
(c/2 + (d*x)/2)^2)/4 + 11*a*tan(c/2 + (d*x)/2)^4))/(32*d) - (7*a*tan(c/2 +
(d*x)/2)^2)/(64*d) + (a*tan(c/2 + (d*x)/2)^4)/(128*d) - (21*a*log(tan(c/2 +
 (d*x)/2)))/(16*d)
```

3.10 $\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx$

Optimal. Leaf size=129

$$ax+\frac{35a\tanh^{-1}(\sin(c+dx))}{128d}-\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d}+\frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d}-$$

```
[Out] a*x+35/128*a*arctanh(sin(d*x+c))/d-1/128*(128*a+35*a*sec(d*x+c))*tan(d*x+c)
/d+1/192*(64*a+35*a*sec(d*x+c))*tan(d*x+c)^3/d-1/240*(48*a+35*a*sec(d*x+c))
*tan(d*x+c)^5/d+1/56*(8*a+7*a*sec(d*x+c))*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3966, 3855}

$$\frac{35a\tanh^{-1}(\sin(c+dx))}{128d}+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}-\frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{240d}+\frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{192d}-\frac{\tan(c+dx)(35a\sec(c+dx)+128a)}{128d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] a*x + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - ((128*a + 35*a*Sec[c + d*x])*T
an[c + d*x])/(128*d) + ((64*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^3)/(192*d)
- ((48*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^5)/(240*d) + ((8*a + 7*a*Sec[c +
 d*x])*Tan[c + d*x]^7)/(56*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx = \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} - \frac{1}{8}\int (8a+7a\sec(c+dx))\tan$$

$$= -\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d} + \frac{(8a+7a\sec(c+dx))\tan}{56d}$$

$$= \frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d} - \frac{(48a+35a\sec(c+dx))\tan}{240d}$$

$$= -\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} + \frac{(64a+35a\sec(c+dx))}{192d}$$

$$= ax - \frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} + \frac{(64a+35a\sec(c+d}{192d}$$

$$= ax + \frac{35a\tanh^{-1}(\sin(c+dx))}{128d} - \frac{(128a+35a\sec(c+dx))\tan(c+}{128d}$$

Mathematica [A]
time = 1.83, size = 115, normalized size = 0.89

$$\frac{a\left(13440\mathrm{ArcTan}(\tan(c+dx)) + 3675\tanh^{-1}(\sin(c+dx)) - \frac{1}{32}(18970 + 223232\cos(c+dx) + 75915\cos(2(c+dx)) + 147968\cos(3(c+dx)) + 12950\cos(4(c+dx)) + 47616\cos(5(c+dx)) + 9765\cos(6(c+dx)) + 11264\cos(7(c+dx)))\sec^7(c+dx)\tan(c+dx)\right)}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] (a*(13440*ArcTan[Tan[c + d*x]] + 3675*ArcTanh[Sin[c + d*x]] - ((18970 + 223
232*Cos[c + d*x] + 75915*Cos[2*(c + d*x)] + 147968*Cos[3*(c + d*x)] + 12950
*Cos[4*(c + d*x)] + 47616*Cos[5*(c + d*x)] + 9765*Cos[6*(c + d*x)] + 11264*
Cos[7*(c + d*x)])*Sec[c + d*x]^7*Tan[c + d*x])/32))/(13440*d)
```

Maple [A]
time = 0.11, size = 180, normalized size = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin^9(dx+c)}{8\cos(dx+c)^8} - \frac{\sin^9(dx+c)}{48\cos(dx+c)^6} + \frac{\sin^9(dx+c)}{64\cos(dx+c)^4} - \frac{5\left(\sin^9(dx+c)\right)}{128\cos(dx+c)^2} - \frac{5\left(\sin^7(dx+c)\right)}{128} - \frac{7\left(\sin^5(dx+c)\right)}{128} - \frac{35\left(\sin^3(dx+c)\right)}{384} - \frac{35\sin(d}{128}\right.}{d}$
default	$\frac{a\left(\frac{\sin^9(dx+c)}{8\cos(dx+c)^8} - \frac{\sin^9(dx+c)}{48\cos(dx+c)^6} + \frac{\sin^9(dx+c)}{64\cos(dx+c)^4} - \frac{5\left(\sin^9(dx+c)\right)}{128\cos(dx+c)^2} - \frac{5\left(\sin^7(dx+c)\right)}{128} - \frac{7\left(\sin^5(dx+c)\right)}{128} - \frac{35\left(\sin^3(dx+c)\right)}{384} - \frac{35\sin(d}{128}\right.}{d}$
risch	$ax + \frac{ia\left(9765\,e^{15i(dx+c)} - 53760\,e^{14i(dx+c)} + 3185\,e^{13i(dx+c)} - 215040\,e^{12i(dx+c)} + 62965\,e^{11i(dx+c)} - 555520\,e^{10i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/8*sin(d*x+c)^9/cos(d*x+c)^8-1/48*sin(d*x+c)^9/cos(d*x+c)^6+1/64*s
in(d*x+c)^9/cos(d*x+c)^4-5/128*sin(d*x+c)^9/cos(d*x+c)^2-5/128*sin(d*x+c)^7
-7/128*sin(d*x+c)^5-35/384*sin(d*x+c)^3-35/128*sin(d*x+c)+35/128*ln(sec(d*x
+c)+tan(d*x+c)))+a*(1/7*tan(d*x+c)^7-1/5*tan(d*x+c)^5+1/3*tan(d*x+c)^3-tan(
d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 164, normalized size = 1.27

$$\frac{256\left(15\tan\left(dx+c\right)^{7}-21\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}+105\,dx+105\,c-105\tan\left(dx+c\right)\right)a+35\,a\left(\frac{2\left(279\sin(dx+c)^{7}-511\sin(dx+c)^{5}+385\sin(dx+c)^{3}-105\sin(dx+c)\right)}{\sin(dx+c)^{8}-4\sin(dx+c)^{6}+6\sin(dx+c)^{4}-4\sin(dx+c)^{2}+1}+105\log\left(\sin\left(dx+c\right)+1\right)-105\log\left(\sin\left(dx+c\right)-1\right)\right)}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/26880*(256*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 1
05*d*x + 105*c - 105*tan(d*x + c))*a + 35*a*(2*(279*sin(d*x + c)^7 - 511*si
n(d*x + c)^5 + 385*sin(d*x + c)^3 - 105*sin(d*x + c))/(sin(d*x + c)^8 - 4*s
in(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) + 105*log(sin(d*x
+ c) + 1) - 105*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.90, size = 156, normalized size = 1.21

$$\frac{26880\,adx\cos\left(dx+c\right)^{8}+3675\,a\cos\left(dx+c\right)^{8}\log\left(\sin\left(dx+c\right)+1\right)-3675\,a\cos\left(dx+c\right)^{8}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(22528\,a\cos\left(dx+c\right)^{7}+9765\,a\cos\left(dx+c\right)^{6}-15616\,a\cos\left(dx+c\right)^{5}-11410\,a\cos\left(dx+c\right)^{4}+8448\,a\cos\left(dx+c\right)^{3}+7000\,a\cos\left(dx+c\right)^{2}-1920\,a\cos\left(dx+c\right)-1680\,a\right)\sin\left(dx+c\right)}{26880\,d\cos\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*a*d*x*cos(d*x + c)^8 + 3675*a*cos(d*x + c)^8*log(sin(d*x + c
) + 1) - 3675*a*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(22528*a*cos(d*x
+ c)^7 + 9765*a*cos(d*x + c)^6 - 15616*a*cos(d*x + c)^5 - 11410*a*cos(d*x +
 c)^4 + 8448*a*cos(d*x + c)^3 + 7000*a*cos(d*x + c)^2 - 1920*a*cos(d*x + c)
 - 1680*a)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\tan^{8}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\tan^{8}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**8,x)
```

```
[Out] a*(Integral(tan(c + d*x)**8*sec(c + d*x), x) + Integral(tan(c + d*x)**8, x)
)
```

Giac [A]
time = 4.94, size = 174, normalized size = 1.35

$$\frac{13440\,(dx+c)a+3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-83825\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+321013\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-724649\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1078359\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-508683\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+140175\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-17115\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{8}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)*a + 3675*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 36
75*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9765*a*tan(1/2*d*x + 1/2*c)^15
 - 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 - 724
649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 - 508683*a*
tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 - 17115*a*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

Mupad [B]
time = 2.47, size = 242, normalized size = 1.88

$$a\,x-\frac{-\frac{93\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{2395\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}-\frac{45859\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960}+\frac{724649\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720}-\frac{359453\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2240}+\frac{24223\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{320}-\frac{1335\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{64}+\frac{163\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}-8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+28\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-56\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+70\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-56\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+28\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}+\frac{35\,a\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - ((163*a*tan(c/2 + (d*x)/2))/64 - (1335*a*tan(c/2 + (d*x)/2)^3)/64 + (
24223*a*tan(c/2 + (d*x)/2)^5)/320 - (359453*a*tan(c/2 + (d*x)/2)^7)/2240 +
(724649*a*tan(c/2 + (d*x)/2)^9)/6720 - (45859*a*tan(c/2 + (d*x)/2)^11)/960
+ (2395*a*tan(c/2 + (d*x)/2)^13)/192 - (93*a*tan(c/2 + (d*x)/2)^15)/64)/(d*
(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6
 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/
2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (35*a*atanh
(tan(c/2 + (d*x)/2)))/(64*d)
```

3.11 $\int (a + a\sec(c + dx)) \tan^6(c + dx)\, dx$

Optimal. Leaf size=102

$$-ax-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a$$

```
[Out] -a*x-5/16*a*arctanh(sin(d*x+c))/d+1/16*(16*a+5*a*sec(d*x+c))*tan(d*x+c)/d-1
/24*(8*a+5*a*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*a*sec(d*x+c))*tan(d*x+c
)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3966, 3855}

$$-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{24d}+\frac{\tan(c+dx)(5a\sec(c+dx)+16a)}{16d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*a*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx &= \frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}-\frac{1}{6}\int(6a+5a\sec(c+dx))\tan \\ &= -\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5a\sec(c+dx))\tan^5(}{30d} \\ &= \frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3(c}{24d} \\ &= -ax+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))}{24d} \\ &= -ax-\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(16a+5a\sec(c+dx))\tan(c+d}{16d}\end{aligned}$$

Mathematica [A]
time = 1.30, size = 95, normalized size = 0.93

$$-\frac{a\left(240\text{ArcTan}(\tan(c+dx))+75\tanh^{-1}(\sin(c+dx))-\frac{1}{8}(295+1168\cos(c+dx)+140\cos(2(c+dx))+568\cos(3(c+dx))+165\cos(4(c+dx))+184\cos(5(c+dx)))\sec^5(c+dx)\tan(c+dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -1/240*(a*(240*ArcTan[Tan[c + d*x]] + 75*ArcTanh[Sin[c + d*x]] - ((295 + 11
68*Cos[c + d*x] + 140*Cos[2*(c + d*x)] + 568*Cos[3*(c + d*x)] + 165*Cos[4*(
c + d*x)] + 184*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Tan[c + d*x])/8))/d
```

Maple [A]
time = 0.10, size = 143, normalized size = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^7(dx+c)}{24\cos(dx+c)^4}+\frac{\sin^7(dx+c)}{16\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{16}+\frac{5\left(\sin^3(dx+c)\right)}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+}{d}$
default	$\frac{a\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^7(dx+c)}{24\cos(dx+c)^4}+\frac{\sin^7(dx+c)}{16\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{16}+\frac{5\left(\sin^3(dx+c)\right)}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+}{d}$
risch	$-ax-\frac{ia\left(165\,e^{11i(dx+c)}-720\,e^{10i(dx+c)}-25\,e^{9i(dx+c)}-2160\,e^{8i(dx+c)}+450\,e^{7i(dx+c)}-3680\,e^{6i(dx+c)}-450\,e^{5i(dx+c}\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*s
in(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c
```

```
)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(
d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 134, normalized size = 1.31

$$\frac{32\left(3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a-5\,a\left(\frac{2\left(33\,\sin\left(dx+c\right)^{5}-40\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{6}-3\,\sin\left(dx+c\right)^{4}+3\,\sin\left(dx+c\right)^{2}-1}+15\,\log\left(\sin\left(dx+c\right)+1\right)-15\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*a*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.84, size = 134, normalized size = 1.31

$$-\frac{480\,adx\cos\left(dx+c\right)^{6}+75\,a\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-75\,a\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(368\,a\cos\left(dx+c\right)^{5}+165\,a\cos\left(dx+c\right)^{4}-176\,a\cos\left(dx+c\right)^{3}-130\,a\cos\left(dx+c\right)^{2}+48\,a\cos\left(dx+c\right)+40\,a\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*a*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*a*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*a*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*a*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*a)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\tan^{6}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] a*(Integral(tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**6, x)
)
```

Giac [A]
time = 2.36, size = 146, normalized size = 1.43

$$-\frac{240\left(dx+c\right)a+75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1095\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+3138\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-5118\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1945\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*a*tan(1/2*d*x + 1/2*c)^11 - 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 - 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 - 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 2.38, size = 188, normalized size = 1.84

$$\frac{-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{73\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}-\frac{523\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{853\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-\frac{389\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{21\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-a\,x-\frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] ((21*a*tan(c/2 + (d*x)/2))/8 - (389*a*tan(c/2 + (d*x)/2)^3)/24 + (853*a*tan
(c/2 + (d*x)/2)^5)/20 - (523*a*tan(c/2 + (d*x)/2)^7)/20 + (73*a*tan(c/2 + (
d*x)/2)^9)/8 - (11*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(15*tan(c/2 + (d*x)/2)^4
- 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^
8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - a*x - (5*a*atan
h(tan(c/2 + (d*x)/2)))/(8*d)
```

3.12 $\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=73

$$ax+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d}$$

```
[Out] a*x+3/8*a*arctanh(sin(d*x+c))/d-1/8*(8*a+3*a*sec(d*x+c))*tan(d*x+c)/d+1/12*
(4*a+3*a*sec(d*x+c))*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3966, 3855}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}-\frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{8d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*a*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx &= \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d}-\frac{1}{4}\int (4a+3a\sec(c+dx))\tan^2\\ &= -\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3a\sec(c+dx))\tan^3(c\\ &= ax-\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3a\sec(c+dx))\tan^3}{12d}\\ &= ax+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d}+\end{aligned}$$

Mathematica [A]
time = 0.44, size = 75, normalized size = 1.03

$$\frac{a\left(24\text{ArcTan}(\tan(c+dx))+9\tanh^{-1}(\sin(c+dx))-\frac{1}{2}(3+32\cos(c+dx)+15\cos(2(c+dx))+16\cos(3(c+dx)))\sec^3(c+dx)\tan(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(24*ArcTan[Tan[c + d*x]] + 9*ArcTanh[Sin[c + d*x]] - ((3 + 32*Cos[c + d*
x] + 15*Cos[2*(c + d*x)] + 16*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan[c + d*x]
)/2))/(24*d)
```

Maple [A]
time = 0.10, size = 104, normalized size = 1.42

method	result
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+c\right.}{d}$
default	$\frac{a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+c\right.}{d}$
risch	$ax+\frac{ia\left(15\,e^{7i(dx+c)}-48\,e^{6i(dx+c)}-9\,e^{5i(dx+c)}-96\,e^{4i(dx+c)}+9\,e^{3i(dx+c)}-80\,e^{2i(dx+c)}-15\,e^{i(dx+c)}-32\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3a\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin
(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/3*tan(d*x+c)^3
-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 102, normalized size = 1.40

$$\frac{16\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a+3\,a\left(\frac{2\left(5\,\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*a*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.21, size = 112, normalized size = 1.53

$$\frac{48\,adx\cos\left(dx+c\right)^4+9\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)^2-8\,a\cos\left(dx+c\right)-6\,a\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*a*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\tan^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int\tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] a*(Integral(tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**4, x)
)
```

Giac [A]
time = 1.06, size = 118, normalized size = 1.62

$$\frac{24\left(dx+c\right)a+9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-71\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+137\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a + 9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 71*a*tan(1/2
*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 - 33*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 1.85, size = 134, normalized size = 1.84

$$a\,x-\frac{-\frac{5\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{71\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{137\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{11\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{3\,a\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - ((11*a*tan(c/2 + (d*x)/2))/4 - (137*a*tan(c/2 + (d*x)/2)^3)/12 + (71*
a*tan(c/2 + (d*x)/2)^5)/12 - (5*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.13 $\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$-ax-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] -a*x-1/2*a*arctanh(sin(d*x+c))/d+1/2*(2*a+a*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3966, 3855}

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + a*Sec[c + d*x])*Tan[c + 
d*x])/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx &= \frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}\int (2a+a\sec(c+dx))\,dx\\ &= -ax+\frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}a\int \sec(c+dx)\,dx\\ &= -ax-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.33

$$-\frac{a\text{ArcTan}(\tan(c+dx))}{d}-\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.07, size = 67, normalized size = 1.49

method	result	size
derivativedivides	$\frac{a\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
default	$\frac{a\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
risch	$-ax-\frac{ia\left(e^{3i(dx+c)}-2\,e^{2i(dx+c)}-e^{i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(
d*x+c)))+a*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 65, normalized size = 1.44

$$-\frac{4\,(dx+c-\tan(dx+c))a+a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.
time = 2.65, size = 87, normalized size = 1.93

$$-\frac{4\,adx\cos(dx+c)^2+a\cos(dx+c)^2\log(\sin(dx+c)+1)-a\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\,(2\,a\cos(dx+c)+a)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + a)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \tan^2(c+dx)\sec(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] a*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**2, x)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(41) = 82.
time = 0.62, size = 88, normalized size = 1.96

$$-\frac{2(dx+c)a + a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.14, size = 80, normalized size = 1.78

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-a\,x-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) - a*x - (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.14 $\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$-ax-\frac{\cot(c+dx)(a+a\sec(c+dx))}{d}$$

```
[Out] -a*x-cot(d*x+c)*(a+a*sec(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot(c+dx)(a\sec(c+dx)+a)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]*(a + a*Sec[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot(c+dx)(a+a\sec(c+dx))}{d}-\int a\,dx\\ &= -ax-\frac{\cot(c+dx)(a+a\sec(c+dx))}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 43, normalized size = 1.65

$$-\frac{a\csc(c+dx)}{d}-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

Maple [A]
time = 0.07, size = 35, normalized size = 1.35

method	result	size
risch	$-ax-\frac{2ia}{d\left(e^{i(dx+c)}-1\right)}$	26
derivativedivides	$\frac{-\frac{a}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35
default	$\frac{-\frac{a}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a/sin(d*x+c)+a*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 31, normalized size = 1.19

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a + a/sin(d*x + c))/d
```

Fricas [A]
time = 2.39, size = 33, normalized size = 1.27

$$-\frac{adx\sin(dx+c)+a\cos(dx+c)+a}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + a)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^2(c+dx)\sec(c+dx)\,dx+\int\cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**2, x)
)
```

Giac [A]
time = 0.47, size = 26, normalized size = 1.00

$$-\frac{(dx+c)a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + a/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 1.07, size = 19, normalized size = 0.73

$$-\frac{a\left(\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(cot(c/2 + (d*x)/2) + d*x))/d
```

3.15 $\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=55

$$ax-\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}$$

```
[Out] a*x-1/3*cot(d*x+c)^3*(a+a*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*a*sec(d*x+c))
/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}+\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{3d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + a*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
a*Sec[c + d*x]))/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{1}{3}\int \cot^2(c+dx)(-3a-2a\sec($$
$$= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx)}{3d}$$
$$= ax-\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 62, normalized size = 1.13

$$\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

Maple [A]
time = 0.10, size = 86, normalized size = 1.56

method	result	size
risch	$ax+\frac{2ia\left(3\,e^{3i(dx+c)}-5\,e^{i(dx+c)}+4\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+1\right)}$	62
derivativedivides	$\frac{a\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	86
default	$\frac{a\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+c
os(d*x+c)^2)*sin(d*x+c))+a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.48, size = 59, normalized size = 1.07

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+\frac{\left(3\,\sin(dx+c)^2-1\right)a}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*a/sin(d*x + c)^3)/d
```

Fricas [A]
time = 2.91, size = 72, normalized size = 1.31

$$\frac{4\,a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+3\left(adx\cos\left(dx+c\right)-adx\right)\sin\left(dx+c\right)-2\,a}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^2 - a*cos(d*x + c) + 3*(a*d*x*cos(d*x + c) - a*d*x)*s
in(d*x + c) - 2*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cot^4(c+dx)\sec(c+dx)\,dx + \int \cot^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**4, x)
)
```

Giac [A]
time = 0.49, size = 56, normalized size = 1.02

$$\frac{12\,(dx+c)a - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(d*x + c)*a - 3*a*tan(1/2*d*x + 1/2*c) + (12*a*tan(1/2*d*x + 1/2*c
)^2 - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 1.20, size = 53, normalized size = 0.96

$$a\,x - \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d} - \frac{\frac{a}{12} - a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - (a*tan(c/2 + (d*x)/2))/(4*d) - (a/12 - a*tan(c/2 + (d*x)/2)^2)/(d*tan
(c/2 + (d*x)/2)^3)
```

3.16 $\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$-ax-\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8a\sec(c+d\ldots}{15d}$$

```
[Out] -a*x-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*a*sec(d*x
+c))/d-1/15*cot(d*x+c)*(15*a+8*a*sec(d*x+c))/d
```

Rubi [A]

time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{15d}-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{15d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + a*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*a*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*a*Sec[c + d*x]))/(15
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{1}{5}\int \cot^4(c+dx)(-5a-4a\,\text{se}\ldots \\
&= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+a\ldots}{15d} \\
&= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+a\ldots}{15d} \\
&= -ax-\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\,\text{sec}\ldots}{15d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 79, normalized size = 0.94

$$-\frac{a\csc(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

Maple [A]
time = 0.09, size = 129, normalized size = 1.54

method	result
risch	$-ax-\frac{2ia\left(15\,e^{7i(dx+c)}+15\,e^{6i(dx+c)}-65\,e^{5i(dx+c)}+25\,e^{4i(dx+c)}+73\,e^{3i(dx+c)}-31\,e^{2i(dx+c)}-31\,e^{i(dx+c)}+23\right)}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{i(dx+c)}+1\right)^3}$
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx\right.}{3}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx\right.}{3}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/s
in(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+
a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 79, normalized size = 0.94

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+\frac{\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a}{\sin(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a/sin(d*x + c)^5)/d
```

Fricas [A]
time = 2.69, size = 139, normalized size = 1.65

$$-\frac{23\,a\cos\left(dx+c\right)^4-8\,a\cos\left(dx+c\right)^3-27\,a\cos\left(dx+c\right)^2+7\,a\cos\left(dx+c\right)+15\left(adx\cos\left(dx+c\right)^3-adx\cos\left(dx+c\right)^2-adx\cos\left(dx+c\right)+adx\right)\sin\left(dx+c\right)+8\,a}{15\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(23*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^3 - 27*a*cos(d*x + c)^2 + 7*a
*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^3 - a*d*x*cos(d*x + c)^2 - a*d*x*cos
(d*x + c) + a*d*x)*sin(d*x + c) + 8*a)/((d*cos(d*x + c)^3 - d*cos(d*x + c)^
2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^6\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**6, x)
)
```

Giac [A]
time = 0.49, size = 83, normalized size = 0.99

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\left(dx+c\right)a-90\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 + 240*(d*x + c)*a - 90*a*tan(1/2*d*x + 1
/2*c) + 3*(80*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/t
an(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 1.45, size = 156, normalized size = 1.86

$$-\frac{a\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-90\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(c+dx\right)\right)}{240\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(3*cos(c/2 + (d*x)/2)^8 + 5*sin(c/2 + (d*x)/2)^8 - 90*cos(c/2 + (d*x)/2
)^2*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 -
30*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 240*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^5*(c + d*x)))/(240*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2
)^5)
```

3.17 $\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$ax-\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}+\frac{\cot(c+dx)(35a+16a\sec(c+dx$$

```
[Out] a*x-1/7*cot(d*x+c)^7*(a+a*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*a*sec(d*x+
c))/d+1/35*cot(d*x+c)*(35*a+16*a*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+24*
a*sec(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{35d}-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{105d}+\frac{\cot(c+dx)(16a\sec(c+dx)+35a)}{35d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + a*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*a*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*a*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*a*Sec[c + d*x]))/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{1}{7}\int \cot^6(c+dx)(-7a-6a\sec($$
$$= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx}{35d}$$
$$= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx}{35d}$$
$$= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+dx}{35d}$$
$$= ax - \frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6a\sec(c+}{35d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 92, normalized size = 0.83

$$\frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d} - \frac{a\cot^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\tan^2(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

Maple [A]
time = 0.12, size = 162, normalized size = 1.46

method	result
risch	$ax + \frac{2ia\left(105\,e^{11i(dx+c)}+210\,e^{10i(dx+c)}-735\,e^{9i(dx+c)}+1638\,e^{7i(dx+c)}-196\,e^{6i(dx+c)}-1882\,e^{5i(dx+c)}+880\,e^{4i(dx+c)}+\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/
sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/7*cot(d*x+c)^7+1/5*c
ot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.47, size = 100, normalized size = 0.90

$$\frac{\left(105\,dx + 105\,c + \frac{105\,\tan\left(dx+c\right)^{6} - 35\,\tan\left(dx+c\right)^{4} + 21\,\tan\left(dx+c\right)^{2} - 15}{\tan\left(dx+c\right)^{7}}\right)a + \frac{3\left(35\,\sin\left(dx+c\right)^{6} - 35\,\sin\left(dx+c\right)^{4} + 21\,\sin\left(dx+c\right)^{2} - 5\right)a}{\sin\left(dx+c\right)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*a/sin(d*x + c)^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(103) = 206.
time = 2.56, size = 210, normalized size = 1.89

$$\frac{176\,a\cos\left(dx+c\right)^{6} - 71\,a\cos\left(dx+c\right)^{5} - 335\,a\cos\left(dx+c\right)^{4} + 125\,a\cos\left(dx+c\right)^{3} + 225\,a\cos\left(dx+c\right)^{2} - 57\,a\cos\left(dx+c\right) + 105\left(adx\cos\left(dx+c\right)^{5} - adx\cos\left(dx+c\right)^{4} - 2\,adx\cos\left(dx+c\right)^{3} + 2\,adx\cos\left(dx+c\right)^{2} + adx\cos\left(dx+c\right) - adx\right)\sin\left(dx+c\right) - 48\,a}{105\left(d\cos\left(dx+c\right)^{5} - d\cos\left(dx+c\right)^{4} - 2\,d\cos\left(dx+c\right)^{3} + 2\,d\cos\left(dx+c\right)^{2} + d\cos\left(dx+c\right) - d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^6 - 71*a*cos(d*x + c)^5 - 335*a*cos(d*x + c)^4 +
125*a*cos(d*x + c)^3 + 225*a*cos(d*x + c)^2 - 57*a*cos(d*x + c) + 105*(a*d*
x*cos(d*x + c)^5 - a*d*x*cos(d*x + c)^4 - 2*a*d*x*cos(d*x + c)^3 + 2*a*d*x*
cos(d*x + c)^2 + a*d*x*cos(d*x + c) - a*d*x)*sin(d*x + c) - 48*a)/((d*cos(d
*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*
cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cot^{8}\left(c + dx\right)\sec\left(c + dx\right)\,dx + \int\cot^{8}\left(c + dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**8*sec(c + d*x), x) + Integral(cot(c + d*x)**8, x)
)
```

Giac [A]
time = 0.51, size = 113, normalized size = 1.02

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6720\,(dx+c)a+3045\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1015\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+168\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
(d*x + c)*a + 3045*a*tan(1/2*d*x + 1/2*c) - (6720*a*tan(1/2*d*x + 1/2*c)^6
- 1015*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 - 15*a)/tan(
1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 1.98, size = 204, normalized size = 1.84

$$-\frac{a\left(15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+3045\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+1015\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-168\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}(c+d\,x)\right)}{6720\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(15*cos(c/2 + (d*x)/2)^12 + 21*sin(c/2 + (d*x)/2)^12 - 280*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 3045*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1015*cos(c/2 + (d*x
)/2)^8*sin(c/2 + (d*x)/2)^4 - 168*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^
2 - 6720*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7*(c + d*x)))/(6720*d*cos(
c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7)
```

3.18 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$-ax-\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}-\frac{\cot^5(c+dx)(21a+16a\sec(c+}{105d}$$

```
[Out] -a*x-1/9*cot(d*x+c)^9*(a+a*sec(d*x+c))/d+1/63*cot(d*x+c)^7*(9*a+8*a*sec(d*x
+c))/d-1/105*cot(d*x+c)^5*(21*a+16*a*sec(d*x+c))/d+1/315*cot(d*x+c)^3*(105*
a+64*a*sec(d*x+c))/d-1/315*cot(d*x+c)*(315*a+128*a*sec(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{63d}-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{105d}+\frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{315d}-\frac{\cot(c+dx)(128a\sec(c+dx)+315a)}{315d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^9*(a + a*Sec[c + d*x]))/(9*d) + (Cot[c + d*x]^7*(9*a
 + 8*a*Sec[c + d*x]))/(63*d) - (Cot[c + d*x]^5*(21*a + 16*a*Sec[c + d*x]))/
(105*d) + (Cot[c + d*x]^3*(105*a + 64*a*Sec[c + d*x]))/(315*d) - (Cot[c + d
*x]*(315*a + 128*a*Sec[c + d*x]))/(315*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{1}{9}\int \cot^8(c+dx)(-9a-8a\sec$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+d}{63d}$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+d}{63d}$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+d}{63d}$$
$$= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+d}{63d}$$
$$= -ax - \frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(}{63d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 111, normalized size = 0.79

$$-\frac{a\csc(c+dx)}{d} + \frac{4a\csc^3(c+dx)}{3d} - \frac{6a\csc^5(c+dx)}{5d} + \frac{4a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{9d} - \frac{a\cot^9(c+dx)\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\tan^2(c+dx)\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (4*a*Csc[c + d*x]^3)/(3*d) - (6*a*Csc[c + d*x]^5)/(
5*d) + (4*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Cot[c + d
*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

Maple [A]
time = 0.16, size = 205, normalized size = 1.46

method	result
derivativedivides	$a\left(-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)^9}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^7}-\frac{\cos^{10}(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^3}-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(\right.}{3}\right.}{9}\right.$
default	$a\left(-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)^9}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^7}-\frac{\cos^{10}(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^{10}(dx+c)}{63\sin(dx+c)^3}-\frac{\cos^{10}(dx+c)}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(\right.}{3}\right.}{9}\right.$
risch	$-ax-\frac{2ia\left(315\,e^{15i(dx+c)}+945\,e^{14i(dx+c)}-3045\,e^{13i(dx+c)}-1155\,e^{12i(dx+c)}+10143\,e^{11i(dx+c)}+1869\,e^{10i(dx+c)}-1899\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c)^7*cos(d*x+c)^10-1/1
05/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*x+c)^10-1/9/sin(d*x+c
)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^
4+64/35*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5
*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 119, normalized size = 0.85

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan\left(dx+c\right)^{8}-105\,\tan\left(dx+c\right)^{6}+63\,\tan\left(dx+c\right)^{4}-45\,\tan\left(dx+c\right)^{2}+35}{\tan\left(dx+c\right)^{9}}\right)a+\frac{\left(315\,\sin\left(dx+c\right)^{8}-420\,\sin\left(dx+c\right)^{6}+378\,\sin\left(dx+c\right)^{4}-180\,\sin\left(dx+c\right)^{2}+35\right)a}{\sin\left(dx+c\right)^{9}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a + (315*sin(d*x + c
)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35)*a/
sin(d*x + c)^9)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(130) = 260.
time = 3.35, size = 279, normalized size = 1.99

$$-\frac{563\,a\cos\left(dx+c\right)^{8}-248\,a\cos\left(dx+c\right)^{7}-1498\,a\cos\left(dx+c\right)^{6}+658\,a\cos\left(dx+c\right)^{5}+1610\,a\cos\left(dx+c\right)^{4}-602\,a\cos\left(dx+c\right)^{3}-763\,a\cos\left(dx+c\right)^{2}+187\,a\cos\left(dx+c\right)+315\,\left(adx\cos\left(dx+c\right)^{7}-adx\cos\left(dx+c\right)^{6}-3\,adx\cos\left(dx+c\right)^{5}+3\,adx\cos\left(dx+c\right)^{4}+3\,adx\cos\left(dx+c\right)^{3}-3\,adx\cos\left(dx+c\right)^{2}-adx\cos\left(dx+c\right)+adx\right)\sin\left(dx+c\right)+128\,a}{315\,\left(d\cos\left(dx+c\right)^{7}-d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{5}+3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{3}-3\,d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(563*a*cos(d*x + c)^8 - 248*a*cos(d*x + c)^7 - 1498*a*cos(d*x + c)^6
 + 658*a*cos(d*x + c)^5 + 1610*a*cos(d*x + c)^4 - 602*a*cos(d*x + c)^3 - 76
3*a*cos(d*x + c)^2 + 187*a*cos(d*x + c) + 315*(a*d*x*cos(d*x + c)^7 - a*d*x
*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^5 + 3*a*d*x*cos(d*x + c)^4 + 3*a*d*x
*cos(d*x + c)^3 - 3*a*d*x*cos(d*x + c)^2 - a*d*x*cos(d*x + c) + a*d*x)*sin(
d*x + c) + 128*a)/((d*cos(d*x + c)^7 - d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^
5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 - 3*d*cos(d*x + c)^2 - d*cos(d*
x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 140, normalized size = 1.00

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80640\,(dx+c)a-40950\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{80640\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-13650\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2898\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-450\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 + 80640*(d*x + c)*a - 40950*a*tan(1/2*d*x + 1/2*c
) + (80640*a*tan(1/2*d*x + 1/2*c)^8 - 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898
*a*tan(1/2*d*x + 1/2*c)^4 - 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*
x + 1/2*c)^9)/d
```

Mupad [B]
time = 3.18, size = 252, normalized size = 1.80

$$-\frac{a\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+45\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+4830\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-40950\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+80640\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-13650\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+2898\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-450\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+80640\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}(c+dx)\right)}{80640\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(35*cos(c/2 + (d*x)/2)^16 + 45*sin(c/2 + (d*x)/2)^16 - 630*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 4830*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^12 - 40950*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 80640*cos(c/2 +
(d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 13650*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^6 + 2898*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 450*cos(c/2 + (
d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 80640*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^9*(c + d*x)))/(80640*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9)
```

3.19 $\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx$

Optimal. Leaf size=192

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{3a^2\sec^2(c+dx)}{2d}-\frac{8a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{2d}+\frac{12a^2\sec^5(c+$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-3/2*a^2*sec(d*x+c)^2/d-8/3*a^2*sec
(d*x+c)^3/d+1/2*a^2*sec(d*x+c)^4/d+12/5*a^2*sec(d*x+c)^5/d+1/3*a^2*sec(d*x+
c)^6/d-8/7*a^2*sec(d*x+c)^7/d-3/8*a^2*sec(d*x+c)^8/d+2/9*a^2*sec(d*x+c)^9/d
+1/10*a^2*sec(d*x+c)^10/d
```

Rubi [A]

time = 0.07, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{a^2\sec^{10}(c+dx)}{10d}+\frac{2a^2\sec^9(c+dx)}{9d}-\frac{3a^2\sec^8(c+dx)}{8d}-\frac{8a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^6(c+dx)}{3d}+\frac{12a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^4(c+dx)}{2d}-\frac{8a^2\sec^3(c+dx)}{3d}-\frac{3a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (3*a^2*Sec[c + d*x]
^2)/(2*d) - (8*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^4)/(2*d) + (12
*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(3*d) - (8*a^2*Sec[c + d*
x]^7)/(7*d) - (3*a^2*Sec[c + d*x]^8)/(8*d) + (2*a^2*Sec[c + d*x]^9)/(9*d) +
 (a^2*Sec[c + d*x]^10)/(10*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^6}{x^{11}}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{11}} + \frac{2a^{10}}{x^{10}} - \frac{3a^{10}}{x^9} - \frac{8a^{10}}{x^8} + \frac{2a^{10}}{x^7} + \frac{12a^{10}}{x^6} + \frac{2a^{10}}{x^5} - \frac{8a^{10}}{x^4} - \cdots\right.\right.}{a^8 d}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d} + \frac{2a^2\sec(c+dx)}{d} - \frac{3a^2\sec^2(c+dx)}{2d} - \frac{8a^2 \mathrm{s}\cdots}{\cdots}$$

Mathematica [A]
time = 0.50, size = 140, normalized size = 0.73

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)(2520\log(\cos(c+dx)) - 5040\sec(c+dx) + 3780\sec^2(c+dx) + 6720\sec^3(c+dx) - 1260\sec^4(c+dx) - 6048\sec^5(c+dx) - 840\sec^6(c+dx) + 2880\sec^7(c+dx) + 945\sec^8(c+dx) - 560\sec^9(c+dx) - 252\sec^{10}(c+dx))}{10080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -1/10080*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(2520*Log[Cos[c + d*x
]] - 5040*Sec[c + d*x] + 3780*Sec[c + d*x]^2 + 6720*Sec[c + d*x]^3 - 1260*S
ec[c + d*x]^4 - 6048*Sec[c + d*x]^5 - 840*Sec[c + d*x]^6 + 2880*Sec[c + d*x
]^7 + 945*Sec[c + d*x]^8 - 560*Sec[c + d*x]^9 - 252*Sec[c + d*x]^10))/d
```

Maple [A]
time = 0.18, size = 225, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{a^2\left(\sin^{10}(dx+c)\right)}{10\cos(dx+c)^{10}} + 2a^2\left(\frac{\sin^{10}(dx+c)}{9\cos(dx+c)^9} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^7} + \frac{\sin^{10}(dx+c)}{105\cos(dx+c)^5} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^3} + \frac{\sin^{10}(dx+c)}{9\cos(dx+c)} + \frac{\left(\frac{128}{35} + \sin^8(dx+c) + \frac{8(\sin \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{a^2\left(\sin^{10}(dx+c)\right)}{10\cos(dx+c)^{10}} + 2a^2\left(\frac{\sin^{10}(dx+c)}{9\cos(dx+c)^9} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^7} + \frac{\sin^{10}(dx+c)}{105\cos(dx+c)^5} - \frac{\sin^{10}(dx+c)}{63\cos(dx+c)^3} + \frac{\sin^{10}(dx+c)}{9\cos(dx+c)} + \frac{\left(\frac{128}{35} + \sin^8(dx+c) + \frac{8(\sin \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	$ia^2x + \frac{2ia^2c}{d} + \frac{2a^2\left(630\,\mathrm{e}^{19i(dx+c)} - 945\,\mathrm{e}^{18i(dx+c)} + 2310\,\mathrm{e}^{17i(dx+c)} - 6300\,\mathrm{e}^{16i(dx+c)} + 11256\,\mathrm{e}^{15i(dx+c)} - 15540\,\mathrm{e}^{14i(\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/10*a^2*sin(d*x+c)^10/cos(d*x+c)^10+2*a^2*(1/9*sin(d*x+c)^10/cos(d*x+
c)^9-1/63*sin(d*x+c)^10/cos(d*x+c)^7+1/105*sin(d*x+c)^10/cos(d*x+c)^5-1/63*
```

```
sin(d*x+c)^10/cos(d*x+c)^3+1/9*sin(d*x+c)^10/cos(d*x+c)+1/9*(128/35+sin(d*x
+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+a
^2*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(
cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 149, normalized size = 0.78

$$-\frac{2520\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,a^2\cos(dx+c)^9-3780\,a^2\cos(dx+c)^8-6720\,a^2\cos(dx+c)^7+1260\,a^2\cos(dx+c)^6+6048\,a^2\cos(dx+c)^5+840\,a^2\cos(dx+c)^4-2880\,a^2\cos(dx+c)^3-945\,a^2\cos(dx+c)^2+560\,a^2\cos(dx+c)+252\,a^2}{\cos(dx+c)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a^2*cos(d*x + c)^9 - 3780*a^2*c
os(d*x + c)^8 - 6720*a^2*cos(d*x + c)^7 + 1260*a^2*cos(d*x + c)^6 + 6048*a^
2*cos(d*x + c)^5 + 840*a^2*cos(d*x + c)^4 - 2880*a^2*cos(d*x + c)^3 - 945*a
^2*cos(d*x + c)^2 + 560*a^2*cos(d*x + c) + 252*a^2)/cos(d*x + c)^10)/d
```

Fricas [A]
time = 4.48, size = 156, normalized size = 0.81

$$-\frac{2520\,a^2\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,a^2\cos\left(dx+c\right)^9+3780\,a^2\cos\left(dx+c\right)^8+6720\,a^2\cos\left(dx+c\right)^7-1260\,a^2\cos\left(dx+c\right)^6-6048\,a^2\cos\left(dx+c\right)^5-840\,a^2\cos\left(dx+c\right)^4+2880\,a^2\cos\left(dx+c\right)^3+945\,a^2\cos\left(dx+c\right)^2-560\,a^2\cos\left(dx+c\right)-252\,a^2}{2520\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a^2*cos(d*x + c
)^9 + 3780*a^2*cos(d*x + c)^8 + 6720*a^2*cos(d*x + c)^7 - 1260*a^2*cos(d*x
+ c)^6 - 6048*a^2*cos(d*x + c)^5 - 840*a^2*cos(d*x + c)^4 + 2880*a^2*cos(d*
x + c)^3 + 945*a^2*cos(d*x + c)^2 - 560*a^2*cos(d*x + c) - 252*a^2)/(d*cos(
d*x + c)^10)
```

Sympy [A]
time = 3.24, size = 314, normalized size = 1.64

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^8\left(c+dx\right)\sec^2\left(c+dx\right)}{10d}+\frac{2a^2\tan^8\left(c+dx\right)\sec\left(c+dx\right)}{9d}+\frac{a^2\tan^8\left(c+dx\right)}{8d}-\frac{a^2\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)}{10d}-\frac{16a^2\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{63d}-\frac{a^2\tan^6\left(c+dx\right)}{6d}+\frac{a^2\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{10d}+\frac{32a^2\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{105d}+\frac{a^2\tan^4\left(c+dx\right)}{4d}-\frac{a^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{10d}-\frac{128a^2\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{315d}-\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{a^2\sec^2\left(c+dx\right)}{10d}+\frac{256a^2\sec\left(c+dx\right)}{315d} & \text{for } d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)^2\tan^9\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8*sec(c
 + d*x)**2/(10*d) + 2*a**2*tan(c + d*x)**8*sec(c + d*x)/(9*d) + a**2*tan(c
+ d*x)**8/(8*d) - a**2*tan(c + d*x)**6*sec(c + d*x)**2/(10*d) - 16*a**2*tan
(c + d*x)**6*sec(c + d*x)/(63*d) - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c
+ d*x)**4*sec(c + d*x)**2/(10*d) + 32*a**2*tan(c + d*x)**4*sec(c + d*x)/(10
5*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(1
```

```
0*d) - 128*a**2*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a**2*tan(c + d*x)**2
/(2*d) + a**2*sec(c + d*x)**2/(10*d) + 256*a**2*sec(c + d*x)/(315*d), Ne(d,
 0)), (x*(a*sec(c) + a)**2*tan(c)**9, True))
```

Giac [A]
time = 6.40, size = 342, normalized size = 1.78

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11477\,a^2+\frac{119810\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{566865\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1605720\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3031770\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{2995020\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{2171610\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{1114200\,a^2(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{382545\,a^2(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}+\frac{78850\,a^2(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}+\frac{7381\,a^2(\cos(dx+c)-1)^{10}}{(\cos(dx+c)+1)^{10}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11477*a^2 + 1
19810*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566865*a^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1605720*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 3031770*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2995020*
a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2171610*a^2*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c)
 + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 78850*a^2*
(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 7381*a^2*(cos(d*x + c) - 1)^10/
(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^10)/d
```

Mupad [B]
time = 5.14, size = 308, normalized size = 1.60

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+\frac{272\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}-\frac{740\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{2252\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{5}-588\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{2000\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{7}-\frac{652\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{7}+\frac{1150\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{63}-\frac{512\,a^2}{315}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+45\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-120\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-252\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-120\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+45\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((1150*a^2*tan(c/2 + (d*x)/2)^2)/63
 - (652*a^2*tan(c/2 + (d*x)/2)^4)/7 + (2000*a^2*tan(c/2 + (d*x)/2)^6)/7 - 5
88*a^2*tan(c/2 + (d*x)/2)^8 + (2252*a^2*tan(c/2 + (d*x)/2)^10)/5 - (740*a^2
*tan(c/2 + (d*x)/2)^12)/3 + (272*a^2*tan(c/2 + (d*x)/2)^14)/3 - 20*a^2*tan(
c/2 + (d*x)/2)^16 + 2*a^2*tan(c/2 + (d*x)/2)^18 - (512*a^2)/315)/(d*(45*tan
(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 21
0*tan(c/2 + (d*x)/2)^8 - 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)
^12 - 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (
d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1))
```

3.20 $\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal. Leaf size=132

$$\frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{d}-\frac{6a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^6(c+dx)}{3d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+a^2*sec(d*x+c)^2/d+2*a^2*sec(d*x+c)
^3/d-6/5*a^2*sec(d*x+c)^5/d-1/3*a^2*sec(d*x+c)^6/d+2/7*a^2*sec(d*x+c)^7/d+1
/8*a^2*sec(d*x+c)^8/d
```

Rubi [A]
time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{a^2\sec^8(c+dx)}{8d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{a^2\sec^6(c+dx)}{3d}-\frac{6a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^3(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2)/d
 + (2*a^2*Sec[c + d*x]^3)/d - (6*a^2*Sec[c + d*x]^5)/(5*d) - (a^2*Sec[c + d
*x]^6)/(3*d) + (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^8)/(8*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^5}{x^9}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{2a^8}{x^8}-\frac{2a^8}{x^7}-\frac{6a^8}{x^6}+\frac{6a^8}{x^4}+\frac{2a^8}{x^3}-\frac{2a^8}{x^2}-\frac{a^8}{x}\right)dx, x, \cos\right.}{a^6 d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d} - \frac{2a^2\sec(c+dx)}{d} + \frac{a^2\sec^2(c+dx)}{d} + \frac{2a^2\sec^3}{d}$$

Mathematica [A]
time = 0.29, size = 110, normalized size = 0.83

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(840\log(\cos(c+dx))-1680\sec(c+dx)+840\sec^2(c+dx)+1680\sec^3(c+dx)-1008\sec^5(c+dx)-280\sec^6(c+dx)+240\sec^7(c+dx)+105\sec^8(c+dx)\right)}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(840*Log[Cos[c + d*x]] - 1680*
Sec[c + d*x] + 840*Sec[c + d*x]^2 + 1680*Sec[c + d*x]^3 - 1008*Sec[c + d*x]
^5 - 280*Sec[c + d*x]^6 + 240*Sec[c + d*x]^7 + 105*Sec[c + d*x]^8))/(3360*d
)
```

Maple [A]
time = 0.13, size = 185, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{a^2\left(\sin^8(dx+c)\right)}{8\cos(dx+c)^8}+2a^2\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(d\right.}{5}\right.}{7}\right.}{d}$
default	$\frac{\frac{a^2\left(\sin^8(dx+c)\right)}{8\cos(dx+c)^8}+2a^2\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(d\right.}{5}\right.}{7}\right.}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{4a^2\left(105\,e^{15i(dx+c)}-105\,e^{14i(dx+c)}+315\,e^{13i(dx+c)}-630\,e^{12i(dx+c)}+1113\,e^{11i(dx+c)}-1015\,e^{10i(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*a^2*sin(d*x+c)^8/cos(d*x+c)^8+2*a^2*(1/7*sin(d*x+c)^8/cos(d*x+c)^7
-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35*sin(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+
```

```
c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*c
os(d*x+c))+a^2*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2+ln(cos(d
*x+c))))
```

Maxima [A]
time = 0.27, size = 110, normalized size = 0.83

$$\frac{840\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{1680\,a^2\cos\left(dx+c\right)^7-840\,a^2\cos\left(dx+c\right)^6-1680\,a^2\cos\left(dx+c\right)^5+1008\,a^2\cos\left(dx+c\right)^3+280\,a^2\cos\left(dx+c\right)^2-240\,a^2\cos\left(dx+c\right)-105\,a^2}{\cos\left(dx+c\right)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a^2*cos(d*x + c)^7 - 840*a^2*cos(d
*x + c)^6 - 1680*a^2*cos(d*x + c)^5 + 1008*a^2*cos(d*x + c)^3 + 280*a^2*cos
(d*x + c)^2 - 240*a^2*cos(d*x + c) - 105*a^2)/cos(d*x + c)^8)/d
```

Fricas [A]
time = 3.22, size = 117, normalized size = 0.89

$$\frac{840\,a^2\cos\left(dx+c\right)^8\log\left(-\cos\left(dx+c\right)\right)-1680\,a^2\cos\left(dx+c\right)^7+840\,a^2\cos\left(dx+c\right)^6+1680\,a^2\cos\left(dx+c\right)^5-1008\,a^2\cos\left(dx+c\right)^3-280\,a^2\cos\left(dx+c\right)^2+240\,a^2\cos\left(dx+c\right)+105\,a^2}{840\,d\cos\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a^2*cos(d*x + c)^7
+ 840*a^2*cos(d*x + c)^6 + 1680*a^2*cos(d*x + c)^5 - 1008*a^2*cos(d*x + c)^
3 - 280*a^2*cos(d*x + c)^2 + 240*a^2*cos(d*x + c) + 105*a^2)/(d*cos(d*x + c
)^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(119) = 238$.
time = 1.73, size = 252, normalized size = 1.91

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{2a^2\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}+\frac{a^2\tan^6\left(c+dx\right)}{6d}-\frac{a^2\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}-\frac{12a^2\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{35d}-\frac{a^2\tan^4\left(c+dx\right)}{4d}+\frac{a^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{16a^2\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}-\frac{a^2\sec^2\left(c+dx\right)}{8d}-\frac{32a^2\sec\left(c+dx\right)}{35d} & \text{for}\: d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)^2\tan^7\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6*sec(
c + d*x)**2/(8*d) + 2*a**2*tan(c + d*x)**6*sec(c + d*x)/(7*d) + a**2*tan(c
+ d*x)**6/(6*d) - a**2*tan(c + d*x)**4*sec(c + d*x)**2/(8*d) - 12*a**2*tan(
c + d*x)**4*sec(c + d*x)/(35*d) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c +
 d*x)**2*sec(c + d*x)**2/(8*d) + 16*a**2*tan(c + d*x)**2*sec(c + d*x)/(35*d
) + a**2*tan(c + d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(8*d) - 32*a**2*sec(c
 + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(124) = 248$.
time = 3.66, size = 292, normalized size = 2.21

$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3819\,a^2+\frac{32232\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{120372\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{261464\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{258370\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{175448\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{77364\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{19944\,a^2(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{2283\,a^2(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3819*a^2 + 3223
2*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120372*a^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 261464*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 258370*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 19944*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2
283*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 1)^8)/d
```

Mupad [B]
time = 4.82, size = 249, normalized size = 1.89

$$\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-16\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+\frac{170\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{3}-\frac{352\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{3}+\frac{2386\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{15}-\frac{336\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{5}+\frac{582\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{35}-\frac{64\,a^2}{35}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((582*a^2*tan(c/2 + (d*x)/2)^2)/35 - (336*a^2*tan(c/2 + (d*x)/2)^4)/5 + (23
86*a^2*tan(c/2 + (d*x)/2)^6)/15 - (352*a^2*tan(c/2 + (d*x)/2)^8)/3 + (170*a
^2*tan(c/2 + (d*x)/2)^10)/3 - 16*a^2*tan(c/2 + (d*x)/2)^12 + 2*a^2*tan(c/2
+ (d*x)/2)^14 - (64*a^2)/35)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x
)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (
d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 +
 (d*x)/2)^16 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.21 $\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=120

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{2d}-\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^5(c+d}{5d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-1/2*a^2*sec(d*x+c)^2/d-4/3*a^2*sec
(d*x+c)^3/d-1/4*a^2*sec(d*x+c)^4/d+2/5*a^2*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c
)^6/d
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{a^2\sec^6(c+dx)}{6d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^4(c+dx)}{4d}-\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (a^2*Sec[c + d*x]^2
)/(2*d) - (4*a^2*Sec[c + d*x]^3)/(3*d) - (a^2*Sec[c + d*x]^4)/(4*d) + (2*a^
2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(6*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^4}{x^7}\,dx, x, \cos(c+dx)\right)}{a^4d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7}+\frac{2a^6}{x^6}-\frac{a^6}{x^5}-\frac{4a^6}{x^4}-\frac{a^6}{x^3}+\frac{2a^6}{x^2}+\frac{a^6}{x}\right)\,dx, x, \cos(c+dx\right.}{a^4d}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{2d}-\frac{4a^2\sec}{}$$

Mathematica [A]
time = 0.39, size = 125, normalized size = 1.04

$$\frac{a^2(312\cos(c+dx)-5(14-28\cos(3(c+dx))+6\cos(4(c+dx))-12\cos(5(c+dx))+30\log(\cos(c+dx))+18\cos(4(c+dx))\log(\cos(c+dx))+3\cos(6(c+dx))\log(\cos(c+dx))+9\cos(2(c+dx))(4+5\log(\cos(c+dx)))))\sec^6(c+dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (a^2*(312*Cos[c + d*x] - 5*(14 - 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)] -
 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Cos[c
 + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4 + 5
*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(480*d)
```

Maple [A]
time = 0.11, size = 149, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{a^2\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+2a^2\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a^2\left(\frac{\left(\tan^4\right.}{}\right.}{d}$
default	$\frac{\frac{a^2\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+2a^2\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a^2\left(\frac{\left(\tan^4\right.}{}\right.}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(30\,e^{11i(dx+c)}-15\,e^{10i(dx+c)}+70\,e^{9i(dx+c)}-90\,e^{8i(dx+c)}+156\,e^{7i(dx+c)}-70\,e^{6i(dx+c)}+156\,e^{5i(dx}\right.}{15d\left(e^{2i(dx+c)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*a^2*sin(d*x+c)^6/cos(d*x+c)^6+2*a^2*(1/5*sin(d*x+c)^6/cos(d*x+c)^5
-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*
```

```
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2
-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.26, size = 97, normalized size = 0.81

$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{120\,a^2\cos\left(dx+c\right)^5-30\,a^2\cos\left(dx+c\right)^4-80\,a^2\cos\left(dx+c\right)^3-15\,a^2\cos\left(dx+c\right)^2+24\,a^2\cos\left(dx+c\right)+10\,a^2}{\cos\left(dx+c\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x
+ c)^4 - 80*a^2*cos(d*x + c)^3 - 15*a^2*cos(d*x + c)^2 + 24*a^2*cos(d*x + c
) + 10*a^2)/cos(d*x + c)^6)/d
```

Fricas [A]
time = 3.05, size = 104, normalized size = 0.87

$$-\frac{60\,a^2\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-120\,a^2\cos\left(dx+c\right)^5+30\,a^2\cos\left(dx+c\right)^4+80\,a^2\cos\left(dx+c\right)^3+15\,a^2\cos\left(dx+c\right)^2-24\,a^2\cos\left(dx+c\right)-10\,a^2}{60\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a^2*cos(d*x + c)^5 +
30*a^2*cos(d*x + c)^4 + 80*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c)^2 - 24*
a^2*cos(d*x + c) - 10*a^2)/(d*cos(d*x + c)^6)
```

Sympy [A]
time = 0.79, size = 189, normalized size = 1.58

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{6d}+\frac{2a^2\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{5d}+\frac{a^2\tan^4\left(c+dx\right)}{4d}-\frac{a^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{6d}-\frac{8a^2\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{15d}-\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{a^2\sec^2\left(c+dx\right)}{6d}+\frac{16a^2\sec\left(c+dx\right)}{15d} & \text{for}\: d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)^2\tan^5\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**5,x)
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4*sec(c
 + d*x)**2/(6*d) + 2*a**2*tan(c + d*x)**4*sec(c + d*x)/(5*d) + a**2*tan(c +
 d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(6*d) - 8*a**2*tan(c
+ d*x)**2*sec(c + d*x)/(15*d) - a**2*tan(c + d*x)**2/(2*d) + a**2*sec(c + d
*x)**2/(6*d) + 16*a**2*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(c) + a)**2
*tan(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(110) = 220.
time = 1.93, size = 242, normalized size = 2.02

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{275\,a^2+\frac{1770\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4845\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4780\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1002\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{147\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (275*a^2 + 1770*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4845*a^2*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 4780*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925
*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

Mupad [B]
time = 5.05, size = 192, normalized size = 1.60

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-12\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{92\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}-44\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{74\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}-\frac{32\,a^2}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((74*a^2*tan(c/2 + (d*x)/2)^2)/5 -
44*a^2*tan(c/2 + (d*x)/2)^4 + (92*a^2*tan(c/2 + (d*x)/2)^6)/3 - 12*a^2*tan(
c/2 + (d*x)/2)^8 + 2*a^2*tan(c/2 + (d*x)/2)^10 - (32*a^2)/15)/(d*(15*tan(c/
2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(
c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.22 $\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+2/3*a^2*sec(d*x+c)^3/d+1/4*a^2*sec(
d*x+c)^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3964, 76}

$$\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]^3)
/(3*d) + (a^2*Sec[c + d*x]^4)/(4*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^3}{x^5}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}+\frac{2a^4}{x^4}-\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{a^2\log(\cos(c+dx))}{d} - \frac{2a^2\sec(c+dx)}{d} + \frac{2a^2\sec^3(c+dx)}{3d} + \frac{a^2\sec^4}{4}$$

Mathematica [A]
time = 0.18, size = 83, normalized size = 1.28

$$\frac{a^2(-20\cos(c+dx)+3(2-4\cos(3(c+dx))+3\log(\cos(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c+dx))))\sec^4(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(-20*Cos[c + d*x] + 3*(2 - 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] +
4*Cos[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))
*Sec[c + d*x]^4)/(24*d)
```

Maple [A]
time = 0.09, size = 109, normalized size = 1.68

method	result	siz
derivativedivides	$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+2a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	10
default	$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+2a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	10
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{4a^2\left(3\,e^{7i(dx+c)}+5\,e^{5i(dx+c)}-3\,e^{4i(dx+c)}+5\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}+\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^2*sin(d*x+c)^4/cos(d*x+c)^4+2*a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/2*tan(
d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.89

$$\frac{12\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{24\,a^2\cos(dx+c)^3-8\,a^2\cos(dx+c)-3\,a^2}{\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a^2*cos(d*x + c)^3 - 8*a^2*cos(d*x + c
) - 3*a^2)/cos(d*x + c)^4)/d
```

Fricas [A]
time = 2.77, size = 65, normalized size = 1.00

$$\frac{12\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-24\,a^2\cos\left(dx+c\right)^3+8\,a^2\cos\left(dx+c\right)+3\,a^2}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a^2*cos(d*x + c)^3 + 8*
a^2*cos(d*x + c) + 3*a^2)/(d*cos(d*x + c)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(58) = 116$.
time = 0.34, size = 126, normalized size = 1.94

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}+\frac{2a^2\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}-\frac{a^2\sec^2\left(c+dx\right)}{4d}-\frac{4a^2\sec\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\left(a\sec\left(c\right)+a\right)^2\tan^3\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2*sec(
c + d*x)**2/(4*d) + 2*a**2*tan(c + d*x)**2*sec(c + d*x)/(3*d) + a**2*tan(c
+ d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(4*d) - 4*a**2*sec(c + d*x)/(3*d), N
e(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(61) = 122$.
time = 0.90, size = 192, normalized size = 2.95

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{57\,a^2+\frac{252\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{124\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{25\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (57*a^2 + 252*a^2*(
```

```
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1)^4)/d
```

Mupad [B]
time = 3.87, size = 133, normalized size = 2.05

$$\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{38\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{8\,a^2}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,a^2\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((38*a^2*tan(c/2 + (d*x)/2)^2)/3 - 8*a^2*tan(c/2 + (d*x)/2)^4 + 2*a^2*tan(c
/2 + (d*x)/2)^6 - (8*a^2)/3)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (2*a^2*atanh(
tan(c/2 + (d*x)/2)^2))/d
```

3.23 $\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=48

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{2d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d+1/2*a^2*sec(d*x+c)^2/d
```

Rubi [A]

time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3964, 45}

$$\frac{a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+ax)^2}{x^3}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a^2}{x^2}+\frac{a^2}{x}\right)\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 1.06

$$-\frac{a^2(-1-4\cos(c+dx)+\log(\cos(c+dx))+\cos(2(c+dx))\log(\cos(c+dx)))\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/2*(a^2*(-1 - 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[C
os[c + d*x]])*Sec[c + d*x]^2)/d
```

Maple [A]
time = 0.06, size = 34, normalized size = 0.71

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\left(\sec^2(dx+c)\right)}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	34
default	$\frac{a^2\left(\frac{\left(\sec^2(dx+c)\right)}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	34
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(2\,\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(1/2*sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.90

$$-\frac{2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{4\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^2)/
d
```

Fricas [A]
time = 3.29, size = 52, normalized size = 1.08

$$-\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,a^2\cos\left(dx+c\right)-a^2}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a^2*cos(d*x + c) - a^2)/(
d*cos(d*x + c)^2)
```

Sympy [A]
time = 0.15, size = 60, normalized size = 1.25

$$\begin{cases} \frac{a^2 \log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{a^2 \sec^2\left(c+dx\right)}{2d} + \frac{2a^2 \sec\left(c+dx\right)}{d} & \text{for } d \neq 0 \\ x(a\sec\left(c\right)+a)^2 \tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*sec(c + d*x)**2/(2*d)
 + 2*a**2*sec(c + d*x)/d, Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(46) = 92.
time = 0.54, size = 142, normalized size = 2.96

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a^2+\frac{10\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a^2 + 10*a^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]
time = 1.21, size = 76, normalized size = 1.58

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2
)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.24 $\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=35

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] 2*a^2*ln(1-cos(d*x+c))/d-a^2*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 78}

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (a^2*Log[Cos[c + d*x]])/d
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\text{Subst}\left(\int \frac{a+ax}{x(a-ax)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.83

$$-\frac{a^2\left(\log(\cos(c+dx)) - 4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
[Out] -((a^2*(Log[Cos[c + d*x]] - 4*Log[Sin[(c + d*x)/2]]))/d)
```

Maple [A]
time = 0.10, size = 28, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{a^2(-2\ln(-1+\sec(dx+c))+\ln(\sec(dx+c)))}{d}$	28
default	$-\frac{a^2(-2\ln(-1+\sec(dx+c))+\ln(\sec(dx+c)))}{d}$	28
risch	$-ia^2x-\frac{2ia^2c}{d}+\frac{4a^2\ln(e^{i(dx+c)}-1)}{d}-\frac{a^2\ln(e^{2i(dx+c)}+1)}{d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/d*a^2*(-2*ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.89

$$\frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-a^2\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - a^2*log(cos(d*x + c)))/d
```

Fricas [A]
time = 4.27, size = 35, normalized size = 1.00

$$-\frac{a^2 \log\left(-\cos\left(dx+c\right)\right) - 2\,a^2 \log\left(-\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(a^2*log(-cos(d*x + c)) - 2*a^2*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cot\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x)*sec(
c + d*x)**2, x) + Integral(cot(c + d*x), x))
```

Giac [A]
time = 0.49, size = 64, normalized size = 1.83

$$\frac{2\,a^2 \log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right) - a^2 \log\left(\left|\frac{\left(\cos\left(dx+c\right)-1\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2} - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] (2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs((cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)))/d
```

Mupad [B]
time = 1.25, size = 36, normalized size = 1.03

$$\frac{a^2\left(4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - \ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4 - 1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^2,x)
[Out] (a^2*(4*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^4 - 1)))/d
```

3.25 $\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=40

$$-\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

```
[Out] -a^2/d/(1-cos(d*x+c))-a^2*ln(1-cos(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$-\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
[Out] -(a^2/(d*(1 - Cos[c + d*x]))) - (a^2*Log[1 - Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{a^4\text{Subst}\left(\int \frac{x}{(a-ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^4\text{Subst}\left(\int \left(\frac{1}{a^2(-1+x)^2}+\frac{1}{a^2(-1+x)}\right)\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 1.40

$$\frac{a^2 \csc^2\left(\frac{1}{2}(c+dx)\right)\left(-1-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^2*(-1 - 2*Log[Sin[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[
Sin[(c + d*x)/2]]))/(2*d)
```

Maple [A]
time = 0.10, size = 39, normalized size = 0.98

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{1}{-1+\sec(dx+c)}-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c))\right)}{d}$	39
default	$\frac{a^2\left(-\frac{1}{-1+\sec(dx+c)}-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c))\right)}{d}$	39
risch	$i a^2 x+\frac{2 i a^2 c}{d}+\frac{2 a^2 e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}-\frac{2 a^2 \ln\left(e^{i(dx+c)}-1\right)}{d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(-1/(-1+sec(d*x+c))-ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 34, normalized size = 0.85

$$-\frac{a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{a^2}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*log(cos(d*x + c) - 1) - a^2/(cos(d*x + c) - 1))/d
```

Fricas [A]
time = 4.55, size = 48, normalized size = 1.20

$$\frac{a^2-\left(a^2\cos\left(dx+c\right)-a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d\cos\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2 - (a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x +
c) - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^3(c+dx)\sec(c+dx)\,dx+\int\cot^3(c+dx)\sec^2(c+dx)\,dx+\int\cot^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3, x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(37) = 74.
time = 0.49, size = 111, normalized size = 2.78

$$-\frac{2\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^2+\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^2*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 2*a^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [**B**]
time = 1.23, size = 50, normalized size = 1.25

$$-\frac{a^2\left(\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/2))/d
```

3.26 $\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=85

$$-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{5a^2}{4d(1-\cos(c+dx))}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(1+\cos(c+dx))}{8d}$$

```
[Out] -1/4*a^2/d/(1-cos(d*x+c))^2+5/4*a^2/d/(1-cos(d*x+c))+7/8*a^2*ln(1-cos(d*x+c
))/d+1/8*a^2*ln(1+cos(d*x+c))/d
```

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{5a^2}{4d(1-\cos(c+dx))}-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*a^2/(d*(1 - Cos[c + d*x])^2) + (5*a^2)/(4*d*(1 - Cos[c + d*x])) + (7*a
^2*Log[1 - Cos[c + d*x]])/(8*d) + (a^2*Log[1 + Cos[c + d*x]])/(8*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^6\text{Subst}\left(\int \frac{x^3}{(a-ax)^3(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{2a^4(-1+x)^3} - \frac{5}{4a^4(-1+x)^2} - \frac{7}{8a^4(-1+x)} - \frac{1}{8a^4(1+x)}\right)\,dx\right.}{d}$$
$$= -\frac{a^2}{4d(1-\cos(c+dx))^2} + \frac{5a^2}{4d(1-\cos(c+dx))} + \frac{7a^2\log(1-\cos(c}{8d}$$

Mathematica [A]
time = 0.29, size = 86, normalized size = 1.01

$$-\frac{a^2(1+\cos(c+dx))^2\left(-10\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)-4\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*(a^2*(1 + Cos[c + d*x])^2*(-10*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^
4 - 4*(Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4
)/d
```

Maple [A]
time = 0.11, size = 63, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{a^2\left(-\frac{\ln(1+\sec(dx+c))}{8}+\frac{1}{4(-1+\sec(dx+c))^2}-\frac{3}{4(-1+\sec(dx+c))}-\frac{7\ln(-1+\sec(dx+c))}{8}+\ln(\sec(dx+c))\right)}{d}$	63
default	$-\frac{a^2\left(-\frac{\ln(1+\sec(dx+c))}{8}+\frac{1}{4(-1+\sec(dx+c))^2}-\frac{3}{4(-1+\sec(dx+c))}-\frac{7\ln(-1+\sec(dx+c))}{8}+\ln(\sec(dx+c))\right)}{d}$	63
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{a^2\left(5\,e^{3i(dx+c)}-8\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-1\right)^4}+\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{4d}+\frac{7a^2\ln\left(e^{i(dx+c)}-1\right)}{4d}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^2*(-1/8*ln(1+sec(d*x+c))+1/4/(-1+sec(d*x+c))^2-3/4/(-1+sec(d*x+c))-7
/8*ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.85

$$\frac{a^2\log\left(\cos\left(dx+c\right)+1\right)+7\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,\left(5\,a^2\cos\left(dx+c\right)-4\,a^2\right)}{\cos\left(dx+c\right)^2-2\,\cos\left(dx+c\right)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*log(cos(d*x + c) + 1) + 7*a^2*log(cos(d*x + c) - 1) - 2*(5*a^2*cos
(d*x + c) - 4*a^2)/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1))/d
```

Fricas [A]
time = 3.73, size = 122, normalized size = 1.44

$$-\frac{10\,a^2\cos\left(dx+c\right)-8\,a^2-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-7\left(a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{8\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a^2*cos(d*x + c) - 8*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)*log(1/2*cos(d*x + c) + 1/2) - 7*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*
x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x
 + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^5\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^5\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5, x))
```

Giac [A]
time = 0.55, size = 138, normalized size = 1.62

$$\frac{14\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-16\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^2+\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(14*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 16*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 8*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 1.26, size = 62, normalized size = 0.73

$$\frac{a^2\left(-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{7\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*((7*log(tan(c/2 + (d*x)/2)))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c
/2 + (d*x)/2)^2/2 - cot(c/2 + (d*x)/2)^4/16))/d
```

3.27 $\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=127

$$-\frac{a^2}{12d(1-\cos(c+dx))^3}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(1+\cos(c+dx))}-\frac{13a^2\log(1-$$

```
[Out] -1/12*a^2/d/(1-cos(d*x+c))^3+1/2*a^2/d/(1-cos(d*x+c))^2-23/16*a^2/d/(1-cos(
d*x+c))-1/16*a^2/d/(1+cos(d*x+c))-13/16*a^2*ln(1-cos(d*x+c))/d-3/16*a^2*ln(
1+cos(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(\cos(c+dx)+1)}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{a^2}{12d(1-\cos(c+dx))^3}-\frac{13a^2\log(1-\cos(c+dx))}{16d}-\frac{3a^2\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*a^2/(d*(1 - Cos[c + d*x])^3) + a^2/(2*d*(1 - Cos[c + d*x])^2) - (23*a
^2)/(16*d*(1 - Cos[c + d*x])) - a^2/(16*d*(1 + Cos[c + d*x])) - (13*a^2*Log
[1 - Cos[c + d*x]])/(16*d) - (3*a^2*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^8\text{Subst}\left(\int \frac{x^5}{(a-ax)^4(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^8\text{Subst}\left(\int \left(\frac{1}{4a^6(-1+x)^4} + \frac{1}{a^6(-1+x)^3} + \frac{23}{16a^6(-1+x)^2} + \frac{13}{16a^6(-1+x)} - \right.\right.}{d}$$

$$= -\frac{a^2}{12d(1-\cos(c+dx))^3} + \frac{a^2}{2d(1-\cos(c+dx))^2} - \frac{23a^2}{16d(1-\cos(c}$$

Mathematica [A]

time = 0.25, size = 114, normalized size = 0.90

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(69\csc^2\left(\frac{1}{2}(c+dx)\right)-12\csc^4\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)+3\left(12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+52\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(69*Csc[(c + d*x)/2]^2
- 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 + 3*(12*Log[Cos[(c + d*x)/2]]
+ 52*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/d
```

Maple [A]

time = 0.12, size = 86, normalized size = 0.68

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{16+16\sec(dx+c)}-\frac{3\ln(1+\sec(dx+c))}{16}-\frac{1}{12(-1+\sec(dx+c))^3}+\frac{1}{4(-1+\sec(dx+c))^2}-\frac{11}{16(-1+\sec(dx+c))}-\frac{13\ln(-1+\sec(dx+c))}{16}\right.}{d}$
default	$\frac{a^2\left(\frac{1}{16+16\sec(dx+c)}-\frac{3\ln(1+\sec(dx+c))}{16}-\frac{1}{12(-1+\sec(dx+c))^3}+\frac{1}{4(-1+\sec(dx+c))^2}-\frac{11}{16(-1+\sec(dx+c))}-\frac{13\ln(-1+\sec(dx+c))}{16}\right.}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{a^2\left(33\,e^{7i(dx+c)}-36\,e^{6i(dx+c)}-49\,e^{5i(dx+c)}+136\,e^{4i(dx+c)}-49\,e^{3i(dx+c)}-36\,e^{2i(dx+c)}+33\,e^{i(dx+c)}\right.}{12d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(1/16/(1+sec(d*x+c))-3/16*ln(1+sec(d*x+c))-1/12/(-1+sec(d*x+c))^3+1
/4/(-1+sec(d*x+c))^2-11/16/(-1+sec(d*x+c))-13/16*ln(-1+sec(d*x+c))+ln(sec(d
*x+c)))
```

Maxima [A]

time = 0.26, size = 109, normalized size = 0.86

$$-\frac{9\,a^2\log\left(\cos\left(dx+c\right)+1\right)+39\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a^2\cos(dx+c)^3-18\,a^2\cos(dx+c)^2-37\,a^2\cos(dx+c)+26\,a^2\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^3+2\,\cos(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(9*a^2*log(cos(d*x + c) + 1) + 39*a^2*log(cos(d*x + c) - 1) - 2*(33*a
^2*cos(d*x + c)^3 - 18*a^2*cos(d*x + c)^2 - 37*a^2*cos(d*x + c) + 26*a^2)/(
cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x + c) - 1))/d
```

Fricas [A]
time = 3.06, size = 191, normalized size = 1.50

$$\frac{66\,a^2\cos\left(dx+c\right)^3-36\,a^2\cos\left(dx+c\right)^2-74\,a^2\cos\left(dx+c\right)+52\,a^2-9\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^3+2\,a^2\cos\left(dx+c\right)-a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-39\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^3+2\,a^2\cos\left(dx+c\right)-a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(66*a^2*cos(d*x + c)^3 - 36*a^2*cos(d*x + c)^2 - 74*a^2*cos(d*x + c) +
 52*a^2 - 9*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)
 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 39*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*
x + c)^3 + 2*a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d
*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^7\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^7\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot^7\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**
7*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**7, x))
```

Giac [A]
time = 0.57, size = 186, normalized size = 1.46

$$-\frac{78\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(a^2+\frac{9\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{143\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(78*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - (a^2 + 9*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
```

```
+ 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 143*a^2*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3)/d
```

Mupad [B]
time = 1.27, size = 113, normalized size = 0.89

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{13\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{a^2}{6}\right)}{16\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (13*a^2*log(tan(c/2 + (d*x)/2)))/(8
*d) - (cot(c/2 + (d*x)/2)^6*(8*a^2*tan(c/2 + (d*x)/2)^4 - (3*a^2*tan(c/2 +
(d*x)/2)^2)/2 + a^2/6))/(16*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(32*d)
```

3.28 $\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=169

$$-\frac{a^2}{32d(1-\cos(c+dx))^4}+\frac{11a^2}{48d(1-\cos(c+dx))^3}-\frac{3a^2}{4d(1-\cos(c+dx))^2}+\frac{51a^2}{32d(1-\cos(c+dx))}-\frac{a^2}{64d(1+\cos$$

```
[Out] -1/32*a^2/d/(1-cos(d*x+c))^4+11/48*a^2/d/(1-cos(d*x+c))^3-3/4*a^2/d/(1-cos(
d*x+c))^2+51/32*a^2/d/(1-cos(d*x+c))-1/64*a^2/d/(1+cos(d*x+c))^2+9/64*a^2/d
/(1+cos(d*x+c))+99/128*a^2*ln(1-cos(d*x+c))/d+29/128*a^2*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{51a^2}{32d(1-\cos(c+dx))}+\frac{9a^2}{64d(\cos(c+dx)+1)}-\frac{3a^2}{4d(1-\cos(c+dx))^2}-\frac{a^2}{64d(\cos(c+dx)+1)^2}+\frac{11a^2}{48d(1-\cos(c+dx))^3}-\frac{a^2}{32d(1-\cos(c+dx))^4}+\frac{99a^2\log(1-\cos(c+dx))}{128d}+\frac{29a^2\log(\cos(c+dx)+1)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/32*a^2/(d*(1 - Cos[c + d*x])^4) + (11*a^2)/(48*d*(1 - Cos[c + d*x])^3) -
 (3*a^2)/(4*d*(1 - Cos[c + d*x])^2) + (51*a^2)/(32*d*(1 - Cos[c + d*x])) -
a^2/(64*d*(1 + Cos[c + d*x])^2) + (9*a^2)/(64*d*(1 + Cos[c + d*x])) + (99*a
^2*Log[1 - Cos[c + d*x]])/(128*d) + (29*a^2*Log[1 + Cos[c + d*x]])/(128*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^{10}\text{Subst}\left(\int \frac{x^7}{(a-ax)^5(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^{10}\text{Subst}\left(\int \left(-\frac{1}{8a^8(-1+x)^5} - \frac{11}{16a^8(-1+x)^4} - \frac{3}{2a^8(-1+x)^3} - \frac{51}{32a^8(-1+x)}\right.\right.}{}$$

$$= -\frac{a^2}{32d(1-\cos(c+dx))^4} + \frac{11a^2}{48d(1-\cos(c+dx))^3} - \frac{3a^2}{4d(1-\cos(c}$$

Mathematica [A]
time = 0.35, size = 146, normalized size = 0.86

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-1224\csc^2\left(\frac{1}{2}(c+dx)\right)+288\csc^4\left(\frac{1}{2}(c+dx)\right)-44\csc^6\left(\frac{1}{2}(c+dx)\right)+3\csc^8\left(\frac{1}{2}(c+dx)\right)-6\left(116\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+396\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+18\sec^2\left(\frac{1}{2}(c+dx)\right)-\sec^4\left(\frac{1}{2}(c+dx)\right)\right)\right)}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-1224*Csc[(c + d*x)/2
]^2 + 288*Csc[(c + d*x)/2]^4 - 44*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
 - 6*(116*Log[Cos[(c + d*x)/2]] + 396*Log[Sin[(c + d*x)/2]] + 18*Sec[(c + d
*x)/2]^2 - Sec[(c + d*x)/2]^4)))/d
```

Maple [A]
time = 0.15, size = 111, normalized size = 0.66

method	result
derivativedivides	$-\frac{a^2\left(\frac{1}{64(1+\sec(dx+c))^2}+\frac{7}{64(1+\sec(dx+c))}-\frac{29\ln(1+\sec(dx+c))}{128}+\frac{1}{32(-1+\sec(dx+c))^4}-\frac{5}{48(-1+\sec(dx+c))^3}+\frac{1}{4(-1+\sec(dx+c))}\right.}{d}$
default	$-\frac{a^2\left(\frac{1}{64(1+\sec(dx+c))^2}+\frac{7}{64(1+\sec(dx+c))}-\frac{29\ln(1+\sec(dx+c))}{128}+\frac{1}{32(-1+\sec(dx+c))^4}-\frac{5}{48(-1+\sec(dx+c))^3}+\frac{1}{4(-1+\sec(dx+c))}\right.}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{a^2\left(279\,e^{11i(dx+c)}-156\,e^{10i(dx+c)}-1141\,e^{9i(dx+c)}+2080\,e^{8i(dx+c)}+670\,e^{7i(dx+c)}-2696\,e^{6i(dx+c}\right.}{96d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^2*(1/64/(1+sec(d*x+c))^2+7/64/(1+sec(d*x+c))-29/128*ln(1+sec(d*x+c))
+1/32/(-1+sec(d*x+c))^4-5/48/(-1+sec(d*x+c))^3+1/4/(-1+sec(d*x+c))^2-21/32/
(-1+sec(d*x+c))-99/128*ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 165, normalized size = 0.98

$$\frac{87\,a^2\log\left(\cos\left(dx+c\right)+1\right)+297\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(279\,a^2\cos(dx+c)^5-78\,a^2\cos(dx+c)^4-634\,a^2\cos(dx+c)^3+338\,a^2\cos(dx+c)^2+343\,a^2\cos(dx+c)-224\,a^2\right)}{\cos(dx+c)^6-2\cos(dx+c)^5-\cos(dx+c)^4+4\cos(dx+c)^3-\cos(dx+c)^2-2\cos(dx+c)+1}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(87*a^2*log(cos(d*x + c) + 1) + 297*a^2*log(cos(d*x + c) - 1) - 2*(27
9*a^2*cos(d*x + c)^5 - 78*a^2*cos(d*x + c)^4 - 634*a^2*cos(d*x + c)^3 + 338
*a^2*cos(d*x + c)^2 + 343*a^2*cos(d*x + c) - 224*a^2)/(cos(d*x + c)^6 - 2*c
os(d*x + c)^5 - cos(d*x + c)^4 + 4*cos(d*x + c)^3 - cos(d*x + c)^2 - 2*cos(
d*x + c) + 1))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(145) = 290$.
time = 3.55, size = 322, normalized size = 1.91

$$-\frac{558\,a^2\cos\left(dx+c\right)^5-156\,a^2\cos\left(dx+c\right)^4-1268\,a^2\cos\left(dx+c\right)^3+676\,a^2\cos\left(dx+c\right)^2+686\,a^2\cos\left(dx+c\right)-448\,a^2-87\,\left(a^2\cos\left(dx+c\right)^6-2\,a^2\cos\left(dx+c\right)^5-a^2\cos\left(dx+c\right)^4+4\,a^2\cos\left(dx+c\right)^3-a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-297\,\left(a^2\cos\left(dx+c\right)^6-2\,a^2\cos\left(dx+c\right)^5-a^2\cos\left(dx+c\right)^4+4\,a^2\cos\left(dx+c\right)^3-a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{384\,\left(d\cos\left(dx+c\right)^6-2\,d\cos\left(dx+c\right)^5-d\cos\left(dx+c\right)^4+4\,d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(558*a^2*cos(d*x + c)^5 - 156*a^2*cos(d*x + c)^4 - 1268*a^2*cos(d*x
+ c)^3 + 676*a^2*cos(d*x + c)^2 + 686*a^2*cos(d*x + c) - 448*a^2 - 87*(a^2*
cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x
+ c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*log(1/2*cos(d*x + c
) + 1/2) - 297*(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c
)^4 + 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*c
os(d*x + c)^4 + 4*d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) +
d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.61, size = 238, normalized size = 1.41

$$\frac{1188\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-1536\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{96\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{\left(3\,a^2+\frac{32\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{174\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{768\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2475\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}\right)(\cos(dx+c)+1)^4}{(\cos(dx+c)-1)^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(1188*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 1536*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 96*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 - (3*a^2 + 32*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 174*a^2*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 768*a^2*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 2475*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d*x
 + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

Mupad [B]
time = 1.59, size = 149, normalized size = 0.88

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{99\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{29\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{a^2}{8}\right)}{64\,d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(16*d) - (a^2*tan(c/2 + (d*x)/2)^4)/(256*d) + (9
9*a^2*log(tan(c/2 + (d*x)/2)))/(64*d) + (cot(c/2 + (d*x)/2)^8*((4*a^2*tan(c
/2 + (d*x)/2)^2)/3 - (29*a^2*tan(c/2 + (d*x)/2)^4)/4 + 32*a^2*tan(c/2 + (d*
x)/2)^6 - a^2/8))/(64*d) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.29 $\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=161

$$-a^2x-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\frac{5a^2\sec(c+}{}$$

```
[Out] -a^2*x-5/8*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a^2*sec(d*x+c)*ta
n(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^
2*tan(d*x+c)^5/d+1/3*a^2*sec(d*x+c)*tan(d*x+c)^5/d+1/7*a^2*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a^2\tan^3(c+dx)\sec(c+dx)}{12d}+\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a^2*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a^2*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx &= \int \left(a^2\tan^6(c+dx)+2a^2\sec(c+dx)\tan^6(c+dx)+a^2\sec^2(c+d\right.\\
&= a^2\int \tan^6(c+dx)\,dx + a^2\int \sec^2(c+dx)\tan^6(c+dx)\,dx + \left(2a^2\right)\\
&= \frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\sec(c+dx)\tan^5(c+dx)}{3d}-a^2\int \tan^4(c+dx\\
&= -\frac{a^2\tan^3(c+dx)}{3d}-\frac{5a^2\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx}{5d}\\
&= \frac{a^2\tan(c+dx)}{d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\\
&= -a^2x-\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5a^2\sec(c+}{}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(161) = 322.
time = 1.47, size = 337, normalized size = 2.09

$$
\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(33600\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec(c)(-14700dx\cos(dx)-14700dx\cos(2c+dx)-8820dx\cos(2c+3dx)-8820dx\cos(4c+3dx)-2940dx\cos(4c+5dx)-2940dx\cos(6c+5dx)-420dx\cos(6c+7dx)-420dx\cos(8c+7dx)+24640\sin(dx)-16240\sin(2c+dx)+2975\sin(c+2dx)+2975\sin(3c+2dx)+14448\sin(2c+3dx)-10080\sin(4c+3dx)+980\sin(3c+4dx)+980\sin(5c+4dx)+6496\sin(4c+5dx)-1680\sin(6c+5dx)+1155\sin(5c+6dx)+1155\sin(7c+6dx)+1168\sin(6c+7dx))\right)}{215040d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^7*(33600*Cos[c +
d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c]*(-14700*d*x*Cos[d*x] - 14700*d*x*Cos[2*c + d*x]
- 8820*d*x*Cos[2*c + 3*d*x] - 8820*d*x*Cos[4*c + 3*d*x] - 2940*d*x*Cos[4*c
+ 5*d*x] - 2940*d*x*Cos[6*c + 5*d*x] - 420*d*x*Cos[6*c + 7*d*x] - 420*d*x*C
os[8*c + 7*d*x] + 24640*Sin[d*x] - 16240*Sin[2*c + d*x] + 2975*Sin[c + 2*d*
x] + 2975*Sin[3*c + 2*d*x] + 14448*Sin[2*c + 3*d*x] - 10080*Sin[4*c + 3*d*x
] + 980*Sin[3*c + 4*d*x] + 980*Sin[5*c + 4*d*x] + 6496*Sin[4*c + 5*d*x] - 1
680*Sin[6*c + 5*d*x] + 1155*Sin[5*c + 6*d*x] + 1155*Sin[7*c + 6*d*x] + 1168
*Sin[6*c + 7*d*x])))/(215040*d)
```

Maple [A]

time = 0.12, size = 169, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{a^2\left(\sin^7(dx+c)\right)}{7\cos(dx+c)^7}+2a^2\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^7(dx+c)}{24\cos(dx+c)^4}+\frac{\sin^7(dx+c)}{16\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{16}+\frac{5\left(\sin^3(dx+c)\right)}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+\cdots}{d}$
default	$\frac{\frac{a^2\left(\sin^7(dx+c)\right)}{7\cos(dx+c)^7}+2a^2\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^7(dx+c)}{24\cos(dx+c)^4}+\frac{\sin^7(dx+c)}{16\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{16}+\frac{5\left(\sin^3(dx+c)\right)}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+\cdots}{d}$
risch	$-a^2x-\frac{ia^2\left(1155\,e^{13i(dx+c)}-1680\,e^{12i(dx+c)}+980\,e^{11i(dx+c)}-10080\,e^{10i(dx+c)}+2975\,e^{9i(dx+c)}-16240\,e^{8i(dx+c)}-24\cdots\right.}{420d\left(e^{2i(dx+c)}\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*a^2*sin(d*x+c)^7/cos(d*x+c)^7+2*a^2*(1/6*sin(d*x+c)^7/cos(d*x+c)^6
-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x
+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a^2
*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A]

time = 0.47, size = 151, normalized size = 0.94

$$\frac{240\,a^2\tan(dx+c)^7+112\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)a^2-35\,a^2\left(\frac{2\left(33\sin(dx+c)^5-40\sin(dx+c)^3+15\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}+15\log(\sin(dx+c)+1)-15\log(\sin(dx+c)-1)\right)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/1680*(240*a^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a^2*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.91, size = 165, normalized size = 1.02

$$-\frac{1680\,a^{2}dx\cos\left(dx+c\right)^{7}+525\,a^{2}\cos\left(dx+c\right)^{7}\log\left(\sin\left(dx+c\right)+1\right)-525\,a^{2}\cos\left(dx+c\right)^{7}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(1168\,a^{2}\cos\left(dx+c\right)^{6}+1155\,a^{2}\cos\left(dx+c\right)^{5}-256\,a^{2}\cos\left(dx+c\right)^{4}-910\,a^{2}\cos\left(dx+c\right)^{3}-192\,a^{2}\cos\left(dx+c\right)^{2}+280\,a^{2}\cos\left(dx+c\right)+120\,a^{2}\right)\sin\left(dx+c\right)}{1680\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a^2*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a^2*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1168*a^2*cos(
d*x + c)^6 + 1155*a^2*cos(d*x + c)^5 - 256*a^2*cos(d*x + c)^4 - 910*a^2*cos
(d*x + c)^3 - 192*a^2*cos(d*x + c)^2 + 280*a^2*cos(d*x + c) + 120*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int \tan^{6}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int \tan^{6}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6, x))
```

Giac [A]
time = 2.66, size = 180, normalized size = 1.12

$$-\frac{840\,(dx+c)a^{2}+525\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-525\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(315\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-2660\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+9863\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-21216\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+29673\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9660\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1365\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{7}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^2*tan(1/2*d*x + 1/2*c)^
13 - 2660*a^2*tan(1/2*d*x + 1/2*c)^11 + 9863*a^2*tan(1/2*d*x + 1/2*c)^9 - 2
1216*a^2*tan(1/2*d*x + 1/2*c)^7 + 29673*a^2*tan(1/2*d*x + 1/2*c)^5 - 9660*a
^2*tan(1/2*d*x + 1/2*c)^3 + 1365*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^7)/d
```

Mupad [B]
time = 2.65, size = 234, normalized size = 1.45

$$-a^{2}\,x-\frac{5\,a^{2}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{3\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}-\frac{19\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{1409\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{60}-\frac{1768\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+\frac{1413\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-23\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\frac{13\,a^{2}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] - a^2*x - (5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((1413*a^2*tan(c/2 + (d
*x)/2)^5)/20 - 23*a^2*tan(c/2 + (d*x)/2)^3 - (1768*a^2*tan(c/2 + (d*x)/2)^7
)/35 + (1409*a^2*tan(c/2 + (d*x)/2)^9)/60 - (19*a^2*tan(c/2 + (d*x)/2)^11)/
3 + (3*a^2*tan(c/2 + (d*x)/2)^13)/4 + (13*a^2*tan(c/2 + (d*x)/2))/4)/(d*(7*
tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 -
35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^1
2 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.30 $\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=119

$$a^2x+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+d}{}$$

```
[Out] a^2*x+3/4*a^2*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a^2*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^2*
tan(d*x+c)^5/d
```

Rubi [A]

time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{4d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a^2
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a^2*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx &= \int \left(a^2\tan^4(c+dx)+2a^2\sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx\right.\\ &= a^2\int \tan^4(c+dx)\,dx+a^2\int \sec^2(c+dx)\tan^4(c+dx)\,dx+\left(2a^2\right)\\ &= \frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\\ &= -\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}\\ &= a^2x+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)}{4d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 558 vs. 2(119) = 238.
time = 5.69, size = 558, normalized size = 4.69

$$\frac{1}{960}a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(240x-\frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{\left(293\cos\left(\frac{dx}{2}\right)+333\cos\left(2c+\frac{3dx}{2}\right)+287\cos\left(2c+\frac{5dx}{2}\right)+67\cos\left(4c+\frac{7dx}{2}\right)+68\cos\left(4c+\frac{9dx}{2}\right)\right)\sec(c)\sec^5(c+dx)\sin\left(\frac{dx}{2}\right)}{2d}-\frac{24\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{149\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{24\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{149\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{\cos\left(\frac{c}{2}\right)\left(\frac{36}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{151}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{36}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{151}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(240*x - (180*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/d + (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
```

```
)/d - ((293*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)
/2] + 67*Cos[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x
]^5*Sin[(d*x)/2])/(2*d) - (24*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^4) + (149*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (24*Sin[c/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (149*Sin[c/2])/(d*(Cos[c/2]
 + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*(36/((Cos
[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) - 151/((Cos[c/2]
 - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 36/((Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 151/((Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)))/d))/960
```

Maple [A]
time = 0.09, size = 130, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{a^2\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+2a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\left(\tan^3(d\right.}{3}\right.}{d}$
default	$\frac{\frac{a^2\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+2a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\left(\tan^3(d\right.}{3}\right.}{d}$
risch	$a^2x+\frac{ia^2\left(75\,e^{9i(dx+c)}-60\,e^{8i(dx+c)}+30\,e^{7i(dx+c)}-360\,e^{6i(dx+c)}-320\,e^{4i(dx+c)}-30\,e^{3i(dx+c)}-280\,e^{2i(dx+c)}-75\,e^{i(\right.}{30d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^2*sin(d*x+c)^5/cos(d*x+c)^5+2*a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 119, normalized size = 1.00

$$\frac{24\,a^2\tan\left(dx+c\right)^5+40\,\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2+15\,a^2\left(\frac{2\,\left(5\,\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(24*a^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a^2*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

Fricas [A]
time = 2.86, size = 139, normalized size = 1.17

$$\frac{120\,a^2dx\cos\left(dx+c\right)^5+45\,a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)-2\left(68\,a^2\cos\left(dx+c\right)^4+75\,a^2\cos\left(dx+c\right)^3+4\,a^2\cos\left(dx+c\right)^2-30\,a^2\cos\left(dx+c\right)-12\,a^2\right)\sin\left(dx+c\right)}{120\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a^2*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*a^2*cos(d*x + c
)^4 + 75*a^2*cos(d*x + c)^3 + 4*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) -
12*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\tan^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\tan^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4, x))
```

Giac [A]
time = 1.24, size = 148, normalized size = 1.24

$$\frac{60\,(dx+c)a^2+45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-110\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+328\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-530\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a^2
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 11
0*a^2*tan(1/2*d*x + 1/2*c)^7 + 328*a^2*tan(1/2*d*x + 1/2*c)^5 - 530*a^2*tan
(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^5)/d
```

Mupad [B]
time = 1.95, size = 174, normalized size = 1.46

$$a^2\,x+\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}-\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{164\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{53\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) + ((164*a^2*tan(c/2 + (d*x)
/2)^5)/15 - (53*a^2*tan(c/2 + (d*x)/2)^3)/3 - (11*a^2*tan(c/2 + (d*x)/2)^7)
/3 + (a^2*tan(c/2 + (d*x)/2)^9)/2 + (7*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.31 $\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=72

$$-a^2x-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}$$

```
[Out] -a^2*x-a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)
/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a^2*Sec[
c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+d\right.$$

$$= a^2\int \tan^2(c+dx)\,dx + a^2\int \sec^2(c+dx)\tan^2(c+dx)\,dx + (2a^2)$$

$$= \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} - a^2\int 1\,dx - a^2\int \mathrm{s}$$

$$= -a^2x - \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 773 vs. 2(72) = 144.
time = 6.33, size = 773, normalized size = 10.74

$$-\frac{1}{4}x\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2+\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{4d}-\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{4d}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{24d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(7\cos\left(\frac{c}{2}\right)-5\sin\left(\frac{c}{2}\right)\right)}{48d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{6d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{24d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-7\cos\left(\frac{c}{2}\right)-5\sin\left(\frac{c}{2}\right)\right)}{48d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{6d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/4*(x*Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2) + (Cos[
c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
4*(a + a*Sec[c + d*x])^2)/(4*d) - (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4*d) + (C
os[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(24
```

```
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Co
s[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(7*Cos[c/2] - 5*Si
n[c/2]))/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d
*x)/2])/(6*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/
2])/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3
) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-7*Cos[c/2
] - 5*Sin[c/2]))/(48*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^
2*Sin[(d*x)/2])/(6*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2]))
```

Maple [A]
time = 0.09, size = 93, normalized size = 1.29

method	result	size
derivativedivides	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$	93
default	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$	93
risch	$-a^2x-\frac{2ia^2\left(3\,e^{5i(dx+c)}-6\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+2*a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2
+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.47, size = 83, normalized size = 1.15

$$\frac{2\,a^2\tan\left(dx+c\right)^3-6\left(dx+c-\tan\left(dx+c\right)\right)a^2-3\,a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a^2*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Fricas [A]
time = 2.37, size = 111, normalized size = 1.54

$$-\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\,a^2\cos\left(dx+c\right)^2+3\,a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^2*cos(d*x + c)^2 +
3*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\tan^2(c+dx)\sec(c+dx)\,dx+\int \tan^2(c+dx)\sec^2(c+dx)\,dx+\int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

Giac [A]
time = 0.73, size = 99, normalized size = 1.38

$$-\frac{3(dx+c)a^2+3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{4\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*(a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 1.21, size = 101, normalized size = 1.40

$$\frac{\frac{4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}-\frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((4*a^2*tan(c/2 + (d*x)/2)^3)/3 - 4*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 +
 (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a^2*
atanh(tan(c/2 + (d*x)/2)))/d - a^2*x
```

3.32 $\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=35

$$-a^2x-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}$$

```
[Out] -a^2*x-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3971, 3554, 8, 2686, 3852}

$$-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+a^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)+2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx\right.\\ &= a^2\int \cot^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\,dx + \left(2a^2\right)\int \cot(c+dx\\ &= -\frac{a^2\cot(c+dx)}{d} - a^2\int 1\,dx - \frac{a^2\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{d} -\\ &= -a^2x - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 46, normalized size = 1.31

$$-\frac{2a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Cot[c/2 + (d*x)/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c/2 + (d*x)
/2]^2])/d
```

Maple [A]
time = 0.08, size = 50, normalized size = 1.43

method	result	size
risch	$-a^2x-\frac{4ia^2}{d\left(e^{i(dx+c)}-1\right)}$	30
derivativedivides	$\frac{-a^2\cot(dx+c)-\frac{2a^2}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	50
default	$\frac{-a^2\cot(dx+c)-\frac{2a^2}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*cot(d*x+c)-2*a^2/sin(d*x+c)+a^2*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 48, normalized size = 1.37

$$-\frac{\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^2 + \frac{2\,a^2}{\sin(dx+c)} + \frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a^2/sin(d*x + c) + a^2/tan(d*x + c))/d
```

Fricas [A]
time = 2.16, size = 42, normalized size = 1.20

$$-\frac{a^2dx\sin\left(dx+c\right) + 2\,a^2\cos\left(dx+c\right) + 2\,a^2}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*sin(d*x + c) + 2*a^2*cos(d*x + c) + 2*a^2)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^2\left(c+dx\right)\sec\left(c+dx\right)dx + \int\cot^2\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int\cot^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

Giac [A]
time = 0.46, size = 31, normalized size = 0.89

$$-\frac{(dx+c)a^2 + \frac{2\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^2 + 2*a^2/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 1.08, size = 24, normalized size = 0.69

$$-a^2\,x-\frac{2\,a^2\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
[Out] - a^2*x - (2*a^2*cot(c/2 + (d*x)/2))/d
```

3.33 $\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=69

$$a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-2/3*a^2*cot(d*x+c)^3/d+2*a^2*csc(d*x+c)/d-2/3*a^2*cs
c(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3971, 3554, 8, 2686, 2687, 30}

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (2*a^2*Csc[c
+ d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^4(c+dx)+2a^2\cot^3(c+dx)\csc(c+dx)+a^2\cot^2(c+dx\right.\\ &= a^2\int \cot^4(c+dx)\,dx+a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\\ &= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^2\,dx,x,-\right.}{d}\\ &= \frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c}{3d}\\ &= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 112, normalized size = 1.62

$$\frac{a^2\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(9dx\cos\left(\frac{dx}{2}\right)-9dx\cos\left(c+\frac{dx}{2}\right)-3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\frac{3dx}{2}\right)-18\sin\left(\frac{dx}{2}\right)-12\sin\left(c+\frac{dx}{2}\right)+10\sin\left(c+\frac{3dx}{2}\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] - 9*d*x*Cos[c + (d*x)/
2] - 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2
] - 12*Sin[c + (d*x)/2] + 10*Sin[c + (3*d*x)/2]))/(24*d)
```

Maple [A]
time = 0.08, size = 112, normalized size = 1.62

method	result

risch	$a^2x+\frac{2ia^2(6\,e^{2i(dx+c)}-9\,e^{i(dx+c)}+5)}{3d(e^{i(dx+c)}-1)^3}$
derivativedivides	$\frac{-\frac{a^2(\cos^3(dx+c))}{3\sin(dx+c)^3}+2a^2\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{(2+\cos^2(dx+c))\sin(dx+c)}{3}\right)+a^2\left(-\frac{(\cot^3(dx+c))}{3}+\cot(dx+c)+dx+c\right.}{d}$
default	$\frac{-\frac{a^2(\cos^3(dx+c))}{3\sin(dx+c)^3}+2a^2\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{(2+\cos^2(dx+c))\sin(dx+c)}{3}\right)+a^2\left(-\frac{(\cot^3(dx+c))}{3}+\cot(dx+c)+dx+c\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/sin(d*x+c)^3*cos(d*x+c)^3+2*a^2*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/3*c
ot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.48, size = 77, normalized size = 1.12

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2+\frac{2\left(3\,\sin(dx+c)^2-1\right)a^2}{\sin(dx+c)^3}-\frac{a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a^2/sin(d*x + c)^3 - a^2/tan(d*x + c)^3)/d
```

Fricas [A]
time = 2.52, size = 81, normalized size = 1.17

$$\frac{5\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)-4\,a^2+3\left(a^2dx\cos\left(dx+c\right)-a^2dx\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(5*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 4*a^2 + 3*(a^2*d*x*cos(d*x +
 c) - a^2*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4, x))
```

Giac [A]
time = 0.48, size = 50, normalized size = 0.72

$$\frac{6\,(dx+c)a^2 + \frac{9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a^2 + (9*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/tan(1/2*d*x + 1
/2*c)^3)/d
```

Mupad [B]
time = 1.10, size = 39, normalized size = 0.57

$$a^2\,x + \frac{a^2\,\left(9\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right) - \cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (a^2*(9*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(6*d)
```

3.34 $\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=107

$$-a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc^5(c+dx)}{5d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/d-2*a^2
*csc(d*x+c)/d+4/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c
 + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^3)/(3*d) -
(2*a^2*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^6(c+dx)+2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx\right.\\
&= a^2\int \cot^6(c+dx)\,dx+a^2\int \cot^4(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\\
&= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^4\,dx,x,-\right.}{d}\\
&= \frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}+a^2\int \cot^2(c+dx)\,dx-\frac{(2a^2}{}\\
&= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c}{d}\\
&= -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a}{}
\end{aligned}$$

Mathematica [A]
time = 0.83, size = 194, normalized size = 1.81

$$\frac{a^2\csc\left(\frac{c}{2}\right)\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-150dx\cos(dx)+150dx\cos(2c+dx)+120dx\cos(c+2dx)-120dx\cos(3c+2dx)-30dx\cos(2c+3dx)+30dx\cos(4c+3dx)-80\sin(c)+280\sin(dx)+445\sin(c+dx)-356\sin(2(c+dx))+89\sin(3(c+dx))+240\sin(2c+dx)-296\sin(c+2dx)-120\sin(3c+2dx)+104\sin(2c+3dx)\right)}{3840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c/2]*Sec[(c + d*x)/2]*(-150*d*x*Cos[d*
x] + 150*d*x*Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*
```

d*x] - 30*d*x*Cos[2*c + 3*d*x] + 30*d*x*Cos[4*c + 3*d*x] - 80*Sin[c] + 280*
Sin[d*x] + 445*Sin[c + d*x] - 356*Sin[2*(c + d*x)] + 89*Sin[3*(c + d*x)] +
240*Sin[2*c + d*x] - 296*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 104*Sin[2*
c + 3*d*x]))/(3840*d)

Maple [A]
time = 0.09, size = 155, normalized size = 1.45

method	result
risch	$-a^{2} x -\frac{4 i a^{2} \left(15 \,{\mathrm e}^{5 i \left(d x +c \right)}-30 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{3 i \left(d x +c \right)}+35 \,{\mathrm e}^{2 i \left(d x +c \right)}-37 \,{\mathrm e}^{i \left(d x +c \right)}+13\right)}{15 d \left({\mathrm e}^{i \left(d x +c \right)}-1\right)^{5} \left({\mathrm e}^{i \left(d x +c \right)}+1\right)}$
derivativedivides	$\frac{-\frac{a^{2} \left(\cos^{5}\left(d x +c \right)\right)}{5 \sin \left(d x +c \right)^{5}}+2 a^{2} \left(-\frac{\cos^{6}\left(d x +c \right)}{5 \sin \left(d x +c \right)^{5}}+\frac{\cos^{6}\left(d x +c \right)}{15 \sin \left(d x +c \right)^{3}}-\frac{\cos^{6}\left(d x +c \right)}{5 \sin \left(d x +c \right)}-\frac{\left(\frac{8}{3}+\cos^{4}\left(d x +c \right)+\frac{4 \left(\cos^{2}\left(d x +c \right)\right)}{3}\right) \sin \left(d x +c \right)}{5}\right)+a^{2} \left(-\cdots\right.}{d}$
default	$\frac{-\frac{a^{2} \left(\cos^{5}\left(d x +c \right)\right)}{5 \sin \left(d x +c \right)^{5}}+2 a^{2} \left(-\frac{\cos^{6}\left(d x +c \right)}{5 \sin \left(d x +c \right)^{5}}+\frac{\cos^{6}\left(d x +c \right)}{15 \sin \left(d x +c \right)^{3}}-\frac{\cos^{6}\left(d x +c \right)}{5 \sin \left(d x +c \right)}-\frac{\left(\frac{8}{3}+\cos^{4}\left(d x +c \right)+\frac{4 \left(\cos^{2}\left(d x +c \right)\right)}{3}\right) \sin \left(d x +c \right)}{5}\right)+a^{2} \left(-\cdots\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/5*a^2/sin(d*x+c)^5*cos(d*x+c)^5+2*a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)
^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c))

Maxima [A]
time = 0.48, size = 97, normalized size = 0.91

$$-\frac{\left(15 \, d x + 15 \, c + \frac{15 \, \tan\left(d x + c\right)^{4} - 5 \, \tan\left(d x + c\right)^{2} + 3}{\tan\left(d x + c\right)^{5}}\right)} a^{2} + \frac{2 \, {\left(15 \, \sin\left(d x + c\right)^{4} - 10 \, \sin\left(d x + c\right)^{2} + 3\right)} a^{2}}{\sin\left(d x + c\right)^{5}} + \frac{3 \, a^{2}}{\tan\left(d x + c\right)^{5}}}{15 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^2/sin(d*x + c
)^5 + 3*a^2/tan(d*x + c)^5)/d

Fricas [A]
time = 3.59, size = 118, normalized size = 1.10

$$-\frac{26 \, a^{2} \cos\left(d x + c\right)^{3} - 22 \, a^{2} \cos\left(d x + c\right)^{2} - 17 \, a^{2} \cos\left(d x + c\right) + 16 \, a^{2} + 15 \, {\left(a^{2} d x \cos\left(d x + c\right)^{2} - 2 \, a^{2} d x \cos\left(d x + c\right) + a^{2} d x\right)} \sin\left(d x + c\right)}{15 \, {\left(d \cos\left(d x + c\right)^{2} - 2 \, d \cos\left(d x + c\right) + d\right)} \sin\left(d x + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*a^2*cos(d*x + c)^3 - 22*a^2*cos(d*x + c)^2 - 17*a^2*cos(d*x + c)
+ 16*a^2 + 15*(a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cot^6\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cot^6\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6, x))
```

Giac [A]
time = 0.51, size = 80, normalized size = 0.75

$$-\frac{120\,(dx+c)a^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{165\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + (165*a^2*tan(1/2*
d*x + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c
)^5)/d
```

Mupad [B]
time = 1.26, size = 78, normalized size = 0.73

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8}-\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{24}+\frac{a^2}{40}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2))/(8*d) - ((11*a^2*tan(c/2 + (d*x)/2)^4)/8 - (5*a^2*
tan(c/2 + (d*x)/2)^2)/24 + a^2/40)/(d*tan(c/2 + (d*x)/2)^5) - a^2*x
```

3.35 $\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=139

$$a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-2/7*a^
2*cot(d*x+c)^7/d+2*a^2*csc(d*x+c)/d-2*a^2*csc(d*x+c)^3/d+6/5*a^2*csc(d*x+c)
^5/d-2/7*a^2*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x])/d - (2*a^
2*Csc[c + d*x]^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)
/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx &= \int \left(a^2\cot^8(c+dx)+2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx\right.\\ &= a^2\int \cot^8(c+dx)\,dx+a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\\ &= -\frac{a^2\cot^7(c+dx)}{7d}-a^2\int \cot^6(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^6\,dx,x,-\right.}{d}\\ &= \frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}+a^2\int \cot^4(c+dx)\,dx-\frac{(2a^2}{}\\ &= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{2a^2\csc(}{d}\\ &= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+}{7d}\\ &= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\mathrm{c}}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 312 vs. 2(139) = 278.

time = 1.13, size = 312, normalized size = 2.24

$$\frac{a^2\csc\left(\frac{c}{2}\right)\csc^7\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(5880dx\cos(dx)-5880dx\cos(2c+dx)-3360dx\cos(c+2dx)+3360dx\cos(3c+2dx)-1260dx\cos(2c+3dx)+1260dx\cos(4c+3dx)+1680dx\cos(3c+4dx)-1680dx\cos(5c+4dx)-420dx\cos(4c+5dx)+420dx\cos(6c+5dx)+4032\sin(c)-9632\sin(dx)-16002\sin(c+dx)+9144\sin(2(c+dx))+3429\sin(3(c+dx))-4572\sin(4(c+dx))+1143\sin(5(c+dx))-11760\sin(2c+dx)+8864\sin(c+2dx)+3360\sin(3c+2dx)+2064\sin(2c+3dx)+2520\sin(4c+3dx)-4432\sin(3c+4dx)-1680\sin(5c+4dx)+1528\sin(4c+5dx)\right)}{860160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c/2]*Csc[(c + d*x)/2]^7*Sec[c/2]*Sec[(c + d*x)/2]^3*(5880*d*x*Cos[
d*x] - 5880*d*x*Cos[2*c + d*x] - 3360*d*x*Cos[c + 2*d*x] + 3360*d*x*Cos[3*c
 + 2*d*x] - 1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] + 1680*d*
x*Cos[3*c + 4*d*x] - 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] +
 420*d*x*Cos[6*c + 5*d*x] + 4032*Sin[c] - 9632*Sin[d*x] - 16002*Sin[c + d*x
] + 9144*Sin[2*(c + d*x)] + 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] +
 1143*Sin[5*(c + d*x)] - 11760*Sin[2*c + d*x] + 8864*Sin[c + 2*d*x] + 3360*
Sin[3*c + 2*d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] - 4432*Sin
[3*c + 4*d*x] - 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(860160*d)
```

Maple [A]

time = 0.12, size = 188, normalized size = 1.35

method	result
risch	$a^2x + \frac{2ia^2\left(210\,\mathrm{e}^{9i(dx+c)}-315\,\mathrm{e}^{8i(dx+c)}-420\,\mathrm{e}^{7i(dx+c)}+1470\,\mathrm{e}^{6i(dx+c)}-504\,\mathrm{e}^{5i(dx+c)}-1204\,\mathrm{e}^{4i(dx+c)}+1108\,\mathrm{e}^{3i(dx+c)}\right.}{105d\left(\mathrm{e}^{i(dx+c)}-1\right)^7\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$
derivativedivides	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8(\cos\right.}{7}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8(\cos\right.}{7}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^2/sin(d*x+c)^7*cos(d*x+c)^7+2*a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)
^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*
x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))+a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d
*x+c)+d*x+c))
```

Maxima [A]

time = 0.48, size = 117, normalized size = 0.84

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^2+\frac{6\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a^2}{\sin(dx+c)^7}-\frac{15\,a^2}{\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^2/sin(d*x + c)^7 - 15*a^2/tan(d*x + c)^7)/d
```

Fricas [A]
time = 3.70, size = 173, normalized size = 1.24

$$\frac{191\,a^2\cos\left(dx+c\right)^5-172\,a^2\cos\left(dx+c\right)^4-253\,a^2\cos\left(dx+c\right)^3+258\,a^2\cos\left(dx+c\right)^2+87\,a^2\cos\left(dx+c\right)-96\,a^2+105\,\left(a^2dx\cos\left(dx+c\right)^4-2\,a^2dx\cos\left(dx+c\right)^3+2\,a^2dx\cos\left(dx+c\right)-a^2dx\right)\sin\left(dx+c\right)}{105\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*a^2*cos(d*x + c)^5 - 172*a^2*cos(d*x + c)^4 - 253*a^2*cos(d*x +
c)^3 + 258*a^2*cos(d*x + c)^2 + 87*a^2*cos(d*x + c) - 96*a^2 + 105*(a^2*d*x
*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^3 + 2*a^2*d*x*cos(d*x + c) - a^2*d
*x)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c
) - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 112, normalized size = 0.81

$$\frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3360\,(dx+c)a^2-735\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4410\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-770\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+147\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 3360*(d*x + c)*a^2 - 735*a^2*tan(1/
2*d*x + 1/2*c) + (4410*a^2*tan(1/2*d*x + 1/2*c)^6 - 770*a^2*tan(1/2*d*x + 1
/2*c)^4 + 147*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/
d
```

Mupad [B]
time = 1.75, size = 182, normalized size = 1.31

$$\frac{a^2\left(35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-735\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4410\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-770\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+147\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3360\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7(c+d\,x)\right)}{3360\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x))^2,x)

[Out] (a^2*(35*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (d*x)/2)^10 - 735*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 4410*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 770*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 147*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7*(c + d*x)))/(3360*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)

3.36 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=179

$$-a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\csc(c+d}{d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d+1/7*a
^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc(d*x+c)/d+8/3*a^2*csc(d*x
+c)^3/d-12/5*a^2*csc(d*x+c)^5/d+8/7*a^2*csc(d*x+c)^7/d-2/9*a^2*csc(d*x+c)^9
/d
```

Rubi [A]
time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{8a^2\csc^7(c+dx)}{7d}-\frac{12a^2\csc^5(c+dx)}{5d}+\frac{8a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d)
- (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^3)/(3*d) - (12*a^2*Csc[c + d
*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^9)/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a^2\cot^{10}(c+dx)+2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+d\right.$$

$$= a^2\int \cot^{10}(c+dx)\,dx + a^2\int \cot^8(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)$$

$$= -\frac{a^2\cot^9(c+dx)}{9d} - a^2\int \cot^8(c+dx)\,dx + \frac{a^2\text{Subst}\left(\int x^8\,dx, x, -\right.}{d}$$

$$= \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} + a^2\int \cot^6(c+dx)\,dx - \frac{(2a^2)}{}$$

$$= -\frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c-}{9d}$$

$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+}{7d}$$

$$= -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2 c}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 428 vs. 2(179) = 358.

time = 1.93, size = 428, normalized size = 2.39

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/330301440*(a^2*Csc[c/2]*Csc[(c + d*x)/2]^9*Sec[c/2]*Sec[(c + d*x)/2]^5*(
453600*d*x*Cos[d*x] - 453600*d*x*Cos[2*c + d*x] - 201600*d*x*Cos[c + 2*d*x]
 + 201600*d*x*Cos[3*c + 2*d*x] - 191520*d*x*Cos[2*c + 3*d*x] + 191520*d*x*C
os[4*c + 3*d*x] + 161280*d*x*Cos[3*c + 4*d*x] - 161280*d*x*Cos[5*c + 4*d*x]
 + 10080*d*x*Cos[4*c + 5*d*x] - 10080*d*x*Cos[6*c + 5*d*x] - 40320*d*x*Cos[
5*c + 6*d*x] + 40320*d*x*Cos[7*c + 6*d*x] + 10080*d*x*Cos[6*c + 7*d*x] - 10
080*d*x*Cos[8*c + 7*d*x] + 259584*Sin[c] - 897024*Sin[d*x] - 1152405*Sin[c
+ d*x] + 512180*Sin[2*(c + d*x)] + 486571*Sin[3*(c + d*x)] - 409744*Sin[4*(
c + d*x)] - 25609*Sin[5*(c + d*x)] + 102436*Sin[6*(c + d*x)] - 25609*Sin[7*
(c + d*x)] - 825216*Sin[2*c + d*x] + 622976*Sin[c + 2*d*x] + 142464*Sin[3*c
 + 2*d*x] + 297088*Sin[2*c + 3*d*x] + 430080*Sin[4*c + 3*d*x] - 424192*Sin[
3*c + 4*d*x] - 188160*Sin[5*c + 4*d*x] + 2048*Sin[4*c + 5*d*x] - 40320*Sin[
6*c + 5*d*x] + 112768*Sin[5*c + 6*d*x] + 40320*Sin[7*c + 6*d*x] - 38272*Sin
[6*c + 7*d*x]))/d
```

Maple [A]
time = 0.14, size = 231, normalized size = 1.29

method	result
risch	$-a^{2} x -\frac{4 i a^{2} \left(315 \,{\mathrm e}^{13 i \left(d x +c \right)}-315 \,{\mathrm e}^{12 i \left(d x +c \right)}-1470 \,{\mathrm e}^{11 i \left(d x +c \right)}+3360 \,{\mathrm e}^{10 i \left(d x +c \right)}+1113 \,{\mathrm e}^{9 i \left(d x +c \right)}-6447 \,{\mathrm e}^{8 i \left(d x +c \right)}+202\right.}{315 d \left({\mathrm e}^{i \left(d x +c \right)}-\right.}$
derivativedivides	$\frac{-\frac{a^{2} \left(\cos^{9}\left(d x +c \right)\right)}{9 \sin \left(d x +c \right)^{9}}+2 a^{2} \left(-\frac{\cos^{10}\left(d x +c \right)}{9 \sin \left(d x +c \right)^{9}}+\frac{\cos^{10}\left(d x +c \right)}{63 \sin \left(d x +c \right)^{7}}-\frac{\cos^{10}\left(d x +c \right)}{105 \sin \left(d x +c \right)^{5}}+\frac{\cos^{10}\left(d x +c \right)}{63 \sin \left(d x +c \right)^{3}}-\frac{\cos^{10}\left(d x +c \right)}{9 \sin \left(d x +c \right)}-\frac{\left(\frac{128}{35}+\cos^{8}\left(d x +c \right)+\right.}{}\right.}{}$
default	$\frac{-\frac{a^{2} \left(\cos^{9}\left(d x +c \right)\right)}{9 \sin \left(d x +c \right)^{9}}+2 a^{2} \left(-\frac{\cos^{10}\left(d x +c \right)}{9 \sin \left(d x +c \right)^{9}}+\frac{\cos^{10}\left(d x +c \right)}{63 \sin \left(d x +c \right)^{7}}-\frac{\cos^{10}\left(d x +c \right)}{105 \sin \left(d x +c \right)^{5}}+\frac{\cos^{10}\left(d x +c \right)}{63 \sin \left(d x +c \right)^{3}}-\frac{\cos^{10}\left(d x +c \right)}{9 \sin \left(d x +c \right)}-\frac{\left(\frac{128}{35}+\cos^{8}\left(d x +c \right)+\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/9*a^2/sin(d*x+c)^9*cos(d*x+c)^9+2*a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)
^10+1/63/sin(d*x+c)^7*cos(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/s
in(d*x+c)^3*cos(d*x+c)^10-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+
c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))+a^
```

```
2*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot
(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 137, normalized size = 0.77

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan\left(dx+c\right)^{8}-105\,\tan\left(dx+c\right)^{6}+63\,\tan\left(dx+c\right)^{4}-45\,\tan\left(dx+c\right)^{2}+35}{\tan\left(dx+c\right)^{9}}\right)a^{2}+\frac{2\left(315\,\sin\left(dx+c\right)^{8}-420\,\sin\left(dx+c\right)^{6}+378\,\sin\left(dx+c\right)^{4}-180\,\sin\left(dx+c\right)^{2}+35\right)a^{2}}{\sin\left(dx+c\right)^{9}}+\frac{35\,a^{2}}{\tan\left(dx+c\right)^{9}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^2 + 2*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^2/sin(d*x + c)^9 + 35*a^2/tan(d*x + c)^9)/d
```

Fricas [A]
time = 2.92, size = 274, normalized size = 1.53

$$-\frac{598\,a^{2}\cos\left(dx+c\right)^{7}-566\,a^{2}\cos\left(dx+c\right)^{6}-1212\,a^{2}\cos\left(dx+c\right)^{5}+1310\,a^{2}\cos\left(dx+c\right)^{4}+860\,a^{2}\cos\left(dx+c\right)^{3}-1014\,a^{2}\cos\left(dx+c\right)^{2}-197\,a^{2}\cos\left(dx+c\right)+256\,a^{2}+315\left(a^{2}dx\cos\left(dx+c\right)^{6}-2\,a^{2}dx\cos\left(dx+c\right)^{5}-a^{2}dx\cos\left(dx+c\right)^{4}+4\,a^{2}dx\cos\left(dx+c\right)^{3}-a^{2}dx\cos\left(dx+c\right)^{2}-2\,a^{2}dx\cos\left(dx+c\right)+a^{2}dx\right)\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)^{6}-2\,d\cos\left(dx+c\right)^{5}-d\cos\left(dx+c\right)^{4}+4\,d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)^{2}-2\,d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*a^2*cos(d*x + c)^7 - 566*a^2*cos(d*x + c)^6 - 1212*a^2*cos(d*x
+ c)^5 + 1310*a^2*cos(d*x + c)^4 + 860*a^2*cos(d*x + c)^3 - 1014*a^2*cos(d*
x + c)^2 - 197*a^2*cos(d*x + c) + 256*a^2 + 315*(a^2*d*x*cos(d*x + c)^6 - 2
*a^2*d*x*cos(d*x + c)^5 - a^2*d*x*cos(d*x + c)^4 + 4*a^2*d*x*cos(d*x + c)^3
 - a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*sin(d*x + c))
/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*cos(d*x + c)^4 + 4*d*cos(d*x +
 c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 145, normalized size = 0.81

$$\frac{63\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-945\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40320\,(dx+c)a^{2}+11655\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{51345\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-9765\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2331\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-405\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 - 945*a^2*tan(1/2*d*x + 1/2*c)^3 - 4
0320*(d*x + c)*a^2 + 11655*a^2*tan(1/2*d*x + 1/2*c) - (51345*a^2*tan(1/2*d*
x + 1/2*c)^8 - 9765*a^2*tan(1/2*d*x + 1/2*c)^6 + 2331*a^2*tan(1/2*d*x + 1/2
*c)^4 - 405*a^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 2.78, size = 230, normalized size = 1.28

$$-\frac{a^2\left(35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-63\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+945\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-11655\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+51345\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-9765\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2331\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-405\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+40320\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(c+d\,x\right)\right)}{40320\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(35*cos(c/2 + (d*x)/2)^14 - 63*sin(c/2 + (d*x)/2)^14 + 945*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 11655*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*
x)/2)^10 + 51345*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 9765*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 2331*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^4 - 405*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 +
(d*x)/2)^5*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(40320*d*cos(c/2 + (d*x)/2)^5*s
in(c/2 + (d*x)/2)^9)
```

3.37 $\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx$

Optimal. Leaf size=210

$$-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{2d}-\frac{11a^3\sec^3(c+dx)}{3d}-\frac{3a^3\sec^4(c+dx)}{2d}+\frac{14a^3\sec^5(c+$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d-1/2*a^3*sec(d*x+c)^2/d-11/3*a^3*se
c(d*x+c)^3/d-3/2*a^3*sec(d*x+c)^4/d+14/5*a^3*sec(d*x+c)^5/d+7/3*a^3*sec(d*x
+c)^6/d-6/7*a^3*sec(d*x+c)^7/d-11/8*a^3*sec(d*x+c)^8/d-1/9*a^3*sec(d*x+c)^9
/d+3/10*a^3*sec(d*x+c)^10/d+1/11*a^3*sec(d*x+c)^11/d
```

Rubi [A]
time = 0.07, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{a^3\sec^{11}(c+dx)}{11d}+\frac{3a^3\sec^{10}(c+dx)}{10d}-\frac{a^3\sec^9(c+dx)}{9d}-\frac{11a^3\sec^8(c+dx)}{8d}-\frac{6a^3\sec^7(c+dx)}{7d}+\frac{7a^3\sec^6(c+dx)}{3d}+\frac{14a^3\sec^5(c+dx)}{5d}-\frac{3a^3\sec^4(c+dx)}{2d}-\frac{11a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2
)/(2*d) - (11*a^3*Sec[c + d*x]^3)/(3*d) - (3*a^3*Sec[c + d*x]^4)/(2*d) + (1
4*a^3*Sec[c + d*x]^5)/(5*d) + (7*a^3*Sec[c + d*x]^6)/(3*d) - (6*a^3*Sec[c +
 d*x]^7)/(7*d) - (11*a^3*Sec[c + d*x]^8)/(8*d) - (a^3*Sec[c + d*x]^9)/(9*d)
 + (3*a^3*Sec[c + d*x]^10)/(10*d) + (a^3*Sec[c + d*x]^11)/(11*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^7}{x^{12}}\,dx, x, \cos(c+dx)\right)}{a^8d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^{11}}{x^{12}} + \frac{3a^{11}}{x^{11}} - \frac{a^{11}}{x^{10}} - \frac{11a^{11}}{x^9} - \frac{6a^{11}}{x^8} + \frac{14a^{11}}{x^7} + \frac{14a^{11}}{x^6} - \frac{6a^{11}}{x^5} \cdots\right.\right.}{a^8d}$$
$$= -\frac{a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} - \frac{a^3\sec^2(c+dx)}{2d} - \frac{11a^3 \cdots}{}$$

Mathematica [A]
time = 0.70, size = 214, normalized size = 1.02

$$-\frac{a^3(-1151740 - 1613260\cos(2(c+dx)) + 960960\cos(3(c+dx)) - 1131504\cos(4(c+dx)) + 314160\cos(5(c+dx)) - 432894\cos(6(c+dx)) + 145530\cos(7(c+dx)) - 106260\cos(8(c+dx)) + 6930\cos(9(c+dx)) - 20790\cos(10(c+dx)) + 1143450\cos(3(c+dx))\log(\cos(c+dx)) + 571725\cos(5(c+dx))\log(\cos(c+dx)) + 190575\cos(7(c+dx))\log(\cos(c+dx)) + 38115\cos(9(c+dx))\log(\cos(c+dx)) + 3465\cos(11(c+dx))\log(\cos(c+dx)) + 462\cos(c+dx)(2606 + 3465\log(\cos(c+dx))))\sec^{11}(c+dx)}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -1/3548160*(a^3*(-1151740 - 1613260*Cos[2*(c + d*x)] + 960960*Cos[3*(c + d*
x)] - 1131504*Cos[4*(c + d*x)] + 314160*Cos[5*(c + d*x)] - 432894*Cos[6*(c
+ d*x)] + 145530*Cos[7*(c + d*x)] - 106260*Cos[8*(c + d*x)] + 6930*Cos[9*(c
 + d*x)] - 20790*Cos[10*(c + d*x)] + 1143450*Cos[3*(c + d*x)]*Log[Cos[c + d
*x]] + 571725*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 190575*Cos[7*(c + d*x)]*
Log[Cos[c + d*x]] + 38115*Cos[9*(c + d*x)]*Log[Cos[c + d*x]] + 3465*Cos[11*
(c + d*x)]*Log[Cos[c + d*x]] + 462*Cos[c + d*x]*(2606 + 3465*Log[Cos[c + d*
x]]))*Sec[c + d*x]^11)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(190) = 380.
time = 0.20, size = 386, normalized size = 1.84

method	result
risch	$ia^3x + \frac{2ia^3c}{d} + \frac{2a^3(10395\,e^{21i(dx+c)} - 3465\,e^{20i(dx+c)} + 53130\,e^{19i(dx+c)} - 72765\,e^{18i(dx+c)} + 216447\,e^{17i(dx+c)} - 15 \cdots}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{11\cos(dx+c)^{11}} + \frac{\sin^{10}(dx+c)}{99\cos(dx+c)^9} - \frac{\sin^{10}(dx+c)}{693\cos(dx+c)^7} + \frac{\sin^{10}(dx+c)}{1155\cos(dx+c)^5} - \frac{\sin^{10}(dx+c)}{693\cos(dx+c)^3} + \frac{\sin^{10}(dx+c)}{99\cos(dx+c)} + \frac{\left(\frac{128}{35} + \sin^8(dx+c) + \cdots\right.}{} \cdots\right.}{}$
default	$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{11\cos(dx+c)^{11}} + \frac{\sin^{10}(dx+c)}{99\cos(dx+c)^9} - \frac{\sin^{10}(dx+c)}{693\cos(dx+c)^7} + \frac{\sin^{10}(dx+c)}{1155\cos(dx+c)^5} - \frac{\sin^{10}(dx+c)}{693\cos(dx+c)^3} + \frac{\sin^{10}(dx+c)}{99\cos(dx+c)} + \frac{\left(\frac{128}{35} + \sin^8(dx+c) + \cdots\right.}{} \cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/11*sin(d*x+c)^10/cos(d*x+c)^11+1/99*sin(d*x+c)^10/cos(d*x+c)^9-
1/693*sin(d*x+c)^10/cos(d*x+c)^7+1/1155*sin(d*x+c)^10/cos(d*x+c)^5-1/693*si
n(d*x+c)^10/cos(d*x+c)^3+1/99*sin(d*x+c)^10/cos(d*x+c)+1/99*(128/35+sin(d*x
+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+3
/10*a^3*sin(d*x+c)^10/cos(d*x+c)^10+3*a^3*(1/9*sin(d*x+c)^10/cos(d*x+c)^9-1
/63*sin(d*x+c)^10/cos(d*x+c)^7+1/105*sin(d*x+c)^10/cos(d*x+c)^5-1/63*sin(d*
x+c)^10/cos(d*x+c)^3+1/9*sin(d*x+c)^10/cos(d*x+c)+1/9*(128/35+sin(d*x+c)^8+
8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+a^3*(1/
8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*
x+c))))
```

Maxima [A]
time = 0.26, size = 162, normalized size = 0.77

$$-\frac{27720\,a^{3}\log\left(\cos\left(dx+c\right)\right)-\frac{83160\,a^{3}\cos\left(dx+c\right)^{10}-13860\,a^{3}\cos\left(dx+c\right)^{9}-101640\,a^{3}\cos\left(dx+c\right)^{8}-41580\,a^{3}\cos\left(dx+c\right)^{7}+77616\,a^{3}\cos\left(dx+c\right)^{6}+64680\,a^{3}\cos\left(dx+c\right)^{5}-23760\,a^{3}\cos\left(dx+c\right)^{4}-38115\,a^{3}\cos\left(dx+c\right)^{3}-3080\,a^{3}\cos\left(dx+c\right)^{2}+8316\,a^{3}\cos\left(dx+c\right)+2520\,a^{3}}{\cos\left(dx+c\right)^{11}}}{27720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/27720*(27720*a^3*log(cos(d*x + c)) - (83160*a^3*cos(d*x + c)^10 - 13860*
a^3*cos(d*x + c)^9 - 101640*a^3*cos(d*x + c)^8 - 41580*a^3*cos(d*x + c)^7 +
 77616*a^3*cos(d*x + c)^6 + 64680*a^3*cos(d*x + c)^5 - 23760*a^3*cos(d*x +
c)^4 - 38115*a^3*cos(d*x + c)^3 - 3080*a^3*cos(d*x + c)^2 + 8316*a^3*cos(d*
x + c) + 2520*a^3)/cos(d*x + c)^11)/d
```

Fricas [A]
time = 3.58, size = 169, normalized size = 0.80

$$-\frac{27720\,a^{3}\cos\left(dx+c\right)^{11}\log\left(-\cos\left(dx+c\right)\right)-83160\,a^{3}\cos\left(dx+c\right)^{10}+13860\,a^{3}\cos\left(dx+c\right)^{9}+101640\,a^{3}\cos\left(dx+c\right)^{8}+41580\,a^{3}\cos\left(dx+c\right)^{7}-77616\,a^{3}\cos\left(dx+c\right)^{6}-64680\,a^{3}\cos\left(dx+c\right)^{5}+23760\,a^{3}\cos\left(dx+c\right)^{4}+38115\,a^{3}\cos\left(dx+c\right)^{3}+3080\,a^{3}\cos\left(dx+c\right)^{2}-8316\,a^{3}\cos\left(dx+c\right)-2520\,a^{3}}{27720\,d\cos\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/27720*(27720*a^3*cos(d*x + c)^11*log(-cos(d*x + c)) - 83160*a^3*cos(d*x
+ c)^10 + 13860*a^3*cos(d*x + c)^9 + 101640*a^3*cos(d*x + c)^8 + 41580*a^3*
cos(d*x + c)^7 - 77616*a^3*cos(d*x + c)^6 - 64680*a^3*cos(d*x + c)^5 + 2376
0*a^3*cos(d*x + c)^4 + 38115*a^3*cos(d*x + c)^3 + 3080*a^3*cos(d*x + c)^2 -
 8316*a^3*cos(d*x + c) - 2520*a^3)/(d*cos(d*x + c)^11)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(190) = 380$.
time = 4.61, size = 439, normalized size = 2.09

$$\begin{cases} \frac{a^{3}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a^{3}\tan^{8}\left(c+dx\right)\sec^{3}\left(c+dx\right)}{11d}+\frac{3a^{3}\tan^{8}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}+\frac{a^{3}\tan^{8}\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a^{3}\tan^{8}\left(c+dx\right)}{8d}-\frac{8a^{3}\tan^{6}\left(c+dx\right)\sec^{3}\left(c+dx\right)}{99d}-\frac{3a^{3}\tan^{6}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}-\frac{8a^{3}\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)}{21d}-\frac{a^{3}\tan^{6}\left(c+dx\right)}{6d}+\frac{16a^{3}\tan^{4}\left(c+dx\right)\sec^{3}\left(c+dx\right)}{231d}+\frac{3a^{3}\tan^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}+\frac{16a^{3}\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{a^{3}\tan^{4}\left(c+dx\right)}{4d}-\frac{64a^{3}\tan^{2}\left(c+dx\right)\sec^{3}\left(c+dx\right)}{1155d}-\frac{3a^{3}\tan^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}-\frac{64a^{3}\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{105d}-\frac{a^{3}\tan^{2}\left(c+dx\right)}{2d}+\frac{128a^{3}\sec^{3}\left(c+dx\right)}{3465d}+\frac{3a^{3}\sec^{2}\left(c+dx\right)}{10d}+\frac{128a^{3}\sec\left(c+dx\right)}{105d} & \text{for}\: d \neq 0 \\ x\left(a\sec\left(c\right)+a\right)^{3}\tan^{9}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**8*sec(c
 + d*x)**3/(11*d) + 3*a**3*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) + a**3*ta
n(c + d*x)**8*sec(c + d*x)/(3*d) + a**3*tan(c + d*x)**8/(8*d) - 8*a**3*tan(
c + d*x)**6*sec(c + d*x)**3/(99*d) - 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2
/(10*d) - 8*a**3*tan(c + d*x)**6*sec(c + d*x)/(21*d) - a**3*tan(c + d*x)**6
/(6*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)**3/(231*d) + 3*a**3*tan(c + d
*x)**4*sec(c + d*x)**2/(10*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d)
 + a**3*tan(c + d*x)**4/(4*d) - 64*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(11
55*d) - 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(10*d) - 64*a**3*tan(c + d*x
)**2*sec(c + d*x)/(105*d) - a**3*tan(c + d*x)**2/(2*d) + 128*a**3*sec(c + d
*x)**3/(3465*d) + 3*a**3*sec(c + d*x)**2/(10*d) + 128*a**3*sec(c + d*x)/(10
5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**9, True))
```

Giac [A]
time = 6.19, size = 367, normalized size = 1.75

$$\frac{27720\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-27720\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{153343\,a^3+\frac{1742213\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9043705\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{28369275\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{59954070\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{67458930\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{57997170\,a^3(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{36975510\,a^3(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{16879995\,a^3(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}+\frac{5213945\,a^3(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}+\frac{976261\,a^3(\cos(dx+c)-1)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{83711\,a^3(\cos(dx+c)-1)^{11}}{(\cos(dx+c)+1)^{11}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{11}}}{27720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/27720*(27720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2
7720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (153343*a^3
 + 1742213*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9043705*a^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28369275*a^3*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 + 59954070*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
67458930*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 57997170*a^3*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 36975510*a^3*(cos(d*x + c) - 1)^7/(c
os(d*x + c) + 1)^7 + 16879995*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8
 + 5213945*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 976261*a^3*(cos(
d*x + c) - 1)^10/(cos(d*x + c) + 1)^10 + 83711*a^3*(cos(d*x + c) - 1)^11/(c
os(d*x + c) + 1)^11)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^11)/d
```

Mupad [B]
time = 5.26, size = 337, normalized size = 1.60

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-22\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+\frac{332\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{3}-\frac{1012\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}+\frac{10456\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{15}-\frac{5192\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{5}+\frac{8164\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{7}-\frac{3676\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{7}+\frac{10090\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{63}-\frac{9334\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{315}+\frac{8704\,a^3}{3465}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{22}-11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-165\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+330\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-462\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+462\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-330\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+165\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((10090*a^3*tan(c/2 + (d*x)/2)^4)/6
3 - (9334*a^3*tan(c/2 + (d*x)/2)^2)/315 - (3676*a^3*tan(c/2 + (d*x)/2)^6)/7
```

```
 + (8164*a^3*tan(c/2 + (d*x)/2)^8)/7 - (5192*a^3*tan(c/2 + (d*x)/2)^10)/5 +
 (10456*a^3*tan(c/2 + (d*x)/2)^12)/15 - (1012*a^3*tan(c/2 + (d*x)/2)^14)/3
+ (332*a^3*tan(c/2 + (d*x)/2)^16)/3 - 22*a^3*tan(c/2 + (d*x)/2)^18 + 2*a^3*
tan(c/2 + (d*x)/2)^20 + (8704*a^3)/3465)/(d*(11*tan(c/2 + (d*x)/2)^2 - 55*t
an(c/2 + (d*x)/2)^4 + 165*tan(c/2 + (d*x)/2)^6 - 330*tan(c/2 + (d*x)/2)^8 +
 462*tan(c/2 + (d*x)/2)^10 - 462*tan(c/2 + (d*x)/2)^12 + 330*tan(c/2 + (d*x
)/2)^14 - 165*tan(c/2 + (d*x)/2)^16 + 55*tan(c/2 + (d*x)/2)^18 - 11*tan(c/2
 + (d*x)/2)^20 + tan(c/2 + (d*x)/2)^22 - 1))
```

3.38 $\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{8a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{2d}-\frac{6a^3\sec^5(c+dx)}{5d}-\frac{4a^3\sec^6(c+}{3d}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d+8/3*a^3*sec(d*x+c)^3/d+3/2*a^3*sec(
d*x+c)^4/d-6/5*a^3*sec(d*x+c)^5/d-4/3*a^3*sec(d*x+c)^6/d+3/8*a^3*sec(d*x+c)
^8/d+1/9*a^3*sec(d*x+c)^9/d
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{a^3\sec^9(c+dx)}{9d}+\frac{3a^3\sec^8(c+dx)}{8d}-\frac{4a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{2d}+\frac{8a^3\sec^3(c+dx)}{3d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d + (8*a^3*Sec[c + d*x]^3)
/(3*d) + (3*a^3*Sec[c + d*x]^4)/(2*d) - (6*a^3*Sec[c + d*x]^5)/(5*d) - (4*a
^3*Sec[c + d*x]^6)/(3*d) + (3*a^3*Sec[c + d*x]^8)/(8*d) + (a^3*Sec[c + d*x]
^9)/(9*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^6}{x^{10}}\,dx,x,\cos(c+dx)\right)}{a^6d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}}+\frac{3a^9}{x^9}-\frac{8a^9}{x^7}-\frac{6a^9}{x^6}+\frac{6a^9}{x^5}+\frac{8a^9}{x^4}-\frac{3a^9}{x^2}-\frac{a^9}{x}\right)dx,x,\text{co}}{a^6d}$$
$$= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{8a^3\sec^3(c+dx)}{3d}+\frac{3a^3\text{sec}}{}$$

Mathematica [A]
time = 0.38, size = 110, normalized size = 0.80

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)(360\log(\cos(c+dx))-1080\sec(c+dx)+960\sec^3(c+dx)+540\sec^4(c+dx)-432\sec^5(c+dx)-480\sec^6(c+dx)+135\sec^8(c+dx)+40\sec^9(c+dx))}{2880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(360*Log[Cos[c + d*x]] - 1080*
Sec[c + d*x] + 960*Sec[c + d*x]^3 + 540*Sec[c + d*x]^4 - 432*Sec[c + d*x]^5
 - 480*Sec[c + d*x]^6 + 135*Sec[c + d*x]^8 + 40*Sec[c + d*x]^9))/(2880*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(125) = 250.
time = 0.15, size = 318, normalized size = 2.32

method	result
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(135\,e^{17i(dx+c)}+600\,e^{15i(dx+c)}-540\,e^{14i(dx+c)}+1764\,e^{13i(dx+c)}-780\,e^{12i(dx+c)}+3816\,e^{11i(dx}\right.}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin^8(dx+c)}{9\cos(dx+c)^9}+\frac{\sin^8(dx+c)}{63\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{315\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{315\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{63\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(d\right.}{5}\right.}{63}\right.}{}$
default	$\frac{a^3\left(\frac{\sin^8(dx+c)}{9\cos(dx+c)^9}+\frac{\sin^8(dx+c)}{63\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{315\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{315\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{63\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(d\right.}{5}\right.}{63}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/9*sin(d*x+c)^8/cos(d*x+c)^9+1/63*sin(d*x+c)^8/cos(d*x+c)^7-1/31
5*sin(d*x+c)^8/cos(d*x+c)^5+1/315*sin(d*x+c)^8/cos(d*x+c)^3-1/63*sin(d*x+c)
```

```
^8/cos(d*x+c)-1/63*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*co
s(d*x+c))+3/8*a^3*sin(d*x+c)^8/cos(d*x+c)^8+3*a^3*(1/7*sin(d*x+c)^8/cos(d*x
+c)^7-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35*sin(d*x+c)^8/cos(d*x+c)^3-1/7*sin
(d*x+c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)
^2)*cos(d*x+c))+a^3*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2+ln(
cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 110, normalized size = 0.80

$$\frac{360\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{1080\,a^3\cos\left(dx+c\right)^8-960\,a^3\cos\left(dx+c\right)^6-540\,a^3\cos\left(dx+c\right)^5+432\,a^3\cos\left(dx+c\right)^4+480\,a^3\cos\left(dx+c\right)^3-135\,a^3\cos\left(dx+c\right)-40\,a^3}{\cos\left(dx+c\right)^9}}{360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/360*(360*a^3*log(cos(d*x + c)) - (1080*a^3*cos(d*x + c)^8 - 960*a^3*cos(d
*x + c)^6 - 540*a^3*cos(d*x + c)^5 + 432*a^3*cos(d*x + c)^4 + 480*a^3*cos(d
*x + c)^3 - 135*a^3*cos(d*x + c) - 40*a^3)/cos(d*x + c)^9)/d
```

Fricas [A]
time = 3.27, size = 117, normalized size = 0.85

$$\frac{360\,a^3\cos\left(dx+c\right)^9\log\left(-\cos\left(dx+c\right)\right)-1080\,a^3\cos\left(dx+c\right)^8+960\,a^3\cos\left(dx+c\right)^6+540\,a^3\cos\left(dx+c\right)^5-432\,a^3\cos\left(dx+c\right)^4-480\,a^3\cos\left(dx+c\right)^3+135\,a^3\cos\left(dx+c\right)+40\,a^3}{360\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(360*a^3*cos(d*x + c)^9*log(-cos(d*x + c)) - 1080*a^3*cos(d*x + c)^8
+ 960*a^3*cos(d*x + c)^6 + 540*a^3*cos(d*x + c)^5 - 432*a^3*cos(d*x + c)^4
- 480*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c) + 40*a^3)/(d*cos(d*x + c)^9
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(126) = 252$.
time = 2.40, size = 350, normalized size = 2.55

$$\begin{cases} -\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^6\left(c+dx\right)\sec^3\left(c+dx\right)}{9d}+\frac{3a^3\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{3a^3\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}+\frac{a^3\tan^6\left(c+dx\right)}{6d}-\frac{2a^3\tan^4\left(c+dx\right)\sec^3\left(c+dx\right)}{21d}-\frac{3a^3\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}-\frac{18a^3\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{35d}-\frac{a^3\tan^4\left(c+dx\right)}{4d}+\frac{8a^3\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)}{105d}+\frac{3a^3\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{24a^3\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{a^3\tan^2\left(c+dx\right)}{2d}-\frac{16a^3\sec^3\left(c+dx\right)}{315d}-\frac{3a^3\sec^2\left(c+dx\right)}{8d}-\frac{48a^3\sec\left(c+dx\right)}{35d} & \text{for}\: d \neq 0 \\ x\left(a\sec\left(c\right)+a\right)^3\tan^7\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**6*sec(
c + d*x)**3/(9*d) + 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) + 3*a**3*t
an(c + d*x)**6*sec(c + d*x)/(7*d) + a**3*tan(c + d*x)**6/(6*d) - 2*a**3*tan
(c + d*x)**4*sec(c + d*x)**3/(21*d) - 3*a**3*tan(c + d*x)**4*sec(c + d*x)**
```

```
2/(8*d) - 18*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d) - a**3*tan(c + d*x)**
4/(4*d) + 8*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(105*d) + 3*a**3*tan(c + d
*x)**2*sec(c + d*x)**2/(8*d) + 24*a**3*tan(c + d*x)**2*sec(c + d*x)/(35*d)
+ a**3*tan(c + d*x)**2/(2*d) - 16*a**3*sec(c + d*x)**3/(315*d) - 3*a**3*sec
(c + d*x)**2/(8*d) - 48*a**3*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
 a)**3*tan(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(125) = 250$.
time = 3.97, size = 317, normalized size = 2.31

$$-\frac{2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{14297\,a^3+\frac{133713\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{560052\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1384068\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{1594782\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1336734\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{781956\,a^3(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{302004\,a^3(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{69201\,a^3(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}+\frac{7129\,a^3(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^9}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (14297*a^3 +
133713*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 560052*a^3*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 + 1384068*a^3*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 + 1594782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734
*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a^3*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 302004*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c)
+ 1)^7 + 69201*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a^3*(co
s(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1)^9)/d
```

Mupad [B]
time = 5.17, size = 278, normalized size = 2.03

$$\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-18\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+\frac{218\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3}-174\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{1382\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{5}-\frac{1558\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{5}+\frac{602\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{5}-\frac{138\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{5}+\frac{128\,a^3}{45}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-1\right)}-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((602*a^3*tan(c/2 + (d*x)/2)^4)/5 - (138*a^3*tan(c/2 + (d*x)/2)^2)/5 - (155
8*a^3*tan(c/2 + (d*x)/2)^6)/5 + (1382*a^3*tan(c/2 + (d*x)/2)^8)/5 - 174*a^3
*tan(c/2 + (d*x)/2)^10 + (218*a^3*tan(c/2 + (d*x)/2)^12)/3 - 18*a^3*tan(c/2
 + (d*x)/2)^14 + 2*a^3*tan(c/2 + (d*x)/2)^16 + (128*a^3)/45)/(d*(9*tan(c/2
+ (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126*tan(
c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^12 + 3
6*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 -
 1)) - (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.39 $\int (a + a\sec(c + dx))^3 \tan^5(c + dx)\,dx$

Optimal. Leaf size=138

$$-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}-\frac{5a^3\sec^3(c+dx)}{3d}-\frac{5a^3\sec^4(c+dx)}{4d}+\frac{a^3\sec^5(c+d}{5d}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d-5/3*a^3*sec
(d*x+c)^3/d-5/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x+c
)^6/d+1/7*a^3*sec(d*x+c)^7/d
```

Rubi [A]

time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{a^3\sec^7(c+dx)}{7d}+\frac{a^3\sec^6(c+dx)}{2d}+\frac{a^3\sec^5(c+dx)}{5d}-\frac{5a^3\sec^4(c+dx)}{4d}-\frac{5a^3\sec^3(c+dx)}{3d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2
)/(2*d) - (5*a^3*Sec[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]^4)/(4*d) + (a^
3*Sec[c + d*x]^5)/(5*d) + (a^3*Sec[c + d*x]^6)/(2*d) + (a^3*Sec[c + d*x]^7)
/(7*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx=-\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^5}{x^8}\,dx,x,\cos(c+dx)\right)}{a^4d}$$
$$=-\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{3a^7}{x^7}+\frac{a^7}{x^6}-\frac{5a^7}{x^5}-\frac{5a^7}{x^4}+\frac{a^7}{x^3}+\frac{3a^7}{x^2}+\frac{a^7}{x}\right)\,dx,x,\cos(\right.}{a^4d}$$
$$=-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}-\frac{5a^3\sec}{}$$

Mathematica [A]
time = 0.41, size = 140, normalized size = 1.01

$$-\frac{a^3(-3732-4522\cos(2(c+dx))+1050\cos(3(c+dx))-2380\cos(4(c+dx))-210\cos(5(c+dx))-630\cos(6(c+dx))+2205\cos(3(c+dx))\log(\cos(c+dx))+735\cos(5(c+dx))\log(\cos(c+dx))+105\cos(7(c+dx))\log(\cos(c+dx))+105\cos(c+dx)(8+35\log(\cos(c+dx))))\sec^7(c+dx)}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/6720*(a^3*(-3732 - 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] - 2380*
Cos[4*(c + d*x)] - 210*Cos[5*(c + d*x)] - 630*Cos[6*(c + d*x)] + 2205*Cos[3
*(c + d*x)]*Log[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 10
5*Cos[7*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c +
 d*x]]))*Sec[c + d*x]^7)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(126) = 252$.
time = 0.12, size = 254, normalized size = 1.84

method	result
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{2a^3\left(315\,e^{13i(dx+c)}+105\,e^{12i(dx+c)}+1190\,e^{11i(dx+c)}-525\,e^{10i(dx+c)}+2261\,e^{9i(dx+c)}-420\,e^{8i(dx+c)}+\right.}{105d(e}$
derivativedivides	$a^3\left(\frac{\sin^6(dx+c)}{7\cos(dx+c)^7}+\frac{\sin^6(dx+c)}{35\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{105\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{35\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{35}\right)+\frac{a^3\left(\sin^6(dx+\right.}{2\cos(dx+c}$
default	$a^3\left(\frac{\sin^6(dx+c)}{7\cos(dx+c)^7}+\frac{\sin^6(dx+c)}{35\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{105\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{35\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{35}\right)+\frac{a^3\left(\sin^6(dx+\right.}{2\cos(dx+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5-1/10
5*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+1/2*a^3*sin(d*x+c)^6/cos(d*x+c)^6+3*a^3*
(1/5*sin(d*x+c)^6/cos(d*x+c)^5-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin(d*x+c
)^6/cos(d*x+c)+1/5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a^3*(1/4
*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 110, normalized size = 0.80

$$-\frac{420\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{1260\,a^3\cos\left(dx+c\right)^6+210\,a^3\cos\left(dx+c\right)^5-700\,a^3\cos\left(dx+c\right)^4-525\,a^3\cos\left(dx+c\right)^3+84\,a^3\cos\left(dx+c\right)^2+210\,a^3\cos\left(dx+c\right)+60\,a^3}{\cos\left(dx+c\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a^3*log(cos(d*x + c)) - (1260*a^3*cos(d*x + c)^6 + 210*a^3*cos(
d*x + c)^5 - 700*a^3*cos(d*x + c)^4 - 525*a^3*cos(d*x + c)^3 + 84*a^3*cos(d
*x + c)^2 + 210*a^3*cos(d*x + c) + 60*a^3)/cos(d*x + c)^7)/d
```

Fricas [A]
time = 2.22, size = 117, normalized size = 0.85

$$-\frac{420\,a^3\cos\left(dx+c\right)^7\log\left(-\cos\left(dx+c\right)\right)-1260\,a^3\cos\left(dx+c\right)^6-210\,a^3\cos\left(dx+c\right)^5+700\,a^3\cos\left(dx+c\right)^4+525\,a^3\cos\left(dx+c\right)^3-84\,a^3\cos\left(dx+c\right)^2-210\,a^3\cos\left(dx+c\right)-60\,a^3}{420\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a^3*cos(d*x + c)^7*log(-cos(d*x + c)) - 1260*a^3*cos(d*x + c)^6
 - 210*a^3*cos(d*x + c)^5 + 700*a^3*cos(d*x + c)^4 + 525*a^3*cos(d*x + c)^3
 - 84*a^3*cos(d*x + c)^2 - 210*a^3*cos(d*x + c) - 60*a^3)/(d*cos(d*x + c)^7
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(121) = 242$.
time = 1.21, size = 255, normalized size = 1.85

$$\begin{cases}\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^4\left(c+dx\right)\sec^3\left(c+dx\right)}{7d}+\frac{a^3\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{2d}+\frac{3a^3\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{5d}+\frac{a^3\tan^4\left(c+dx\right)}{4d}-\frac{4a^3\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)}{35d}-\frac{a^3\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{2d}-\frac{4a^3\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{5d}-\frac{a^3\tan^2\left(c+dx\right)}{2d}+\frac{8a^3\sec^3\left(c+dx\right)}{105d}+\frac{a^3\sec^2\left(c+dx\right)}{2d}+\frac{8a^3\sec\left(c+dx\right)}{5d} & \text{for}\: d\neq 0\\x\left(a\sec\left(c\right)+a\right)^3\tan^5\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**4*sec(c
 + d*x)**3/(7*d) + a**3*tan(c + d*x)**4*sec(c + d*x)**2/(2*d) + 3*a**3*tan(
c + d*x)**4*sec(c + d*x)/(5*d) + a**3*tan(c + d*x)**4/(4*d) - 4*a**3*tan(c
+ d*x)**2*sec(c + d*x)**3/(35*d) - a**3*tan(c + d*x)**2*sec(c + d*x)**2/(2*
```

```
d) - 4*a**3*tan(c + d*x)**2*sec(c + d*x)/(5*d) - a**3*tan(c + d*x)**2/(2*d)
 + 8*a**3*sec(c + d*x)**3/(105*d) + a**3*sec(c + d*x)**2/(2*d) + 8*a**3*sec
(c + d*x)/(5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(126) = 252$.
time = 2.13, size = 267, normalized size = 1.93

$$\frac{420\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2497\,a^3+\frac{18319\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{58317\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{69475\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{28749\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{8463\,a^3(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{1089\,a^3(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2497*a^3 + 18319
*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 58317*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 69475*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 56035*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^3*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a^3*(cos(d*x + c) - 1)^6/(cos(d*x
+ c) + 1)^6 + 1089*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d
```

Mupad [B]
time = 5.48, size = 221, normalized size = 1.60

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-\frac{224\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{422\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}-\frac{382\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{352\,a^3}{105}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((422*a^3*tan(c/2 + (d*x)/2)^4)/5 -
 (382*a^3*tan(c/2 + (d*x)/2)^2)/15 - (224*a^3*tan(c/2 + (d*x)/2)^6)/3 + (12
8*a^3*tan(c/2 + (d*x)/2)^8)/3 - 14*a^3*tan(c/2 + (d*x)/2)^10 + 2*a^3*tan(c/
2 + (d*x)/2)^12 + (352*a^3)/105)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 +
(d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2
 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.40 $\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=99

$$\frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{d}+\frac{2a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{a^3\sec^5(c+dx)}{5d}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d-a^3*sec(d*x+c)^2/d+2/3*a^3*sec(d*x+
c)^3/d+3/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3964, 76}

$$\frac{a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{2a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2)/d
 + (2*a^3*Sec[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]^4)/(4*d) + (a^3*Sec[c
 + d*x]^5)/(5*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^4}{x^6}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{3a^5}{x^5}+\frac{2a^5}{x^4}-\frac{2a^5}{x^3}-\frac{3a^5}{x^2}-\frac{a^5}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= \frac{a^3\log(\cos(c+dx))}{d} - \frac{3a^3\sec(c+dx)}{d} - \frac{a^3\sec^2(c+dx)}{d} + \frac{2a^3\sec^3}{3}$$

Mathematica [A]
time = 0.29, size = 92, normalized size = 0.93

$$-\frac{a^3(142+280\cos(2(c+dx))+90\cos(4(c+dx))+\cos(3(c+dx))(60-75\log(\cos(c+dx)))-150\cos(c+dx)\log(\cos(c+dx))-15\cos(5(c+dx))\log(\cos(c+dx)))\sec^5(c+dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/240*(a^3*(142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + Cos[3*(c + 
d*x)]*(60 - 75*Log[Cos[c + d*x]]) - 150*Cos[c + d*x]*Log[Cos[c + d*x]] - 15
*Cos[5*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^5)/d
```

Maple [A]
time = 0.10, size = 186, normalized size = 1.88

method	result
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(45\,e^{9i(dx+c)}+30\,e^{8i(dx+c)}+140\,e^{7i(dx+c)}+142\,e^{5i(dx+c)}+140\,e^{3i(dx+c)}+30\,e^{2i(dx+c)}+45\,e^{i(dx}\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+\frac{3a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx}{3\cos(dx}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+\frac{3a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx}{3\cos(dx}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15
*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+3/4*a^3*sin(d*x+
c)^4/cos(d*x+c)^4+3*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos
(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c
))))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 0.85

$$\frac{60\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{180\,a^3\cos\left(dx+c\right)^4+60\,a^3\cos\left(dx+c\right)^3-40\,a^3\cos\left(dx+c\right)^2-45\,a^3\cos\left(dx+c\right)-12\,a^3}{\cos\left(dx+c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/60*(60*a^3*log(cos(d*x + c)) - (180*a^3*cos(d*x + c)^4 + 60*a^3*cos(d*x +
 c)^3 - 40*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 12*a^3)/cos(d*x + c)^
5)/d
```

Fricas [A]
time = 2.93, size = 91, normalized size = 0.92

$$\frac{60\,a^3\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-180\,a^3\cos\left(dx+c\right)^4-60\,a^3\cos\left(dx+c\right)^3+40\,a^3\cos\left(dx+c\right)^2+45\,a^3\cos\left(dx+c\right)+12\,a^3}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^3*cos(d*x + c)^4 - 6
0*a^3*cos(d*x + c)^3 + 40*a^3*cos(d*x + c)^2 + 45*a^3*cos(d*x + c) + 12*a^3
)/(d*cos(d*x + c)^5)
```

Sympy [A]
time = 0.53, size = 165, normalized size = 1.67

$$\begin{cases} -\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)}{5d}+\frac{3a^3\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}+\frac{a^3\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{d}+\frac{a^3\tan^2\left(c+dx\right)}{2d}-\frac{2a^3\sec^3\left(c+dx\right)}{15d}-\frac{3a^3\sec^2\left(c+dx\right)}{4d}-\frac{2a^3\sec\left(c+dx\right)}{d} & \text{for}\ d\neq 0 \\ x\left(a\sec\left(c\right)+a\right)^3\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**2*sec(
c + d*x)**3/(5*d) + 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) + a**3*tan
(c + d*x)**2*sec(c + d*x)/d + a**3*tan(c + d*x)**2/(2*d) - 2*a**3*sec(c + d
*x)**3/(15*d) - 3*a**3*sec(c + d*x)**2/(4*d) - 2*a**3*sec(c + d*x)/d, Ne(d,
 0)), (x*(a*sec(c) + a)**3*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(93) = 186.
time = 1.06, size = 217, normalized size = 2.19

$$-\frac{60\,a^3\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)-60\,a^3\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)+\frac{393\,a^3+\frac{2085\,a^3\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{2610\,a^3\left(\cos\left(dx+c\right)-1\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}+\frac{1970\,a^3\left(\cos\left(dx+c\right)-1\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{805\,a^3\left(\cos\left(dx+c\right)-1\right)^4}{\left(\cos\left(dx+c\right)+1\right)^4}+\frac{137\,a^3\left(\cos\left(dx+c\right)-1\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}}{\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (393*a^3 + 2085*a^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a^3*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 1970*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805
*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

Mupad [B]
time = 5.55, size = 162, normalized size = 1.64

$$\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{70\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{64\,a^3}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((62*a^3*tan(c/2 + (d*x)/2)^4)/3 - (70*a^3*tan(c/2 + (d*x)/2)^2)/3 - 10*a^3
*tan(c/2 + (d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)^8 + (64*a^3)/15)/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (2*a^3*atanh(tan(c/2 +
(d*x)/2)^2))/d
```

3.41 $\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=66

$$-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)}{3d}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+3/2*a^3*sec(d*x+c)^2/d+1/3*a^3*sec
(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 45}

$$\frac{a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (3*a^3*Sec[c + d*x]
^2)/(2*d) + (a^3*Sec[c + d*x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int (a + a\sec(c+dx))^3 \tan(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+ax)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4} + \frac{3a^3}{x^3} + \frac{3a^3}{x^2} + \frac{a^3}{x}\right)\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{3a^3\sec^2(c+dx)}{2d} + \frac{a^3\sec$$

Mathematica [A]
time = 0.17, size = 64, normalized size = 0.97

$$-\frac{a^3(-22 - 18\cos(2(c+dx)) + 9\cos(c+dx)(-2+\log(\cos(c+dx))) + 3\cos(3(c+dx))\log(\cos(c+dx)))\sec^3(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
[Out] -1/12*(a^3*(-22 - 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*
x]]) + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/d
```

Maple [A]
time = 0.07, size = 44, normalized size = 0.67

method	result	size
derivativedivides	$\frac{a^3\left(\frac{\left(\sec^3(dx+c)\right)}{3}+\frac{3\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	44
default	$\frac{a^3\left(\frac{\left(\sec^3(dx+c)\right)}{3}+\frac{3\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	44
risch	$ia^3x + \frac{2ia^3c}{d} + \frac{2a^3\left(9\,e^{5i(dx+c)}+9\,e^{4i(dx+c)}+22\,e^{3i(dx+c)}+9\,e^{2i(dx+c)}+9\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3} - \frac{a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*a^3*(1/3*sec(d*x+c)^3+3/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.88

$$-\frac{6\,a^3\log\left(\cos\left(dx+c\right)\right) - \frac{18\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\left(dx+c\right)+2\,a^3}{\cos\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*log(cos(d*x + c)) - (18*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c)
 + 2*a^3)/cos(d*x + c)^3)/d
```

Fricas [A]
time = 3.27, size = 65, normalized size = 0.98

$$-\frac{6\,a^3\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-18\,a^3\cos\left(dx+c\right)^2-9\,a^3\cos\left(dx+c\right)-2\,a^3}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*cos(d*x + c)^3*log(-cos(d*x + c)) - 18*a^3*cos(d*x + c)^2 - 9*a
^3*cos(d*x + c) - 2*a^3)/(d*cos(d*x + c)^3)
```

Sympy [A]
time = 0.21, size = 76, normalized size = 1.15

$$\begin{cases}\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\sec^3\left(c+dx\right)}{3d}+\frac{3a^3\sec^2\left(c+dx\right)}{2d}+\frac{3a^3\sec\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a\sec\left(c\right)+a)^3\tan\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*sec(c + d*x)**3/(3*d)
 + 3*a**3*sec(c + d*x)**2/(2*d) + 3*a**3*sec(c + d*x)/d, Ne(d, 0)), (x*(a*s
ec(c) + a)**3*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(62) = 124.
time = 0.59, size = 167, normalized size = 2.53

$$\frac{6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{51\,a^3+\frac{69\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a^3*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (51*a^3 + 69*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 45*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
```

```
 + 1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

Mupad [B]
time = 1.94, size = 105, normalized size = 1.59

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{20\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3
*tan(c/2 + (d*x)/2)^2 + (20*a^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
+ (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.42 $\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}$$

```
[Out] 4*a^3*ln(1-cos(d*x+c))/d-3*a^3*ln(cos(d*x+c))/d+a^3*sec(d*x+c)/d
```

Rubi [A]

time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (3*a^3*Log[Cos[c + d*x]])/d + (a^3*Sec[c
+ d*x])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^2\text{Subst}\left(\int \frac{(a+ax)^2}{x^2(a-ax)}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{a^2\text{Subst}\left(\int \left(-\frac{4a}{-1+x}+\frac{a}{x^2}+\frac{3a}{x}\right)\,dx,x,\cos(c+dx)\right)}{d}\\
&= \frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 36, normalized size = 0.75

$$\frac{a^3\left(-3\log(\cos(c+dx))+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
[Out] (a^3*(-3*Log[Cos[c + d*x]] + 8*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.75

method	result	size
derivativedivides	$-\frac{a^3(-\sec(dx+c)-4\ln(-1+\sec(dx+c))+\ln(\sec(dx+c)))}{d}$	36
default	$-\frac{a^3(-\sec(dx+c)-4\ln(-1+\sec(dx+c))+\ln(\sec(dx+c)))}{d}$	36
risch	$-ia^3x-\frac{2ia^3c}{d}+\frac{2a^3e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{8a^3\ln(e^{i(dx+c)}-1)}{d}-\frac{3a^3\ln(e^{2i(dx+c)}+1)}{d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/d*a^3*(-sec(d*x+c)-4*ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.90

$$\frac{4\,a^3\log\left(\cos\left(dx+c\right)-1\right)-3\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{a^3}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] (4*a^3*log(cos(d*x + c) - 1) - 3*a^3*log(cos(d*x + c)) + a^3/cos(d*x + c))/
d
```

Fricas [A]
time = 3.33, size = 61, normalized size = 1.27

$$-\frac{3\,a^3\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-4\,a^3\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^3}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*cos(d*x + c)*log(-cos(d*x + c)) - 4*a^3*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^3)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot(c+dx)\sec(c+dx)\,dx+\int 3\cot(c+dx)\sec^2(c+dx)\,dx+\int \cot(c+dx)\sec^3(c+dx)\,dx+\int \cot(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)*sec(c + d*x), x) + Integral(3*cot(c + d*x)*se
c(c + d*x)**2, x) + Integral(cot(c + d*x)*sec(c + d*x)**3, x) + Integral(co
t(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(48) = 96.
time = 0.51, size = 145, normalized size = 3.02

$$\frac{4a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-3a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{5a^3+\frac{3a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (4*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^3*log(abs(-(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 1)) + (5*a^3 + 3*a^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B]
time = 1.20, size = 86, normalized size = 1.79

$$\frac{8a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2a^3}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}-\frac{3a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{d}-\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (8*a^3*log(tan(c/2 + (d*x)/2)))/d - (2*a^3)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
- (3*a^3*log(tan(c/2 + (d*x)/2)^2 - 1))/d - (a^3*log(tan(c/2 + (d*x)/2)^2 +
 1))/d
```

3.43 $\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=40

$$-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -2*a^3/d/(1-cos(d*x+c))-a^3*ln(1-cos(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
[Out] (-2*a^3)/(d*(1 - Cos[c + d*x])) - (a^3*Log[1 - Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^4\text{Subst}\left(\int \frac{a+ax}{(a-ax)^2}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{a^4\text{Subst}\left(\int \left(\frac{2}{a(-1+x)^2}+\frac{1}{a(-1+x)}\right)\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 1.15

$$-\frac{a^3\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+2\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
[Out] -((a^3*(Cot[(c + d*x)/2]^2 + 2*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2
]])))/d)
```

Maple [A]
time = 0.11, size = 39, normalized size = 0.98

method	result	size
derivativedivides	$\frac{a^3\left(-\frac{2}{-1+\sec(dx+c)}-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c))\right)}{d}$	39
default	$\frac{a^3\left(-\frac{2}{-1+\sec(dx+c)}-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c))\right)}{d}$	39
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{4a^3e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}-\frac{2a^3\ln\left(e^{i(dx+c)}-1\right)}{d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*a^3*(-2/(-1+sec(d*x+c))-ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 34, normalized size = 0.85

$$-\frac{a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,a^3}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -(a^3*log(cos(d*x + c) - 1) - 2*a^3/(cos(d*x + c) - 1))/d
```

Fricas [A]
time = 2.37, size = 50, normalized size = 1.25

$$\frac{2\,a^3-\left(a^3\cos\left(dx+c\right)-a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{d\cos\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (2*a^3 - (a^3*cos(d*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
+ c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cot^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot^3\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\cot^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**3*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(38) = 76.
time = 0.52, size = 109, normalized size = 2.72

$$-\frac{a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^3+\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B]
time = 1.23, size = 48, normalized size = 1.20

$$-\frac{a^3\left(\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2))/d
```

3.44 $\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=61

$$-\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -1/2*a^3/d/(1-cos(d*x+c))^2+2*a^3/d/(1-cos(d*x+c))+a^3*ln(1-cos(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 45}

$$\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*a^3/(d*(1 - Cos[c + d*x])^2) + (2*a^3)/(d*(1 - Cos[c + d*x])) + (a^3*L
og[1 - Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{a^6\text{Subst}\left(\int \frac{x^2}{(a-ax)^3}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x)^3}-\frac{2}{a^3(-1+x)^2}-\frac{1}{a^3(-1+x)}\right)\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))}+\frac{a^3\log(1-\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 72, normalized size = 1.18

$$-\frac{a^3(1+\cos(c+dx))^3\left(-8\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)-16\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/64*(a^3*(1 + Cos[c + d*x])^3*(-8*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4
 - 16*Log[Sin[(c + d*x)/2]])*Sec[(c + d*x)/2]^6)/d
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{a^3\left(-\frac{1}{-1+\sec(dx+c)}+\frac{1}{2(-1+\sec(dx+c))^2}-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c))\right)}{d}$	52
default	$-\frac{a^3\left(-\frac{1}{-1+\sec(dx+c)}+\frac{1}{2(-1+\sec(dx+c))^2}-\ln(-1+\sec(dx+c))+\ln(\sec(dx+c))\right)}{d}$	52
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(2\,e^{3i(dx+c)}-3\,e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^4}+\frac{2a^3\ln\left(e^{i(dx+c)}-1\right)}{d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^3*(-1/(-1+sec(d*x+c))+1/2/(-1+sec(d*x+c))^2-ln(-1+sec(d*x+c))+ln(sec
(d*x+c)))
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.97

$$\frac{2\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{4\,a^3\cos(dx+c)-3\,a^3}{\cos(dx+c)^2-2\,\cos(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(cos(d*x + c) - 1) - (4*a^3*cos(d*x + c) - 3*a^3)/(cos(d*x +
c)^2 - 2*cos(d*x + c) + 1))/d
```

Fricas [A]
time = 2.47, size = 82, normalized size = 1.34

$$-\frac{4\,a^3\cos\left(dx+c\right)-3\,a^3-2\left(a^3\cos\left(dx+c\right)^2-2\,a^3\cos\left(dx+c\right)+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*cos(d*x + c) - 3*a^3 - 2*(a^3*cos(d*x + c)^2 - 2*a^3*cos(d*x +
c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c
) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot^5(c+dx)\sec(c+dx)\,dx+\int 3\cot^5(c+dx)\sec^2(c+dx)\,dx+\int\cot^5(c+dx)\sec^3(c+dx)\,dx+\int\cot^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**5*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**5, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(55) = 110.
time = 0.55, size = 138, normalized size = 2.26

$$\frac{8\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^3+\frac{6\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a^3*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + 6*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 12*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 1.22, size = 78, normalized size = 1.28

$$\frac{2\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{a^3}{8}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*log(tan(c/2 + (d*x)/2)))/d + ((3*a^3*tan(c/2 + (d*x)/2)^2)/4 - a^3/8
)/(d*tan(c/2 + (d*x)/2)^4) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.45 $\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{a^3}{6d(1-\cos(c+dx))^3}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{17a^3}{8d(1-\cos(c+dx))}-\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(1+$$

```
[Out] -1/6*a^3/d/(1-cos(d*x+c))^3+7/8*a^3/d/(1-cos(d*x+c))^2-17/8*a^3/d/(1-cos(d*
x+c))-15/16*a^3*ln(1-cos(d*x+c))/d-1/16*a^3*ln(1+cos(d*x+c))/d
```

Rubi [A]

time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$-\frac{17a^3}{8d(1-\cos(c+dx))}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{a^3}{6d(1-\cos(c+dx))^3}-\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6*a^3/(d*(1 - Cos[c + d*x])^3) + (7*a^3)/(8*d*(1 - Cos[c + d*x])^2) - (1
7*a^3)/(8*d*(1 - Cos[c + d*x])) - (15*a^3*Log[1 - Cos[c + d*x]])/(16*d) - (
a^3*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^8\text{Subst}\left(\int \frac{x^4}{(a-ax)^4(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^8\text{Subst}\left(\int \left(\frac{1}{2a^5(-1+x)^4} + \frac{7}{4a^5(-1+x)^3} + \frac{17}{8a^5(-1+x)^2} + \frac{15}{16a^5(-1+x)} + \right.\right.}{d}$$

$$= -\frac{a^3}{6d(1-\cos(c+dx))^3} + \frac{7a^3}{8d(1-\cos(c+dx))^2} - \frac{17a^3}{8d(1-\cos(c+}$$

Mathematica [A]
time = 0.69, size = 102, normalized size = 0.95

$$-\frac{a^3(1+\cos(c+dx))^3\left(102\csc^2\left(\frac{1}{2}(c+dx)\right) - 21\csc^4\left(\frac{1}{2}(c+dx)\right) + 2\csc^6\left(\frac{1}{2}(c+dx)\right) + 12\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(a^3*(1 + Cos[c + d*x])^3*(102*Csc[(c + d*x)/2]^2 - 21*Csc[(c + d*x)
/2]^4 + 2*Csc[(c + d*x)/2]^6 + 12*(Log[Cos[(c + d*x)/2]] + 15*Log[Sin[(c +
d*x)/2]]))*Sec[(c + d*x)/2]^6)/d
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.69

method	result
derivativedivides	$\frac{a^3\left(-\frac{\ln(1+\sec(dx+c))}{16} - \frac{1}{6(-1+\sec(dx+c))^3} + \frac{3}{8(-1+\sec(dx+c))^2} - \frac{7}{8(-1+\sec(dx+c))} - \frac{15\ln(-1+\sec(dx+c))}{16} + \ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^3\left(-\frac{\ln(1+\sec(dx+c))}{16} - \frac{1}{6(-1+\sec(dx+c))^3} + \frac{3}{8(-1+\sec(dx+c))^2} - \frac{7}{8(-1+\sec(dx+c))} - \frac{15\ln(-1+\sec(dx+c))}{16} + \ln(\sec(dx+c))\right)}{d}$
risch	$ia^3x + \frac{2ia^3c}{d} + \frac{a^3\left(51\,e^{5i(dx+c)} - 162\,e^{4i(dx+c)} + 238\,e^{3i(dx+c)} - 162\,e^{2i(dx+c)} + 51\,e^{i(dx+c)}\right)}{12d\left(e^{i(dx+c)}-1\right)^6} - \frac{15a^3\ln\left(e^{i(dx+c)}-1\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(-1/16*ln(1+sec(d*x+c))-1/6/(-1+sec(d*x+c))^3+3/8/(-1+sec(d*x+c))^2
-7/8/(-1+sec(d*x+c))-15/16*ln(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 96, normalized size = 0.90

$$-\frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right) + 45\,a^3\log\left(\cos\left(dx+c\right)-1\right) - \frac{2\left(51\,a^3\cos\left(dx+c\right)^2 - 81\,a^3\cos\left(dx+c\right) + 34\,a^3\right)}{\cos\left(dx+c\right)^3 - 3\,\cos\left(dx+c\right)^2 + 3\,\cos\left(dx+c\right) - 1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*log(cos(d*x + c) + 1) + 45*a^3*log(cos(d*x + c) - 1) - 2*(51*a
^3*cos(d*x + c)^2 - 81*a^3*cos(d*x + c) + 34*a^3)/(cos(d*x + c)^3 - 3*cos(d
*x + c)^2 + 3*cos(d*x + c) - 1))/d
```

Fricas [A]
time = 2.19, size = 178, normalized size = 1.66

$$\frac{102\,a^3\cos\left(dx+c\right)^2-162\,a^3\cos\left(dx+c\right)+68\,a^3-3\left(a^3\cos\left(dx+c\right)^3-3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-45\left(a^3\cos\left(dx+c\right)^3-3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)-a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^3-3\,d\cos\left(dx+c\right)^2+3\,d\cos\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*cos(d*x + c)^2 - 162*a^3*cos(d*x + c) + 68*a^3 - 3*(a^3*cos(d
*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) - a^3)*log(1/2*cos(d*
x + c) + 1/2) - 45*(a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d
*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + 3*d*cos(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 165, normalized size = 1.54

$$-\frac{90\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a^3+\frac{15\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{165\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(90*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^3*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a^3 + 15*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 66*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 165*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c)
+ 1)^3/(cos(d*x + c) - 1)^3)/d
```

Mupad [B]
time = 1.29, size = 94, normalized size = 0.88

$$\frac{a^3 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{a^3}{6}}{8\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}-\frac{15\,a^3 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((11*a^3*tan(c/2 + (d*x)/2)^4)/2 -
(5*a^3*tan(c/2 + (d*x)/2)^2)/4 + a^3/6)/(8*d*tan(c/2 + (d*x)/2)^6) - (15*a^
3*log(tan(c/2 + (d*x)/2)))/(8*d)
```

3.46 $\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=149

$$-\frac{a^3}{16d(1-\cos(c+dx))^4}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(1+\cos$$

```
[Out] -1/16*a^3/d/(1-cos(d*x+c))^4+5/12*a^3/d/(1-cos(d*x+c))^3-39/32*a^3/d/(1-cos
(d*x+c))^2+9/4*a^3/d/(1-cos(d*x+c))+1/32*a^3/d/(1+cos(d*x+c))+57/64*a^3*ln(
1-cos(d*x+c))/d+7/64*a^3*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(\cos(c+dx)+1)}-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{5a^3}{12d(1-\cos(c+dx))^3}-\frac{a^3}{16d(1-\cos(c+dx))^4}+\frac{57a^3\log(1-\cos(c+dx))}{64d}+\frac{7a^3\log(\cos(c+dx)+1)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*a^3/(d*(1 - Cos[c + d*x])^4) + (5*a^3)/(12*d*(1 - Cos[c + d*x])^3) -
(39*a^3)/(32*d*(1 - Cos[c + d*x])^2) + (9*a^3)/(4*d*(1 - Cos[c + d*x])) + a
^3/(32*d*(1 + Cos[c + d*x])) + (57*a^3*Log[1 - Cos[c + d*x]])/(64*d) + (7*a
^3*Log[1 + Cos[c + d*x]])/(64*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^{10}\mathrm{Subst}\left(\int \frac{x^6}{(a-ax)^5(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^{10}\mathrm{Subst}\left(\int \left(-\frac{1}{4a^7(-1+x)^5} - \frac{5}{4a^7(-1+x)^4} - \frac{39}{16a^7(-1+x)^3} - \frac{9}{4a^7(-1+x)^2}\right.\right.}{d}$$

$$= -\frac{a^3}{16d(1-\cos(c+dx))^4} + \frac{5a^3}{12d(1-\cos(c+dx))^3} - \frac{39a^3}{32d(1-\cos(}$$

Mathematica [A]

time = 0.37, size = 130, normalized size = 0.87

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(864\csc^2\left(\frac{1}{2}(c+dx)\right)-234\csc^4\left(\frac{1}{2}(c+dx)\right)+40\csc^6\left(\frac{1}{2}(c+dx)\right)-3\csc^8\left(\frac{1}{2}(c+dx)\right)+12\left(14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+114\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\right)}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(864*Csc[(c + d*x)/2]^2 - 234*
Csc[(c + d*x)/2]^4 + 40*Csc[(c + d*x)/2]^6 - 3*Csc[(c + d*x)/2]^8 + 12*(14*
Log[Cos[(c + d*x)/2]] + 114*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/(
6144*d)
```

Maple [A]

time = 0.15, size = 99, normalized size = 0.66

method	result
derivativedivides	$-\frac{a^3\left(\frac{1}{32+32\sec(dx+c)}-\frac{7\ln(1+\sec(dx+c))}{64}+\frac{1}{16(-1+\sec(dx+c))^4}-\frac{1}{6(-1+\sec(dx+c))^3}+\frac{11}{32(-1+\sec(dx+c))^2}-\frac{13}{16(-1+\sec(dx+c)}\right.}{d}$
default	$-\frac{a^3\left(\frac{1}{32+32\sec(dx+c)}-\frac{7\ln(1+\sec(dx+c))}{64}+\frac{1}{16(-1+\sec(dx+c))^4}-\frac{1}{6(-1+\sec(dx+c))^3}+\frac{11}{32(-1+\sec(dx+c))^2}-\frac{13}{16(-1+\sec(dx+c)}\right.}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{a^3\left(213\,e^{9i(dx+c)}-606\,e^{8i(dx+c)}+472\,e^{7i(dx+c)}+846\,e^{6i(dx+c)}-1658\,e^{5i(dx+c)}+846\,e^{4i(dx+c)}+4\right.}{48d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^3*(1/32/(1+sec(d*x+c))-7/64*ln(1+sec(d*x+c))+1/16/(-1+sec(d*x+c))^4-
1/6/(-1+sec(d*x+c))^3+11/32/(-1+sec(d*x+c))^2-13/16/(-1+sec(d*x+c))-57/64*l
n(-1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]

time = 0.27, size = 142, normalized size = 0.95

$$\frac{21\,a^3\log\left(\cos\left(dx+c\right)+1\right)+171\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(213\,a^3\cos(dx+c)^4-303\,a^3\cos(dx+c)^3-95\,a^3\cos(dx+c)^2+333\,a^3\cos(dx+c)-136\,a^3\right)}{\cos(dx+c)^5-3\,\cos(dx+c)^4+2\,\cos(dx+c)^3+2\,\cos(dx+c)^2-3\,\cos(dx+c)+1}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(21*a^3*log(cos(d*x + c) + 1) + 171*a^3*log(cos(d*x + c) - 1) - 2*(21
3*a^3*cos(d*x + c)^4 - 303*a^3*cos(d*x + c)^3 - 95*a^3*cos(d*x + c)^2 + 333
*a^3*cos(d*x + c) - 136*a^3)/(cos(d*x + c)^5 - 3*cos(d*x + c)^4 + 2*cos(d*x
 + c)^3 + 2*cos(d*x + c)^2 - 3*cos(d*x + c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(127) = 254$.
time = 3.01, size = 272, normalized size = 1.83

$$-\frac{426\,a^3\cos\left(dx+c\right)^4-606\,a^3\cos\left(dx+c\right)^3-190\,a^3\cos\left(dx+c\right)^2+666\,a^3\cos\left(dx+c\right)-272\,a^3-21\left(a^3\cos\left(dx+c\right)^5-3\,a^3\cos\left(dx+c\right)^4+2\,a^3\cos\left(dx+c\right)^3+2\,a^3\cos\left(dx+c\right)^2-3\,a^3\cos\left(dx+c\right)+a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-171\left(a^3\cos\left(dx+c\right)^5-3\,a^3\cos\left(dx+c\right)^4+2\,a^3\cos\left(dx+c\right)^3+2\,a^3\cos\left(dx+c\right)^2-3\,a^3\cos\left(dx+c\right)+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{192\left(d\cos\left(dx+c\right)^5-3\,d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)^2-3\,d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*a^3*cos(d*x + c)^4 - 606*a^3*cos(d*x + c)^3 - 190*a^3*cos(d*x +
 c)^2 + 666*a^3*cos(d*x + c) - 272*a^3 - 21*(a^3*cos(d*x + c)^5 - 3*a^3*cos
(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x +
 c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - 171*(a^3*cos(d*x + c)^5 - 3*a^3*co
s(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x
+ c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x +
 c)^4 + 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 213, normalized size = 1.43

$$\frac{684\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-768\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(3\,a^3+\frac{28\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{504\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{1425\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}\right)(\cos(dx+c)+1)^4}{(\cos(dx+c)-1)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(684*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 768*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^3*(cos(d*x + c)
```

```
 - 1)/(cos(d*x + c) + 1) - (3*a^3 + 28*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 132*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 504*a^3*(cos(d*
x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1425*a^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

Mupad [B]
time = 1.20, size = 130, normalized size = 0.87

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{57\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,d}+\frac{21\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}-\frac{a^3}{8}}{32\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(64*d) + (57*a^3*log(tan(c/2 + (d*x)/2)))/(32*d)
 + ((7*a^3*tan(c/2 + (d*x)/2)^2)/6 - (11*a^3*tan(c/2 + (d*x)/2)^4)/2 + 21*a
^3*tan(c/2 + (d*x)/2)^6 - a^3/8)/(32*d*tan(c/2 + (d*x)/2)^8) - (a^3*log(tan
(c/2 + (d*x)/2)^2 + 1))/d
```

3.47 $\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx$

Optimal. Leaf size=237

$$-a^3x-\frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d}+\frac{a^3\tan(c+dx)}{d}+\frac{115a^3\sec(c+dx)\tan(c+dx)}{128d}+\frac{5a^3\sec^3(c+dx)\tan(c+$$

```
[Out] -a^3*x-125/128*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+115/128*a^3*sec(d
*x+c)*tan(d*x+c)/d+5/64*a^3*sec(d*x+c)^3*tan(d*x+c)/d-1/3*a^3*tan(d*x+c)^3/
d-5/8*a^3*sec(d*x+c)*tan(d*x+c)^3/d-5/48*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+1/
5*a^3*tan(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)^5/d+1/8*a^3*sec(d*x+c)^3
*tan(d*x+c)^5/d+3/7*a^3*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.23, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\frac{3a^3\tan^7(c+dx)}{7d}+\frac{a^3\tan^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}-\frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d}+\frac{a^3\tan^5(c+dx)\sec^3(c+dx)}{8d}-\frac{5a^3\tan^3(c+dx)\sec^3(c+dx)}{48d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{a^3\tan^5(c+dx)\sec(c+dx)}{2d}-\frac{5a^3\tan^3(c+dx)\sec(c+dx)}{8d}+\frac{115a^3\tan(c+dx)\sec(c+dx)}{128d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] -(a^3*x) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])/d +
 (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c
+ d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]*Tan[c + d
*x]^3)/(8*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*Tan[c +
d*x]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) + (a^3*Sec[c + d*x]
^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx &= \int \left(a^3\tan^6(c+dx)+3a^3\sec(c+dx)\tan^6(c+dx)+3a^3\sec^2(c+d\right. \\
&= a^3\int \tan^6(c+dx)\,dx + a^3\int \sec^3(c+dx)\tan^6(c+dx)\,dx + \left(3a^3\right) \\
&= \frac{a^3\tan^5(c+dx)}{5d} + \frac{a^3\sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3\sec^3(c+dx)\tan}{8d} \\
&= -\frac{a^3\tan^3(c+dx)}{3d} - \frac{5a^3\sec(c+dx)\tan^3(c+dx)}{8d} - \frac{5a^3\sec^3(c+dx)}{48} \\
&= \frac{a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a^3\sec^3(c+dx)\,t}{64d} \\
&= -a^3x - \frac{15a^3\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^3\tan(c+dx)}{d} + \frac{115a^3\sec(c+}{} \\
&= -a^3x - \frac{125a^3\tanh^{-1}(\sin(c+dx))}{128d} + \frac{a^3\tan(c+dx)}{d} + \frac{115a^3\sec(c}{}
\end{aligned}
$$

Mathematica [A]
time = 2.13, size = 363, normalized size = 1.53

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^8(c+dx)\left(1680000\cos^8(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(470400dx\cos(c)+376320dx\cos(c+2dx)+376320dx\cos(3c+2dx)+188160dx\cos(3c+4dx)+188160dx\cos(5c+4dx)+53760dx\cos(5c+6dx)+53760dx\cos(7c+6dx)+6720dx\cos(7c+8dx)+6720dx\cos(9c+8dx)+519680\sin(c)-133175\sin(dx)-133175\sin(2c+dx)-544768\sin(c+2dx)+286720\sin(3c+2dx)-63595\sin(2c+3dx)-63595\sin(4c+3dx)-254464\sin(3c+4dx)+161280\sin(5c+4dx)-65135\sin(4c+5dx)-65135\sin(6c+5dx)-118784\sin(5c+6dx)-27195\sin(6c+7dx)-27195\sin(8c+7dx)-14848\sin(7c+8dx))\right)}{13762560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^8*(1680000*Cos[c
+ d*x]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) - Sec[c]*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x]
 + 376320*d*x*Cos[3*c + 2*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*C
os[5*c + 4*d*x] + 53760*d*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] +
 6720*d*x*Cos[7*c + 8*d*x] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] - 13
3175*Sin[d*x] - 133175*Sin[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[
3*c + 2*d*x] - 63595*Sin[2*c + 3*d*x] - 63595*Sin[4*c + 3*d*x] - 254464*Sin
[3*c + 4*d*x] + 161280*Sin[5*c + 4*d*x] - 65135*Sin[4*c + 5*d*x] - 65135*Si
n[6*c + 5*d*x] - 118784*Sin[5*c + 6*d*x] - 27195*Sin[6*c + 7*d*x] - 27195*S
in[8*c + 7*d*x] - 14848*Sin[7*c + 8*d*x])))/(13762560*d)
```

Maple [A]
time = 0.14, size = 290, normalized size = 1.22

method	result
risch	$-a^3x - \frac{ia^3\left(27195\,\mathrm{e}^{15i(dx+c)}+65135\,\mathrm{e}^{13i(dx+c)}-161280\,\mathrm{e}^{12i(dx+c)}+63595\,\mathrm{e}^{11i(dx+c)}-286720\,\mathrm{e}^{10i(dx+c)}+133175\,\mathrm{e}^{9i(}\right.}{}$

derivativedivides	$\frac{a^{3}\left(\frac{\sin^{7}(dx+c)}{8\cos(dx+c)^{8}}+\frac{\sin^{7}(dx+c)}{48\cos(dx+c)^{6}}-\frac{\sin^{7}(dx+c)}{192\cos(dx+c)^{4}}+\frac{\sin^{7}(dx+c)}{128\cos(dx+c)^{2}}+\frac{\left(\sin^{5}(dx+c)\right)}{128}+\frac{5\left(\sin^{3}(dx+c)\right)}{384}+\frac{5\sin(dx+c)}{128}-\frac{5\ln(\sec(dx}{}\right.}{}$
default	$\frac{a^{3}\left(\frac{\sin^{7}(dx+c)}{8\cos(dx+c)^{8}}+\frac{\sin^{7}(dx+c)}{48\cos(dx+c)^{6}}-\frac{\sin^{7}(dx+c)}{192\cos(dx+c)^{4}}+\frac{\sin^{7}(dx+c)}{128\cos(dx+c)^{2}}+\frac{\left(\sin^{5}(dx+c)\right)}{128}+\frac{5\left(\sin^{3}(dx+c)\right)}{384}+\frac{5\sin(dx+c)}{128}-\frac{5\ln(\sec(dx}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/8*sin(d*x+c)^7/cos(d*x+c)^8+1/48*sin(d*x+c)^7/cos(d*x+c)^6-1/19
2*sin(d*x+c)^7/cos(d*x+c)^4+1/128*sin(d*x+c)^7/cos(d*x+c)^2+1/128*sin(d*x+c
)^5+5/384*sin(d*x+c)^3+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(d*x+c)))+3/
7*a^3*sin(d*x+c)^7/cos(d*x+c)^7+3*a^3*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*s
in(d*x+c)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5
/48*sin(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a^3*(1/5*t
an(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 262, normalized size = 1.11

$$\frac{11520\,a^{3}\tan\left(dx+c\right)^{7}+1792\left(3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^{3}+35\,a^{3}\left(\frac{2\left(15\,\sin(dx+c)^{7}+73\,\sin(dx+c)^{5}-55\,\sin(dx+c)^{3}+15\,\sin(dx+c)\right)}{\sin(dx+c)^{8}-4\,\sin(dx+c)^{6}+6\,\sin(dx+c)^{4}-4\,\sin(dx+c)^{2}+1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-840\,a^{3}\left(\frac{2\left(33\,\sin(dx+c)^{5}-40\,\sin(dx+c)^{3}+15\,\sin(dx+c)\right)}{\sin(dx+c)^{6}-3\,\sin(dx+c)^{4}+3\,\sin(dx+c)^{2}-1}+15\,\log\left(\sin\left(dx+c\right)+1\right)-15\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/26880*(11520*a^3*tan(d*x + c)^7 + 1792*(3*tan(d*x + c)^5 - 5*tan(d*x + c)
^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 + 35*a^3*(2*(15*sin(d*x + c)^7 +
73*sin(d*x + c)^5 - 55*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 -
4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*
x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*a^3*(2*(33*sin(d*x + c)^5 - 4
0*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))
/d
```

Fricas [A]
time = 2.61, size = 178, normalized size = 0.75

$$-\frac{26880\,a^{3}dx\cos\left(dx+c\right)^{8}+13125\,a^{3}\cos\left(dx+c\right)^{8}\log\left(\sin\left(dx+c\right)+1\right)-13125\,a^{3}\cos\left(dx+c\right)^{8}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(14848\,a^{3}\cos\left(dx+c\right)^{7}+27195\,a^{3}\cos\left(dx+c\right)^{6}+7424\,a^{3}\cos\left(dx+c\right)^{5}-17710\,a^{3}\cos\left(dx+c\right)^{4}-14592\,a^{3}\cos\left(dx+c\right)^{3}+1960\,a^{3}\cos\left(dx+c\right)^{2}+5760\,a^{3}\cos\left(dx+c\right)+1680\,a^{3}\right)\sin\left(dx+c\right)}{26880\,d\cos\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 + 13125*a^3*cos(d*x + c)^8*log(sin(d
*x + c) + 1) - 13125*a^3*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*a
^3*cos(d*x + c)^7 + 27195*a^3*cos(d*x + c)^6 + 7424*a^3*cos(d*x + c)^5 - 17
```

```
710*a^3*cos(d*x + c)^4 - 14592*a^3*cos(d*x + c)^3 + 1960*a^3*cos(d*x + c)^2
 + 5760*a^3*cos(d*x + c) + 1680*a^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\tan^6(c+dx)\sec(c+dx)\,dx+\int 3\tan^6(c+dx)\sec^2(c+dx)\,dx+\int \tan^6(c+dx)\sec^3(c+dx)\,dx+\int \tan^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**6,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**6*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**6, x))
```

Giac [A]
time = 2.68, size = 196, normalized size = 0.83

$$-\frac{13440\,(dx+c)a^3+13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(315\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-11375\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+79723\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-269879\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+550089\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-749973\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+212625\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-26565\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^8}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/13440*(13440*(d*x + c)*a^3 + 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^3*tan(1/2*d*x +
 1/2*c)^15 - 11375*a^3*tan(1/2*d*x + 1/2*c)^13 + 79723*a^3*tan(1/2*d*x + 1/
2*c)^11 - 269879*a^3*tan(1/2*d*x + 1/2*c)^9 + 550089*a^3*tan(1/2*d*x + 1/2*
c)^7 - 749973*a^3*tan(1/2*d*x + 1/2*c)^5 + 212625*a^3*tan(1/2*d*x + 1/2*c)^
3 - 26565*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

Mupad [B]
time = 2.43, size = 263, normalized size = 1.11

$$-a^3\,x-\frac{125\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d}-\frac{\frac{3\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{64}-\frac{325\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{192}+\frac{11389\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{960}-\frac{269879\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{6720}+\frac{183363\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}}{2240}-\frac{35713\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{320}+\frac{2025\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{64}-\frac{253\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}-8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-56\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+70\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-56\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] - a^3*x - (125*a^3*atanh(tan(c/2 + (d*x)/2)))/(64*d) - ((2025*a^3*tan(c/2 +
 (d*x)/2)^3)/64 - (35713*a^3*tan(c/2 + (d*x)/2)^5)/320 + (183363*a^3*tan(c/
2 + (d*x)/2)^7)/2240 - (269879*a^3*tan(c/2 + (d*x)/2)^9)/6720 + (11389*a^3*
tan(c/2 + (d*x)/2)^11)/960 - (325*a^3*tan(c/2 + (d*x)/2)^13)/192 + (3*a^3*t
an(c/2 + (d*x)/2)^15)/64 - (253*a^3*tan(c/2 + (d*x)/2))/64)/(d*(28*tan(c/2
+ (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/
2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*ta
n(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.48 $\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=169

$$a^3x+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx)\tan(c+dx)}{16d}-\frac{a^3\sec^3(c+dx)\tan(c+dx)}{8d}$$

```
[Out] a^3*x+19/16*a^3*arctanh(sin(d*x+c))/d-a^3*tan(d*x+c)/d-17/16*a^3*sec(d*x+c)
*tan(d*x+c)/d-1/8*a^3*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d+3/4*
a^3*sec(d*x+c)*tan(d*x+c)^3/d+1/6*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+3/5*a^3*t
an(d*x+c)^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{a^3\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{3a^3\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{17a^3\tan(c+dx)\sec(c+dx)}{16d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] a^3*x + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d - (17*
a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
8*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*
d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]^5)/(5*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx &= \int \left(a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)+3a^3\sec^2(c+d\right.\\ &= a^3\int\tan^4(c+dx)\,dx+a^3\int\sec^3(c+dx)\tan^4(c+dx)\,dx+\left(3a^3\right)\\ &= \frac{a^3\tan^3(c+dx)}{3d}+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d}+\frac{a^3\sec^3(c+dx)\,\mathrm{ta}}{6d}\\ &= -\frac{a^3\tan(c+dx)}{d}-\frac{9a^3\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^3\sec^3(c+dx)\,\mathrm{ta}}{8d}\\ &= a^3x+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{8d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx}{16d}\\ &= a^3x+\frac{19a^3\tanh^{-1}(\sin(c+dx))}{16d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+d}{16}\end{aligned}$$

Mathematica [A]
time = 1.17, size = 303, normalized size = 1.79

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(-9120\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec(c)(2400dx\cos(c)+1800dx\cos(c+2dx)+1800dx\cos(3c+2dx)+720dx\cos(3c+4dx)+720dx\cos(5c+4dx)+120dx\cos(5c+6dx)+120dx\cos(7c+6dx)+1760\sin(c)+210\sin(dx)+210\sin(2c+dx)-1440\sin(c+2dx)+1200\sin(3c+2dx)-865\sin(2c+3dx)-865\sin(4c+3dx)-1296\sin(3c+4dx)-240\sin(5c+4dx)-435\sin(4c+5dx)-435\sin(6c+5dx)-176\sin(5c+6dx))\right)}{61440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(-9120*Cos[c +
d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + Sec[c]*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 180
0*d*x*Cos[3*c + 2*d*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x
] + 120*d*x*Cos[5*c + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] + 210
*Sin[d*x] + 210*Sin[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x
] - 865*Sin[2*c + 3*d*x] - 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 2
40*Sin[5*c + 4*d*x] - 435*Sin[4*c + 5*d*x] - 435*Sin[6*c + 5*d*x] - 176*Sin
[5*c + 6*d*x])))/(61440*d)
```

Maple [A]
time = 0.11, size = 223, normalized size = 1.32

method	result
risch	$a^3x+\frac{ia^3\left(435\,e^{11i(dx+c)}+240\,e^{10i(dx+c)}+865\,e^{9i(dx+c)}-1200\,e^{8i(dx+c)}-210\,e^{7i(dx+c)}-1760\,e^{6i(dx+c)}+210\,e^{5i(dx+\cdots\right)}{120d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{a^3\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^3\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}\cdots}{}$
default	$\frac{a^3\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^3\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/6*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48
*sin(d*x+c)^5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*
x+c)+tan(d*x+c)))+3/5*a^3*sin(d*x+c)^5/cos(d*x+c)^5+3*a^3*(1/4*sin(d*x+c)^5
/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)
+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 210, normalized size = 1.24

$$\frac{288\,a^3\tan(dx+c)^5+160\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^3-5\,a^3\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)+90\,a^3\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log(\sin(dx+c)+1)-3\,\log(\sin(dx+c)-1)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/480*(288*a^3*tan(d*x + c)^5 + 160*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d
*x + c))*a^3 - 5*a^3*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 90*a^3*(2*(5*sin(d*x + c)^3 - 3*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c)
+ 1) - 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.08, size = 152, normalized size = 0.90

$$\frac{480\,a^3dx\cos\left(dx+c\right)^6+285\,a^3\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-285\,a^3\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)-2\left(176\,a^3\cos\left(dx+c\right)^5+435\,a^3\cos\left(dx+c\right)^4+208\,a^3\cos\left(dx+c\right)^3-110\,a^3\cos\left(dx+c\right)^2-144\,a^3\cos\left(dx+c\right)-40\,a^3\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 + 285*a^3*cos(d*x + c)^6*log(sin(d*x + c)
 + 1) - 285*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*a^3*cos(d*x
+ c)^5 + 435*a^3*cos(d*x + c)^4 + 208*a^3*cos(d*x + c)^3 - 110*a^3*cos(d*x
+ c)^2 - 144*a^3*cos(d*x + c) - 40*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\tan^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \tan^4\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**4*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**4, x))
```

Giac [A]
time = 1.37, size = 164, normalized size = 0.97

$$\frac{240\left(dx+c\right)a^3+285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-95\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-366\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1746\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3135\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)*a^3 + 285*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 285
*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*a^3*tan(1/2*d*x + 1/2*c)^11
 - 95*a^3*tan(1/2*d*x + 1/2*c)^9 - 366*a^3*tan(1/2*d*x + 1/2*c)^7 + 1746*a^
```

```
3*tan(1/2*d*x + 1/2*c)^5 - 3135*a^3*tan(1/2*d*x + 1/2*c)^3 + 525*a^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 2.44, size = 203, normalized size = 1.20

$$a^3\,x+\frac{19\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{19\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\frac{61\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{20}-\frac{291\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{209\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}-\frac{35\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (19*a^3*atanh(tan(c/2 + (d*x)/2)))/(8*d) + ((209*a^3*tan(c/2 + (d*x
)/2)^3)/8 - (291*a^3*tan(c/2 + (d*x)/2)^5)/20 + (61*a^3*tan(c/2 + (d*x)/2)^
7)/20 + (19*a^3*tan(c/2 + (d*x)/2)^9)/24 - (3*a^3*tan(c/2 + (d*x)/2)^11)/8
- (35*a^3*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
(d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.49 $\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=98

$$-a^3x-\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)}{d}+\frac{11a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] -a^3*x-13/8*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+11/8*a^3*sec(d*x+c)*
tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{11a^3\tan(c+dx)\sec(c+dx)}{8d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d + (1
1*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Tan[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx &= \int \left(a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)+3a^3\sec^2(c+\right.\\
&= a^3\int \tan^2(c+dx)\,dx+a^3\int \sec^3(c+dx)\tan^2(c+dx)\,dx+\left(3a^3\right)\\
&= \frac{a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)}{4d}\\
&= -a^3x-\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)}{d}+\frac{11a^3}{}\\
&= -a^3x-\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)}{d}+
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(98) = 196.
time = 0.88, size = 230, normalized size = 2.35

$$-\frac{a^3\sec^4(c+dx)\left(24dx-39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\cos(2(c+dx))\left(8dx-13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos(4(c+dx))\left(8dx-13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-38\sin(c+dx)-32\sin(2(c+dx))-22\sin(3(c+dx))\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -1/64*(a^3*Sec[c + d*x]^4*(24*d*x - 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*Cos[2*(c + d*x)]*(8*
d*x - 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 13*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x - 13*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 13*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 38*Sin[
c + d*x] - 32*Sin[2*(c + d*x)] - 22*Sin[3*(c + d*x)]))/d
```

Maple [A]
time = 0.09, size = 157, normalized size = 1.60

method	result
risch	$-a^3 x - \frac{i a^3 \left(11\,e^{7 i (dx+c)} + 16\,e^{6 i (dx+c)} + 19\,e^{5 i (dx+c)} - 19\,e^{3 i (dx+c)} - 16\,e^{2 i (dx+c)} - 11\,e^{i (dx+c)}\right)}{4 d \left(e^{2 i (dx+c)} + 1\right)^4} + \frac{13 a^3 \ln\left(e^{i (dx+c)} - i\right.}{8d}$
derivativedivides	$\frac{a^3 \left(\frac{\sin^3(dx+c)}{4 \cos(dx+c)^4} + \frac{\sin^3(dx+c)}{8 \cos(dx+c)^2} + \frac{\sin(dx+c)}{8} - \frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + \frac{a^3 \left(\sin^3(dx+c)\right)}{\cos(dx+c)^3} + 3 a^3 \left(\frac{\sin^3(dx+c)}{2 \cos(dx+c)^2} + \frac{\sin(dx+c)}{2} - \ln\right.}{d}$
default	$\frac{a^3 \left(\frac{\sin^3(dx+c)}{4 \cos(dx+c)^4} + \frac{\sin^3(dx+c)}{8 \cos(dx+c)^2} + \frac{\sin(dx+c)}{8} - \frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + \frac{a^3 \left(\sin^3(dx+c)\right)}{\cos(dx+c)^3} + 3 a^3 \left(\frac{\sin^3(dx+c)}{2 \cos(dx+c)^2} + \frac{\sin(dx+c)}{2} - \ln\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*s
in(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+a^3*sin(d*x+c)^3/cos(d*x+c)^3+3*a^
3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c
)))+a^3*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 147, normalized size = 1.50

$$\frac{16\,a^3 \tan(dx+c)^3 - 16\,(dx + c - \tan(dx+c))a^3 + a^3 \left(\frac{2\left(\sin(dx+c)^3 + \sin(dx+c)\right)}{\sin(dx+c)^4 - 2\,\sin(dx+c)^2 + 1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 12\,a^3 \left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2 - 1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*tan(d*x + c)^3 - 16*(d*x + c - tan(d*x + c))*a^3 + a^3*(2*(sin
(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.12, size = 113, normalized size = 1.15

$$-\frac{16\,a^3 dx \cos(dx+c)^4 + 13\,a^3 \cos(dx+c)^4 \log(\sin(dx+c)+1) - 13\,a^3 \cos(dx+c)^4 \log(-\sin(dx+c)+1) - 2\left(11\,a^3 \cos(dx+c)^2 + 8\,a^3 \cos(dx+c) + 2\,a^3\right) \sin(dx+c)}{16\,d \cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 + 13*a^3*cos(d*x + c)^4*log(sin(d*x + c) +
 1) - 13*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(11*a^3*cos(d*x + c)
^2 + 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\tan^2(c+dx)\sec(c+dx)\,dx + \int 3\tan^2(c+dx)\sec^2(c+dx)\,dx + \int \tan^2(c+dx)\sec^3(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**2*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**2, x))
```

Giac [A]
time = 0.82, size = 132, normalized size = 1.35

$$-\frac{8(dx+c)a^3 + 13a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 13a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 13a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 21a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)*a^3 + 13*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^3*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 13*a
^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 1.91, size = 146, normalized size = 1.49

$$\frac{\frac{5a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} - \frac{13a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} + \frac{3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4} + \frac{21a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} - a^3x - \frac{13a^3\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*tan(c/2 + (d*x)/2)^3)/4 - (13*a^3*tan(c/2 + (d*x)/2)^5)/4 + (5*a^3*
tan(c/2 + (d*x)/2)^7)/4 + (21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^8 + 1)) - a^3*x - (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.50 $\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=49

$$-a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}$$

```
[Out] -a^3*x+a^3*arctanh(sin(d*x+c))/d-4*a^3*cot(d*x+c)/d-4*a^3*csc(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3971, 3554, 8, 2686, 3852, 2701, 327, 213}

$$-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+a^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d - (4*a^3*
Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)+3a^3\csc^2(c+d\right.$$

$$= a^3\int \cot^2(c+dx)\,dx + a^3\int \csc^2(c+dx)\sec(c+dx)\,dx + \left(3a^3\right)\int$$

$$= -\frac{a^3\cot(c+dx)}{d} - a^3\int 1\,dx - \frac{a^3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx\right.}{d}$$

$$= -a^3x - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+dx)}{d} - \frac{a^3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,\right.}{d}$$

$$= -a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+d}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(49) = 98.
time = 0.25, size = 109, normalized size = 2.22

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/8*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(d*x + Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*Csc[c
/2]*Csc[(c + d*x)/2]*Sin[(d*x)/2]))/d
```

Maple [A]
time = 0.08, size = 79, normalized size = 1.61

method	result	size
risch	$-a^{3} x-\frac{8 i a^{3}}{d \left({\mathrm e}^{i \left(d x +c \right)}-1\right)}-\frac{a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{d}+\frac{a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{d}$	71
derivativedivides	$\frac{a^{3} \left(-\frac{1}{\sin \left(d x +c \right)}+\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)\right)-3 a^{3} \cot \left(d x +c \right)-\frac{3 a^{3}}{\sin \left(d x +c \right)}+a^{3} \left(-\cot \left(d x +c \right)-d x -c \right)}{d}$	79
default	$\frac{a^{3} \left(-\frac{1}{\sin \left(d x +c \right)}+\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)\right)-3 a^{3} \cot \left(d x +c \right)-\frac{3 a^{3}}{\sin \left(d x +c \right)}+a^{3} \left(-\cot \left(d x +c \right)-d x -c \right)}{d}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-3*a^3*cot(d*x+c)-3*a^3/s
in(d*x+c)+a^3*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 1.73

$$-\frac{2 \left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a^{3}+a^{3}\left(\frac{2}{\sin\left(dx+c\right)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6\,a^{3}}{\sin\left(dx+c\right)}+\frac{6\,a^{3}}{\tan\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2/sin(d*x + c) - log(sin(d*x
+ c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3/sin(d*x + c) + 6*a^3/tan(d*x + c
))/d
```

Fricas [A]
time = 3.16, size = 84, normalized size = 1.71

$$-\frac{2\,a^{3}dx\sin\left(dx+c\right)-a^{3}\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+a^{3}\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+8\,a^{3}\cos\left(dx+c\right)+8\,a^{3}}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x*sin(d*x + c) - a^3*log(sin(d*x + c) + 1)*sin(d*x + c) + a^3
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 8*a^3*cos(d*x + c) + 8*a^3)/(d*sin(d
*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot^2(c+dx)\sec(c+dx)\,dx + \int 3\cot^2(c+dx)\sec^2(c+dx)\,dx + \int \cot^2(c+dx)\sec^3(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**2*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**2, x))
```

Giac [A]
time = 0.52, size = 66, normalized size = 1.35

$$-\frac{(dx+c)a^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + a^3*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 4*a^3/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 1.20, size = 35, normalized size = 0.71

$$-\frac{a^3\left(4\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(4*cot(c/2 + (d*x)/2) - 2*atanh(tan(c/2 + (d*x)/2)) + d*x))/d
```

3.51 $\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=69

$$a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-4/3*a^3*cot(d*x+c)^3/d+3*a^3*csc(d*x+c)/d-4/3*a^3*cs
c(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3971, 3554, 8, 2686, 2687, 30}

$$-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) + (3*a^3*Csc[c
+ d*x])/d - (4*a^3*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)+3a^3\cot^2(c+\right.\\ &= a^3\int \cot^4(c+dx)\,dx+a^3\int \cot(c+dx)\csc^3(c+dx)\,dx+\left(3a^3\right)\\ &= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int \cot^2(c+dx)\,dx-\frac{a^3\text{Subst}\left(\int x^2\,dx,x,\text{cs}\right.}{d}\\ &= \frac{a^3\cot(c+dx)}{d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}-\frac{4a^3\csc^3(c}{3d}\\ &= a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}-\frac{4a^3}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 112, normalized size = 1.62

$$\frac{a^3\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(9dx\cos\left(\frac{dx}{2}\right)-9dx\cos\left(c+\frac{dx}{2}\right)-3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\frac{3dx}{2}\right)-24\sin\left(\frac{dx}{2}\right)-18\sin\left(c+\frac{dx}{2}\right)+14\sin\left(c+\frac{3dx}{2}\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] - 9*d*x*Cos[c + (d*x)/
2] - 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 24*Sin[(d*x)/2
] - 18*Sin[c + (d*x)/2] + 14*Sin[c + (3*d*x)/2]))/(24*d)
```

Maple [A]
time = 0.10, size = 125, normalized size = 1.81

method	result

risch	$a^3x+\frac{2ia^3(9\,e^{2i(dx+c)}-12\,e^{i(dx+c)}+7)}{3d(e^{i(dx+c)}-1)^3}$
derivativedivides	$\frac{-\frac{a^3}{3\sin(dx+c)^3}-\frac{a^3(\cos^3(dx+c))}{\sin(dx+c)^3}+3a^3\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{(2+\cos^2(dx+c))\sin(dx+c)}{3}\right)+a^3\left(-\frac{(\cot^3(dx+c))}{3}+\cot(\right.}{d}$
default	$\frac{-\frac{a^3}{3\sin(dx+c)^3}-\frac{a^3(\cos^3(dx+c))}{\sin(dx+c)^3}+3a^3\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{(2+\cos^2(dx+c))\sin(dx+c)}{3}\right)+a^3\left(-\frac{(\cot^3(dx+c))}{3}+\cot(\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/sin(d*x+c)^3-a^3/sin(d*x+c)^3*cos(d*x+c)^3+3*a^3*(-1/3/sin(d*
x+c)^3*cos(d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*
x+c))+a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.48, size = 90, normalized size = 1.30

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+\frac{3\left(3\,\sin(dx+c)^2-1\right)a^3}{\sin(dx+c)^3}-\frac{a^3}{\sin(dx+c)^3}-\frac{3\,a^3}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 3*(3*sin(d
*x + c)^2 - 1)*a^3/sin(d*x + c)^3 - a^3/sin(d*x + c)^3 - 3*a^3/tan(d*x + c)
^3)/d
```

Fricas [A]
time = 2.69, size = 82, normalized size = 1.19

$$\frac{7\,a^3\cos\left(dx+c\right)^2+2\,a^3\cos\left(dx+c\right)-5\,a^3+3\left(a^3dx\cos\left(dx+c\right)-a^3dx\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(7*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 5*a^3 + 3*(a^3*d*x*cos(d*x
 + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cot^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cot^4\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**4*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**4, x))
```

Giac [A]
time = 0.53, size = 50, normalized size = 0.72

$$\frac{3\,(dx+c)a^3+\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + (6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3)/tan(1/2*d*x + 1
/2*c)^3)/d
```

Mupad [B]
time = 1.20, size = 39, normalized size = 0.57

$$a^3\,x+\frac{a^3\,\left(6\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (a^3*(6*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(3*d)
```

3.52 $\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-a^3x-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{3a^3\csc(c+dx)}{d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{4a^3\csc^5(c+dx)}{5d}$$

```
[Out] -a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-4/5*a^3*cot(d*x+c)^5/d-3*a^3
*csc(d*x+c)/d+7/3*a^3*csc(d*x+c)^3/d-4/5*a^3*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30, 14}

$$-\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (4*a^3*Cot[c
 + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x]^3)/(3*d) -
(4*a^3*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\cot^6(c+dx)+3a^3\cot^5(c+dx)\csc(c+dx)+3a^3\cot^4(c+
\\
&= a^3\int \cot^6(c+dx)\,dx+a^3\int \cot^3(c+dx)\csc^3(c+dx)\,dx+\left(3a^3\right)
\\
&= -\frac{a^3\cot^5(c+dx)}{5d}-a^3\int \cot^4(c+dx)\,dx-\frac{a^3\mathrm{Subst}\left(\int x^2(-1+x
\\
&= \frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}+a^3\int \cot^2(c+dx)\,dx-\frac{a^3\mathrm{Su}
\\
&= -\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{3a^3\csc(c}{d}
\\
&= -a^3x-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{3a
\end{aligned}$$

Mathematica [A]
time = 0.70, size = 112, normalized size = 1.05

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(60dx+(-10+13\cos(c+dx))\cot\left(\frac{c}{2}\right)\csc^4\left(\frac{1}{2}(c+dx)\right)+(-38+51\cos(c+dx)-16\cos(2(c+dx)))\csc\left(\frac{c}{2}\right)\csc^5\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(60*d*x + (-10 + 13*Cos
[c + d*x])*Cot[c/2]*Csc[(c + d*x)/2]^4 + (-38 + 51*Cos[c + d*x] - 16*Cos[2*
(c + d*x)])*Csc[c/2]*Csc[(c + d*x)/2]^5*Sin[(d*x)/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(99) = 198$.
time = 0.11, size = 232, normalized size = 2.17

method	result
risch	$-a^3x - \frac{2ia^3\left(45\,e^{4i(dx+c)}-135\,e^{3i(dx+c)}+185\,e^{2i(dx+c)}-115\,e^{i(dx+c)}+32\right)}{15d\left(e^{i(dx+c)}-1\right)^5}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^4(dx+c)}{5\sin(dx+c)^5}-\frac{\cos^4(dx+c)}{15\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{15\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^3\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+3a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos}{15\sin}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^4(dx+c)}{5\sin(dx+c)^5}-\frac{\cos^4(dx+c)}{15\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{15\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^3\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+3a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos}{15\sin}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^4-1/15/sin(d*x+c)^3*cos(d*x+c)^4+1/1
5/sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^3/sin(d*x
+c)^5*cos(d*x+c)^5+3*a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*
cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))+a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)
)
```

Maxima [A]
time = 0.48, size = 122, normalized size = 1.14

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+\frac{3\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a^3}{\sin(dx+c)^5}-\frac{\left(5\,\sin(dx+c)^2-3\right)a^3}{\sin(dx+c)^5}+\frac{9\,a^3}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^3 + 3*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^3/sin(d*x + c
)^5 - (5*sin(d*x + c)^2 - 3)*a^3/sin(d*x + c)^5 + 9*a^3/tan(d*x + c)^5)/d
```

Fricas [A]
time = 3.45, size = 118, normalized size = 1.10

$$-\frac{32 a^3 \cos(dx+c)^3 - 19 a^3 \cos(dx+c)^2 - 29 a^3 \cos(dx+c) + 22 a^3 + 15 \left(a^3 dx \cos(dx+c)^2 - 2 a^3 dx \cos(dx+c) + a^3 dx\right) \sin(dx+c)}{15 \left(d \cos(dx+c)^2 - 2 d \cos(dx+c) + d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(32*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 29*a^3*cos(d*x + c)
+ 22*a^3 + 15*(a^3*d*x*cos(d*x + c)^2 - 2*a^3*d*x*cos(d*x + c) + a^3*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int 3 \cot^6(c+dx) \sec(c+dx)\, dx + \int 3 \cot^6(c+dx) \sec^2(c+dx)\, dx + \int \cot^6(c+dx) \sec^3(c+dx)\, dx + \int \cot^6(c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**6*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**6, x))
```

Giac [A]
time = 0.53, size = 66, normalized size = 0.62

$$-\frac{60 (dx+c) a^3 + \frac{105 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 20 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 3 a^3}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^3 + (105*a^3*tan(1/2*d*x + 1/2*c)^4 - 20*a^3*tan(1/2*
d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 1.45, size = 62, normalized size = 0.58

$$\frac{a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3 d} - a^3 x - \frac{a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20 d} - \frac{7 a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(3*d) - a^3*x - (a^3*cot(c/2 + (d*x)/2)^5)/(20*d
) - (7*a^3*cot(c/2 + (d*x)/2))/(4*d)
```

3.53 $\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=141

$$a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{3a^3\csc(c+dx)}{d}-\frac{10a^3\csc^3(c+dx}{3d}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-4/7*a^
3*cot(d*x+c)^7/d+3*a^3*csc(d*x+c)/d-10/3*a^3*csc(d*x+c)^3/d+11/5*a^3*csc(d*
x+c)^5/d-4/7*a^3*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$,
Rules used = {3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a
^3*Csc[c + d*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d
*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)+3a^3\cot^6(c+\right.\\&= a^3\int \cot^8(c+dx)\,dx+a^3\int \cot^5(c+dx)\csc^3(c+dx)\,dx+\left(3a^3\right)\\&= -\frac{a^3\cot^7(c+dx)}{7d}-a^3\int \cot^6(c+dx)\,dx-\frac{a^3\mathrm{Subst}\left(\int x^2(-1+x\right.}{}\\&= \frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}+a^3\int \cot^4(c+dx)\,dx-\frac{a^3\mathrm{Su}}{}\\&= -\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{3a^3\csc(}{}\\&= \frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+}{7d}\\&= a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot}{}\end{aligned}$$

Mathematica [A]
time = 0.99, size = 252, normalized size = 1.79

$$\frac{a^3 \csc\left(\frac{c}{2}\right) \csc^7\left(\frac{1}{2}(c+dx)\right) \sec\left(\frac{c}{2}\right) \sec\left(\frac{1}{2}(c+dx)\right) (5880dx\cos(dx) - 5880dx\cos(2c+dx) - 5880dx\cos(c+2dx) + 5880dx\cos(3c+2dx) + 2520dx\cos(2c+3dx) - 2520dx\cos(4c+3dx) - 420dx\cos(3c+4dx) + 420dx\cos(5c+4dx) + 4200\sin(c) - 11032\sin(dx) - 23282\sin(c+dx) + 23282\sin(2(c+dx)) - 9978\sin(3(c+dx)) + 1663\sin(4(c+dx)) - 13720\sin(2c+dx) + 15512\sin(c+2dx) + 9240\sin(3c+2dx) - 8088\sin(2c+3dx) - 2520\sin(4c+3dx) + 1768\sin(3c+4dx))}{215040d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^7*Sec[c/2]*Sec[(c + d*x)/2]*(5880*d*x*Cos[d*
x] - 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c +
 2*d*x] + 2520*d*x*Cos[2*c + 3*d*x] - 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*C
os[3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] - 11032*Sin[d*x] -
 23282*Sin[c + d*x] + 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] + 1663
*Sin[4*(c + d*x)] - 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[
3*c + 2*d*x] - 8088*Sin[2*c + 3*d*x] - 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c
 + 4*d*x]))/(215040*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(129) = 258$.
time = 0.11, size = 293, normalized size = 2.08

method	result
risch	$a^3x + \frac{2ia^3(315\,e^{7i(dx+c)} - 1155\,e^{6i(dx+c)} + 1715\,e^{5i(dx+c)} - 525\,e^{4i(dx+c)} - 1379\,e^{3i(dx+c)} + 1939\,e^{2i(dx+c)} - 1011\,e^{i(dx+c)}}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7} - \frac{\cos^6(dx+c)}{35\sin(dx+c)^5} + \frac{\cos^6(dx+c)}{105\sin(dx+c)^3} - \frac{\cos^6(dx+c)}{35\sin(dx+c)} - \frac{\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right) - \frac{3a^3\left(\cos^7(\right.}{7\sin(dx-}}{}$
default	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7} - \frac{\cos^6(dx+c)}{35\sin(dx+c)^5} + \frac{\cos^6(dx+c)}{105\sin(dx+c)^3} - \frac{\cos^6(dx+c)}{35\sin(dx+c)} - \frac{\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right) - \frac{3a^3\left(\cos^7(\right.}{7\sin(dx-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/1
05/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x
+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-3/7*a^3/sin(d*x+c)^7*cos(d*x+c)^7+3*a^3
*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/sin(d*
x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+6/5*
cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a^3*(-1/7*cot(d*x+c)^7+1/5*cot(d
*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 152, normalized size = 1.08

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^3+\frac{9\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a^3}{\sin(dx+c)^7}-\frac{\left(35\,\sin(dx+c)^4-42\,\sin(dx+c)^2+15\right)a^3}{\sin(dx+c)^7}-\frac{45\,a^3}{\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^3 + 9*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^3/sin(d*x + c)^7 - (35*sin(d*x + c)^4 - 42*
sin(d*x + c)^2 + 15)*a^3/sin(d*x + c)^7 - 45*a^3/tan(d*x + c)^7)/d
```

Fricas [A]
time = 3.14, size = 160, normalized size = 1.13

$$\frac{221\,a^3\cos\left(dx+c\right)^4-348\,a^3\cos\left(dx+c\right)^3-25\,a^3\cos\left(dx+c\right)^2+303\,a^3\cos\left(dx+c\right)-136\,a^3+105\left(a^3dx\cos\left(dx+c\right)^3-3\,a^3dx\cos\left(dx+c\right)^2+3\,a^3dx\cos\left(dx+c\right)-a^3dx\right)\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^3-3\,d\cos\left(dx+c\right)^2+3\,d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(221*a^3*cos(d*x + c)^4 - 348*a^3*cos(d*x + c)^3 - 25*a^3*cos(d*x + c
)^2 + 303*a^3*cos(d*x + c) - 136*a^3 + 105*(a^3*d*x*cos(d*x + c)^3 - 3*a^3*
d*x*cos(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*co
s(d*x + c)^3 - 3*d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 96, normalized size = 0.68

$$\frac{1680\,(dx+c)a^3-105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2730\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+126\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1680*(1680*(d*x + c)*a^3 - 105*a^3*tan(1/2*d*x + 1/2*c) + (2730*a^3*tan(1
/2*d*x + 1/2*c)^6 - 560*a^3*tan(1/2*d*x + 1/2*c)^4 + 126*a^3*tan(1/2*d*x +
1/2*c)^2 - 15*a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 1.70, size = 91, normalized size = 0.65

$$\frac{a^3\left(126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-560\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2730\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+1680\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(c+dx)-15\right)}{1680\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*(126*tan(c/2 + (d*x)/2)^2 - 560*tan(c/2 + (d*x)/2)^4 + 2730*tan(c/2 +
(d*x)/2)^6 - 105*tan(c/2 + (d*x)/2)^8 + 1680*tan(c/2 + (d*x)/2)^7*(c + d*x)
 - 15))/(1680*d*tan(c/2 + (d*x)/2)^7)
```

3.54 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=179

$$-a^3x-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{3a^3\csc(c+d}{d}$$

```
[Out] -a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+1/7*a
^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-3*a^3*csc(d*x+c)/d+13/3*a^3*csc(d*
x+c)^3/d-21/5*a^3*csc(d*x+c)^5/d+15/7*a^3*csc(d*x+c)^7/d-4/9*a^3*csc(d*x+c)
^9/d
```

Rubi [A]
time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc^7(c+dx)}{7d}-\frac{21a^3\csc^5(c+dx)}{5d}+\frac{13a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c +
 d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d)
- (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]^3)/(3*d) - (21*a^3*Csc[c +
d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) - (4*a^3*Csc[c + d*x]^9)/(9*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cot^{10}(c+dx) + 3a^3\cot^9(c+dx)\csc(c+dx) + 3a^3\cot^8(c+\right.$$

$$= a^3\int \cot^{10}(c+dx)\,dx + a^3\int \cot^7(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right.$$

$$= -\frac{a^3\cot^9(c+dx)}{9d} - a^3\int \cot^8(c+dx)\,dx - \frac{a^3\mathrm{Subst}\left(\int x^2(-1+\right.}{}$$

$$= \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} + a^3\int \cot^6(c+dx)\,dx - \frac{a^3\mathrm{S}}{}$$

$$= -\frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{3a^3\csc}{}$$

$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c}{9d}$$

$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c}{7d}$$

$$= -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 370 vs. 2(179) = 358.
time = 1.36, size = 370, normalized size = 2.07

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Csc[c/2]*Csc[(c + d*x)/2]^9*Sec[c/2]*Sec[(c + d*x)/2]^3*(-181440*d*x*C
os[d*x] + 181440*d*x*Cos[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*
x*Cos[3*c + 2*d*x] + 10080*d*x*Cos[2*c + 3*d*x] - 10080*d*x*Cos[4*c + 3*d*x
] - 60480*d*x*Cos[3*c + 4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] + 30240*d*x*Cos
[4*c + 5*d*x] - 30240*d*x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 50
40*d*x*Cos[7*c + 6*d*x] - 169344*Sin[c] + 338112*Sin[d*x] + 675036*Sin[c +
d*x] - 506277*Sin[2*(c + d*x)] - 37502*Sin[3*(c + d*x)] + 225012*Sin[4*(c +
 d*x)] - 112506*Sin[5*(c + d*x)] + 18751*Sin[6*(c + d*x)] + 431424*Sin[2*c
+ d*x] - 375552*Sin[c + 2*d*x] - 201600*Sin[3*c + 2*d*x] + 41248*Sin[2*c +
3*d*x] - 84000*Sin[4*c + 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c
+ 4*d*x] - 98016*Sin[4*c + 5*d*x] - 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c
+ 6*d*x]))/(41287680*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(163) = 326.
time = 0.13, size = 364, normalized size = 2.03

method	result
risch	$-a^3x - \frac{2ia^3\left(945\,\mathrm{e}^{11i(dx+c)} - 3150\,\mathrm{e}^{10i(dx+c)} + 2625\,\mathrm{e}^{9i(dx+c)} + 6300\,\mathrm{e}^{8i(dx+c)} - 13482\,\mathrm{e}^{7i(dx+c)} + 5292\,\mathrm{e}^{6i(dx+c)} + 1056\right.}{315d\left(\mathrm{e}^{i(dx+c)}-1\right)^9\left(\mathrm{e}^{i(dx+c)}+\right.}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^8(dx+c)}{9\sin(dx+c)^9} - \frac{\cos^8(dx+c)}{63\sin(dx+c)^7} + \frac{\cos^8(dx+c)}{315\sin(dx+c)^5} - \frac{\cos^8(dx+c)}{315\sin(dx+c)^3} + \frac{\cos^8(dx+c)}{63\sin(dx+c)} + \frac{\left(\frac{16}{5} + \cos^6(dx+c) + \frac{6\left(\cos^4(dx+c)\right)}{5} + \frac{8\left(\cos^2\right.}{}\right.}{63}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^8(dx+c)}{9\sin(dx+c)^9} - \frac{\cos^8(dx+c)}{63\sin(dx+c)^7} + \frac{\cos^8(dx+c)}{315\sin(dx+c)^5} - \frac{\cos^8(dx+c)}{315\sin(dx+c)^3} + \frac{\cos^8(dx+c)}{63\sin(dx+c)} + \frac{\left(\frac{16}{5} + \cos^6(dx+c) + \frac{6\left(\cos^4(dx+c)\right)}{5} + \frac{8\left(\cos^2\right.}{}\right.}{63}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x+c)^8+1/3
15/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/sin(d*x+c
)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*s
in(d*x+c))-1/3*a^3/sin(d*x+c)^9*cos(d*x+c)^9+3*a^3*(-1/9/sin(d*x+c)^9*cos(d
*x+c)^10+1/63/sin(d*x+c)^7*cos(d*x+c)^10-1/105/sin(d*x+c)^5*cos(d*x+c)^10+1
/63/sin(d*x+c)^3*cos(d*x+c)^10-1/9/sin(d*x+c)*cos(d*x+c)^10-1/9*(128/35+cos
(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c
))+a^3*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^
3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 182, normalized size = 1.02

$$-\frac{\left(315\,dx + 315\,c + \frac{315\,\tan(dx+c)^8 - 105\,\tan(dx+c)^6 + 63\,\tan(dx+c)^4 - 45\,\tan(dx+c)^2 + 35}{\tan(dx+c)^9}\right)a^3 + \frac{3\left(315\,\sin(dx+c)^8 - 420\,\sin(dx+c)^6 + 378\,\sin(dx+c)^4 - 180\,\sin(dx+c)^2 + 35\right)a^3}{\sin(dx+c)^9} - \frac{\left(105\,\sin(dx+c)^6 - 189\,\sin(dx+c)^4 + 135\,\sin(dx+c)^2 - 35\right)a^3}{\sin(dx+c)^9} + \frac{105\,a^3}{\tan(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^3 + 3*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^3/sin(d*x + c)^9 - (105*sin(d*x + c)^6 - 189*sin(d*x + c)^4 + 135*sin(d
*x + c)^2 - 35)*a^3/sin(d*x + c)^9 + 105*a^3/tan(d*x + c)^9)/d
```

Fricas [A]
time = 2.98, size = 235, normalized size = 1.31

$$-\frac{668\,a^3\cos(dx+c)^6 - 1059\,a^3\cos(dx+c)^5 - 573\,a^3\cos(dx+c)^4 + 1813\,a^3\cos(dx+c)^3 - 393\,a^3\cos(dx+c)^2 - 789\,a^3\cos(dx+c) + 368\,a^3 + 315\left(a^3dx\cos(dx+c)^5 - 3\,a^3dx\cos(dx+c)^4 + 2\,a^3dx\cos(dx+c)^3 + 2\,a^3dx\cos(dx+c)^2 - 3\,a^3dx\cos(dx+c) + a^3dx\right)\sin(dx+c)}{315\left(d\cos(dx+c)^5 - 3\,d\cos(dx+c)^4 + 2\,d\cos(dx+c)^3 + 2\,d\cos(dx+c)^2 - 3\,d\cos(dx+c) + d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(668*a^3*cos(d*x + c)^6 - 1059*a^3*cos(d*x + c)^5 - 573*a^3*cos(d*x
+ c)^4 + 1813*a^3*cos(d*x + c)^3 - 393*a^3*cos(d*x + c)^2 - 789*a^3*cos(d*x
 + c) + 368*a^3 + 315*(a^3*d*x*cos(d*x + c)^5 - 3*a^3*d*x*cos(d*x + c)^4 +
2*a^3*d*x*cos(d*x + c)^3 + 2*a^3*d*x*cos(d*x + c)^2 - 3*a^3*d*x*cos(d*x + c
) + a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 2*d*co
s(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**A**]
time = 0.61, size = 128, normalized size = 0.72

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20160\,(dx+c)a^3-2520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{31185\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-6720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1827\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 + 20160*(d*x + c)*a^3 - 2520*a^3*t
an(1/2*d*x + 1/2*c) + (31185*a^3*tan(1/2*d*x + 1/2*c)^8 - 6720*a^3*tan(1/2*
d*x + 1/2*c)^6 + 1827*a^3*tan(1/2*d*x + 1/2*c)^4 - 360*a^3*tan(1/2*d*x + 1/
2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [**B**]
time = 1.94, size = 206, normalized size = 1.15

$$-\frac{a^3\left(35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+105\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-2520\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+31185\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-6720\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+1827\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-360\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+20160\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9(c+d\,x)\right)}{20160\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(35*cos(c/2 + (d*x)/2)^12 + 105*sin(c/2 + (d*x)/2)^12 - 2520*cos(c/2
+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 31185*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1827*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 360*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^2 + 20160*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(20160
*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9)
```

3.55 $\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=213

$$a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-1/7*a^
3*cot(d*x+c)^7/d+1/9*a^3*cot(d*x+c)^9/d-4/11*a^3*cot(d*x+c)^11/d+3*a^3*csc(
d*x+c)/d-16/3*a^3*csc(d*x+c)^3/d+34/5*a^3*csc(d*x+c)^5/d-36/7*a^3*csc(d*x+c
)^7/d+19/9*a^3*csc(d*x+c)^9/d-4/11*a^3*csc(d*x+c)^11/d
```

Rubi [A]
time = 0.17, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^{11}(c+dx)}{11d}+\frac{19a^3\csc^9(c+dx)}{9d}-\frac{36a^3\csc^7(c+dx)}{7d}+\frac{34a^3\csc^5(c+dx)}{5d}-\frac{16a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d) - (4*
a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c + d*x]
^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*d) +
 (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)+3a^3\cot^{10}(c+\right. \\
&= a^3\int \cot^{12}(c+dx)\,dx + a^3\int \cot^9(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right) \\
&= -\frac{a^3\cot^{11}(c+dx)}{11d} - a^3\int \cot^{10}(c+dx)\,dx - \frac{a^3\mathrm{Subst}\left(\int x^2(-1+\right.}{} \\
&= \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + a^3\int \cot^8(c+dx)\,dx - \frac{a^3\mathrm{Su}}{} \\
&= -\frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} + \frac{3a^3\csc(}{} \\
&= \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c}{11d} \\
&= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\cot^9(c}{9d} \\
&= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+}{7d} \\
&= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot}{}
\end{aligned}
$$

Mathematica [**A**]
time = 6.19, size = 268, normalized size = 1.26

$$
-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(20(2786111-4528480\cos(c+dx)+2388316\cos(2(c+dx))-750112\cos(3(c+dx))+112229\cos(4(c+dx)))\cot^2\left(\frac{c}{2}\right)\csc^{10}\left(\frac{1}{2}(c+dx)\right)-5\cot\left(\frac{c}{2}\right)\left(90832896dx+(-32611198+54812150\cos(c+dx)-32118776\cos(2(c+dx))+12626567\cos(3(c+dx))-3023754\cos(4(c+dx))+347267\cos(5(c+dx)))\csc\left(\frac{c}{2}\right)\csc^{11}\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)+7392\csc\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(4370\sin\left(\frac{dx}{2}\right)-3060\sin\left(c+\frac{dx}{2}\right)+2860\sin\left(c+\frac{3dx}{2}\right)-855\sin\left(2c+\frac{3dx}{2}\right)+743\sin\left(2c+\frac{5dx}{2}\right)\right)\tan\left(\frac{c}{2}\right)\right)}{3633315840d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/3633315840*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(20*(2786111 - 4
528480*Cos[c + d*x] + 2388316*Cos[2*(c + d*x)] - 750112*Cos[3*(c + d*x)] +
112229*Cos[4*(c + d*x)])*Cot[c/2]^2*Csc[(c + d*x)/2]^10 - 5*Cot[c/2]*(90832
896*d*x + (-32611198 + 54812150*Cos[c + d*x] - 32118776*Cos[2*(c + d*x)] +
12626567*Cos[3*(c + d*x)] - 3023754*Cos[4*(c + d*x)] + 347267*Cos[5*(c + d*
x)])*Csc[c/2]*Csc[(c + d*x)/2]^11*Sin[(d*x)/2]) + 7392*Csc[c/2]*Sec[(c + d*
x)/2]^5*(4370*Sin[(d*x)/2] - 3060*Sin[c + (d*x)/2] + 2860*Sin[c + (3*d*x)/2
] - 855*Sin[2*c + (3*d*x)/2] + 743*Sin[2*c + (5*d*x)/2]))*Tan[c/2])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(193) = 386$.
time = 0.17, size = 425, normalized size = 2.00

method	result
risch	$a^3x+\frac{2ia^3(10395\,e^{15i(dx+c)}-31185\,e^{14i(dx+c)}+1155\,e^{13i(dx+c)}+148995\,e^{12i(dx+c)}-190113\,e^{11i(dx+c)}-117117\,e^{10i}}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^{10}(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^{10}(dx+c)}{99\sin(dx+c)^9}+\frac{\cos^{10}(dx+c)}{693\sin(dx+c)^7}-\frac{\cos^{10}(dx+c)}{1155\sin(dx+c)^5}+\frac{\cos^{10}(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^{10}(dx+c)}{99\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos^8(dx+c)+\right.}{}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^{10}(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^{10}(dx+c)}{99\sin(dx+c)^9}+\frac{\cos^{10}(dx+c)}{693\sin(dx+c)^7}-\frac{\cos^{10}(dx+c)}{1155\sin(dx+c)^5}+\frac{\cos^{10}(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^{10}(dx+c)}{99\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos^8(dx+c)+\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/11/sin(d*x+c)^11*cos(d*x+c)^10-1/99/sin(d*x+c)^9*cos(d*x+c)^10
+1/693/sin(d*x+c)^7*cos(d*x+c)^10-1/1155/sin(d*x+c)^5*cos(d*x+c)^10+1/693/s
in(d*x+c)^3*cos(d*x+c)^10-1/99/sin(d*x+c)*cos(d*x+c)^10-1/99*(128/35+cos(d*
x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-
3/11*a^3/sin(d*x+c)^11*cos(d*x+c)^11+3*a^3*(-1/11/sin(d*x+c)^11*cos(d*x+c)^
12+1/99/sin(d*x+c)^9*cos(d*x+c)^12-1/231/sin(d*x+c)^7*cos(d*x+c)^12+1/231/s
in(d*x+c)^5*cos(d*x+c)^12-1/99/sin(d*x+c)^3*cos(d*x+c)^12+1/11/sin(d*x+c)*c
os(d*x+c)^12+1/11*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^
6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))+a^3*(-1/11*cot(d*x+c)
^11+1/9*cot(d*x+c)^9-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot
(d*x+c)+d*x+c))
```

Maxima [A]

time = 0.49, size = 212, normalized size = 1.00

$$\frac{\left(3465\,dx+3465\,c+\frac{3465\,\tan(dx+c)^{10}-1155\,\tan(dx+c)^8+693\,\tan(dx+c)^6-495\,\tan(dx+c)^4+385\,\tan(dx+c)^2-315}{\tan(dx+c)^{11}}\right)a^3+\frac{15\left(693\,\sin(dx+c)^{10}-1155\,\sin(dx+c)^8+1386\,\sin(dx+c)^6-990\,\sin(dx+c)^4+385\,\sin(dx+c)^2-63\right)a^3}{\sin(dx+c)^{11}}-\frac{\left(1155\,\sin(dx+c)^8-2772\,\sin(dx+c)^6+2970\,\sin(dx+c)^4-1540\,\sin(dx+c)^2+315\right)a^3}{\sin(dx+c)^{11}}-\frac{945\,a^3}{\tan(dx+c)^{11}}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3465*((3465*d*x + 3465*c + (3465*tan(d*x + c)^10 - 1155*tan(d*x + c)^8 +
693*tan(d*x + c)^6 - 495*tan(d*x + c)^4 + 385*tan(d*x + c)^2 - 315)/tan(d*x
 + c)^11)*a^3 + 15*(693*sin(d*x + c)^10 - 1155*sin(d*x + c)^8 + 1386*sin(d*
x + c)^6 - 990*sin(d*x + c)^4 + 385*sin(d*x + c)^2 - 63)*a^3/sin(d*x + c)^1
1 - (1155*sin(d*x + c)^8 - 2772*sin(d*x + c)^6 + 2970*sin(d*x + c)^4 - 1540
*sin(d*x + c)^2 + 315)*a^3/sin(d*x + c)^11 - 945*a^3/tan(d*x + c)^11)/d
```

Fricas [A]

time = 3.11, size = 314, normalized size = 1.47

$$\frac{7453\,a^3\cos(dx+c)^8-11964\,a^3\cos(dx+c)^7-11866\,a^3\cos(dx+c)^6+30542\,a^3\cos(dx+c)^5+90\,a^3\cos(dx+c)^4-26438\,a^3\cos(dx+c)^3+8539\,a^3\cos(dx+c)^2+7671\,a^3\cos(dx+c)-3712\,a^3+3465\left(a^3dx\cos(dx+c)^7-3\,a^3dx\cos(dx+c)^6+a^3dx\cos(dx+c)^5+5\,a^3dx\cos(dx+c)^4-5\,a^3dx\cos(dx+c)^3-a^3dx\cos(dx+c)^2+3\,a^3dx\cos(dx+c)-a^3dx\right)\sin(dx+c)}{3465\left(d\cos(dx+c)^7-3\,d\cos(dx+c)^6+d\cos(dx+c)^5+5\,d\cos(dx+c)^4-5\,d\cos(dx+c)^3-d\cos(dx+c)^2+3\,d\cos(dx+c)-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*a^3*cos(d*x + c)^8 - 11964*a^3*cos(d*x + c)^7 - 11866*a^3*cos(
d*x + c)^6 + 30542*a^3*cos(d*x + c)^5 + 90*a^3*cos(d*x + c)^4 - 26438*a^3*c
os(d*x + c)^3 + 8539*a^3*cos(d*x + c)^2 + 7671*a^3*cos(d*x + c) - 3712*a^3
+ 3465*(a^3*d*x*cos(d*x + c)^7 - 3*a^3*d*x*cos(d*x + c)^6 + a^3*d*x*cos(d*x
 + c)^5 + 5*a^3*d*x*cos(d*x + c)^4 - 5*a^3*d*x*cos(d*x + c)^3 - a^3*d*x*cos
(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x +
 c)^7 - 3*d*cos(d*x + c)^6 + d*cos(d*x + c)^5 + 5*d*cos(d*x + c)^4 - 5*d*co
s(d*x + c)^3 - d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**12*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [A]
time = 0.66, size = 161, normalized size = 0.76

$$-\frac{693\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11550\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-887040\,(dx+c)a^3+159390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{5\left(264726\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-59136\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+18018\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-4554\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+770\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-63\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(693*a^3*tan(1/2*d*x + 1/2*c)^5 - 11550*a^3*tan(1/2*d*x + 1/2*c)^
3 - 887040*(d*x + c)*a^3 + 159390*a^3*tan(1/2*d*x + 1/2*c) - 5*(264726*a^3*
tan(1/2*d*x + 1/2*c)^10 - 59136*a^3*tan(1/2*d*x + 1/2*c)^8 + 18018*a^3*tan(
1/2*d*x + 1/2*c)^6 - 4554*a^3*tan(1/2*d*x + 1/2*c)^4 + 770*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 63*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

Mupad [B]
time = 3.03, size = 254, normalized size = 1.19

$$-\frac{a^3\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+693\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-11550\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+159390\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-1323630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+295680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-90090\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+22770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3850\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-887040\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}(c+dx)\right)}{887040\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^12*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(315*cos(c/2 + (d*x)/2)^16 + 693*sin(c/2 + (d*x)/2)^16 - 11550*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 159390*cos(c/2 + (d*x)/2)^4*sin(c/2
```

```
+ (d*x)/2)^12 - 1323630*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680
*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 90090*cos(c/2 + (d*x)/2)^10*si
n(c/2 + (d*x)/2)^6 + 22770*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 385
0*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 - 887040*cos(c/2 + (d*x)/2)^5*
sin(c/2 + (d*x)/2)^11*(c + d*x)))/(887040*d*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2)^11)
```

3.56 $\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^4(c+dx)}{4ad}-\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^6(c+dx)}{6ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-3/2*sec(d*x+c)^2/a/d+sec(d*x+c)^3/a/d+3/
4*sec(d*x+c)^4/a/d-3/5*sec(d*x+c)^5/a/d-1/6*sec(d*x+c)^6/a/d+1/7*sec(d*x+c)
^7/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{\sec^7(c+dx)}{7ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\sec^5(c+dx)}{5ad}+\frac{3\sec^4(c+dx)}{4ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - (3*Sec[c + d*x]^2)/(2*a*d
) + Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^4)/(4*a*d) - (3*Sec[c + d*x]^5)/
(5*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]^7/(7*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^3}{x^8}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{a^7}{x^7}-\frac{3a^7}{x^6}+\frac{3a^7}{x^5}+\frac{3a^7}{x^4}-\frac{3a^7}{x^3}-\frac{a^7}{x^2}+\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^4(c+d}{4ad}$$

Mathematica [A]
time = 0.53, size = 137, normalized size = 1.01

$$-\frac{(35\cos(c+dx)(104+105\log(\cos(c+dx)))+3(212+602\cos(2(c+dx))+140\cos(4(c+dx))+210\cos(5(c+dx))+70\cos(6(c+dx))+245\cos(5(c+dx))\log(\cos(c+dx))+35\cos(7(c+dx))\log(\cos(c+dx))+105\cos(3(c+dx))(6+7\log(\cos(c+dx)))))\sec^7(c+dx)}{6720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6720*((35*Cos[c + d*x]*(104 + 105*Log[Cos[c + d*x]]) + 3*(212 + 602*Cos[
2*(c + d*x)] + 140*Cos[4*(c + d*x)] + 210*Cos[5*(c + d*x)] + 70*Cos[6*(c +
d*x)] + 245*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 35*Cos[7*(c + d*x)]*Log[Co
s[c + d*x]] + 105*Cos[3*(c + d*x)]*(6 + 7*Log[Cos[c + d*x]])))*Sec[c + d*x]
^7)/(a*d)
```

Maple [A]
time = 0.15, size = 82, normalized size = 0.61

method	result
derivativedivides	$\frac{\frac{\left(\sec^7(dx+c)\right)}{7}-\frac{\left(\sec^6(dx+c)\right)}{6}-\frac{3\left(\sec^5(dx+c)\right)}{5}+\frac{3\left(\sec^4(dx+c)\right)}{4}+\sec^3(dx+c)-\frac{3\left(\sec^2(dx+c)\right)}{2}-\sec(dx+c)+\ln(\sec(dx+c))}{da}$
default	$\frac{\frac{\left(\sec^7(dx+c)\right)}{7}-\frac{\left(\sec^6(dx+c)\right)}{6}-\frac{3\left(\sec^5(dx+c)\right)}{5}+\frac{3\left(\sec^4(dx+c)\right)}{4}+\sec^3(dx+c)-\frac{3\left(\sec^2(dx+c)\right)}{2}-\sec(dx+c)+\ln(\sec(dx+c))}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2\left(105\,e^{13i(dx+c)}+315\,e^{12i(dx+c)}+210\,e^{11i(dx+c)}+945\,e^{10i(dx+c)}+903\,e^{9i(dx+c)}+1820\,e^{8i(dx+c)}+636\,e^{7i}\right.}{105da\left(e^{2i(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/7*sec(d*x+c)^7-1/6*sec(d*x+c)^6-3/5*sec(d*x+c)^5+3/4*sec(d*x+c)^4+
sec(d*x+c)^3-3/2*sec(d*x+c)^2-sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.29, size = 90, normalized size = 0.67

$$-\frac{\frac{420\log(\cos(dx+c))}{a}+\frac{420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315\cos(dx+c)^3+252\cos(dx+c)^2+70\cos(dx+c)-60}{a\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*log(cos(d*x + c))/a + (420*cos(d*x + c)^6 + 630*cos(d*x + c)^5
- 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2 + 70*cos(d*x
 + c) - 60)/(a*cos(d*x + c)^7))/d
```

Fricas [A]
time = 4.93, size = 95, normalized size = 0.70

$$-\frac{420 \cos(dx+c)^7 \log(-\cos(dx+c)) + 420 \cos(dx+c)^6 + 630 \cos(dx+c)^5 - 420 \cos(dx+c)^4 - 315 \cos(dx+c)^3 + 252 \cos(dx+c)^2 + 70 \cos(dx+c) - 60}{420\, ad \cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 420*cos(d*x + c)^6 + 630*co
s(d*x + c)^5 - 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2
 + 70*cos(d*x + c) - 60)/(a*d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^9(c+dx)}{\sec(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 5.55, size = 245, normalized size = 1.81

$$\frac{\frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{5775(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{20685(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{42595(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + \frac{56035(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{28749(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} + \frac{8463(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} + \frac{1089(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7} + 705}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 420*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (5775*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 20685*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
42595*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*(cos(d*x + c) - 1)^
4/(cos(d*x + c) + 1)^4 + 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*(cos(d*x + c) - 1)^7/
```

```
(cos(d*x + c) + 1)^7 + 705)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
7))/d
```

Mupad [B]
time = 5.14, size = 208, normalized size = 1.54

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d}-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}-\frac{22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{32}{35}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-21\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) - ((26*tan(c/2 + (d*x)/2)^4)/5 - (22*
tan(c/2 + (d*x)/2)^2)/5 + (32*tan(c/2 + (d*x)/2)^6)/3 - (128*tan(c/2 + (d*x
)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 32/35)/(d*
(a - 7*a*tan(c/2 + (d*x)/2)^2 + 21*a*tan(c/2 + (d*x)/2)^4 - 35*a*tan(c/2 +
(d*x)/2)^6 + 35*a*tan(c/2 + (d*x)/2)^8 - 21*a*tan(c/2 + (d*x)/2)^10 + 7*a*t
an(c/2 + (d*x)/2)^12 - a*tan(c/2 + (d*x)/2)^14))
```

3.57 $\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^2(c+dx)}{ad}-\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^4(c+dx)}{4ad}+\frac{\sec^5(c+dx)}{5ad}$$

```
[Out] ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d+sec(d*x+c)^2/a/d-2/3*sec(d*x+c)^3/a/d-1/4
*sec(d*x+c)^4/a/d+1/5*sec(d*x+c)^5/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^4(c+dx)}{4ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec^2(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^2/(a*d) - (2*Se
c[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^4/(4*a*d) + Sec[c + d*x]^5/(5*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(a-ax)^3(a+ax)^2}{x^6}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{a^5}{x^5}-\frac{2a^5}{x^4}+\frac{2a^5}{x^3}+\frac{a^5}{x^2}-\frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^2(c+dx)}{ad}-\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^4(c+dx)}{4ad}$$

Mathematica [A]
time = 0.26, size = 103, normalized size = 1.06

$$\frac{(58+40\cos(2(c+dx))+60\cos(3(c+dx))+30\cos(4(c+dx))+75\cos(3(c+dx))\log(\cos(c+dx))+15\cos(5(c+dx))\log(\cos(c+dx))+30\cos(c+dx)(4+5\log(\cos(c+dx))))\sec^5(c+dx)}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((58 + 40*Cos[2*(c + d*x)] + 60*Cos[3*(c + d*x)] + 30*Cos[4*(c + d*x)] + 75
*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]]
 + 30*Cos[c + d*x]*(4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(240*a*d)
```

Maple [A]
time = 0.11, size = 65, normalized size = 0.67

method	result
derivativedivides	$-\frac{-\frac{\left(\sec^5(dx+c)\right)}{5}+\frac{\left(\sec^4(dx+c)\right)}{4}+\frac{2\left(\sec^3(dx+c)\right)}{3}-\left(\sec^2(dx+c)\right)-\sec(dx+c)+\ln(\sec(dx+c))}{da}$
default	$-\frac{-\frac{\left(\sec^5(dx+c)\right)}{5}+\frac{\left(\sec^4(dx+c)\right)}{4}+\frac{2\left(\sec^3(dx+c)\right)}{3}-\left(\sec^2(dx+c)\right)-\sec(dx+c)+\ln(\sec(dx+c))}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\,e^{9i(dx+c)}+4\,e^{8i(dx+c)}+\frac{8\,e^{7i(dx+c)}}{3}+8\,e^{6i(dx+c)}+\frac{116\,e^{5i(dx+c)}}{15}+8\,e^{4i(dx+c)}+\frac{8\,e^{3i(dx+c)}}{3}+4\,e^{2i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a*(-1/5*sec(d*x+c)^5+1/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3-sec(d*x+c)^2-se
c(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 70, normalized size = 0.72

$$\frac{\frac{60\,\log(\cos(dx+c))}{a}+\frac{60\,\cos(dx+c)^4+60\,\cos(dx+c)^3-40\,\cos(dx+c)^2-15\,\cos(dx+c)+12}{a\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/60*(60*log(cos(d*x + c))/a + (60*cos(d*x + c)^4 + 60*cos(d*x + c)^3 - 40*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*cos(d*x + c)^5))/d

Fricas [A]
time = 3.76, size = 75, normalized size = 0.77

$$\frac{60\,\cos\left(dx+c\right)^{5}\log\left(-\cos\left(dx+c\right)\right)+60\,\cos\left(dx+c\right)^{4}+60\,\cos\left(dx+c\right)^{3}-40\,\cos\left(dx+c\right)^{2}-15\,\cos\left(dx+c\right)+12}{60\,ad\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 60*cos(d*x + c)^4 + 60*cos(d*x + c)^3 - 40*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*d*cos(d*x + c)^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^{7}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c)),x)

[Out] Integral(tan(c + d*x)**7/(sec(c + d*x) + 1), x)/a

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(91) = 182.
time = 3.36, size = 201, normalized size = 2.07

$$-\frac{\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{485\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1330\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1970\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{805\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{137\,(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}+73}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] -1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (485*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1330*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 73)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d

Mupad [B]
time = 6.02, size = 153, normalized size = 1.58

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{16}{15}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^4)/3 - (10*tan(c/2 + (d*x)/2)^2)/3 + 10*tan(c/2 + (d
*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^8 + 16/15)/(d*(a - 5*a*tan(c/2 + (d*x)/2)^2
 + 10*a*tan(c/2 + (d*x)/2)^4 - 10*a*tan(c/2 + (d*x)/2)^6 + 5*a*tan(c/2 + (d
*x)/2)^8 - a*tan(c/2 + (d*x)/2)^10)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d
)
```

3.58 $\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=66

$$-\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{3ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-1/2*sec(d*x+c)^2/a/d+1/3*sec(d*x+c)^3/a/
d
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 76}

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - Sec[c + d*x]^2/(2*a*d) +
Sec[c + d*x]^3/(3*a*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx, x, \cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.19, size = 65, normalized size = 0.98

$$-\frac{(2+6\cos(2(c+dx))+3\cos(3(c+dx))\log(\cos(c+dx))+\cos(c+dx)(6+9\log(\cos(c+dx))))\sec^3(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
[Out] -1/12*((2 + 6*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + Cos
[c + d*x]*(6 + 9*Log[Cos[c + d*x]]))*Sec[c + d*x]^3)/(a*d)
```

Maple [A]
time = 0.10, size = 44, normalized size = 0.67

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^3(dx+c)\right)}{3}-\frac{\left(\sec^2(dx+c)\right)}{2}-\sec(dx+c)+\ln(\sec(dx+c))}{da}$	44
default	$\frac{\frac{\left(\sec^3(dx+c)\right)}{3}-\frac{\left(\sec^2(dx+c)\right)}{2}-\sec(dx+c)+\ln(\sec(dx+c))}{da}$	44
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2\left(3\,e^{5i(dx+c)}+3\,e^{4i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{3da\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{ad}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/3*sec(d*x+c)^3-1/2*sec(d*x+c)^2-sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.76

$$-\frac{\frac{6\,\log(\cos(dx+c))}{a}+\frac{6\,\cos(dx+c)^2+3\,\cos(dx+c)-2}{a\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*log(cos(d*x + c))/a + (6*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)/(a*co
s(d*x + c)^3))/d
```

Fricas [A]
time = 3.79, size = 55, normalized size = 0.83

$$-\frac{6 \cos\left(dx+c\right)^{3} \log\left(-\cos\left(dx+c\right)\right)+6 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)-2}{6 \, ad \cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 6*cos(d*x + c)^2 + 3*cos(d*x +
c) - 2)/(a*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{5}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(62) = 124.
time = 1.64, size = 157, normalized size = 2.38

$$\frac{\frac{6 \log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{a}-\frac{6 \log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)}{a}+\frac{\frac{21\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{45\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{11\left(\cos\left(dx+c\right)-1\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+3}{a\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right)^{3}}}{6 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 6*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (21*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 45*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3)/(a*((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1)^3))/d
```

Mupad [B]
time = 2.19, size = 99, normalized size = 1.50

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{4}{3}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) + (2*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2
 + (d*x)/2)^4 - 4/3)/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)
/2)^4 - a*tan(c/2 + (d*x)/2)^6))
```

3.59 $\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 45}

$$\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx&=-\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx,x,\cos(c+dx)\right)}{a^2d}\\&=-\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)\,dx,x,\cos(c+dx)\right)}{a^2d}\\&=\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 0.75

$$\frac{\log(\cos(c+dx))+\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Log[Cos[c + d*x]] + Sec[c + d*x])/(a*d)
```

Maple [A]
time = 0.06, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$-\frac{-\sec(dx+c)+\ln(\sec(dx+c))}{da}$	25
default	$-\frac{-\sec(dx+c)+\ln(\sec(dx+c))}{da}$	25
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{ad}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a*(-sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 28, normalized size = 1.00

$$\frac{\frac{\log(\cos(dx+c))}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(cos(d*x + c))/a + 1/(a*cos(d*x + c)))/d
```

Fricas [A]
time = 2.91, size = 33, normalized size = 1.18

$$\frac{\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)*log(-cos(d*x + c)) + 1)/(a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(28) = 56.
time = 0.74, size = 111, normalized size = 3.96

$$-\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - log(abs(-(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + ((cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 1.22, size = 44, normalized size = 1.57

$$\frac{2}{d\,\left(a-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] 2/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d)
```

3.60 $\int \frac{\tan(c+dx)}{a+a\sec(c+dx)} \, dx$

Optimal. Leaf size=17

$$-\frac{\log(1+\cos(c+dx))}{ad}$$

```
[Out] -ln(1+cos(d*x+c))/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 31}

$$-\frac{\log(\cos(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[1 + Cos[c + d*x]]/(a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)}{a+a\sec(c+dx)} \, dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+ax} \, dx, x, \cos(c+dx)\right)}{d} \\ &= -\frac{\log(1+\cos(c+dx))}{ad} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.12

$$-\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] (-2*Log[Cos[(c + d*x)/2]])/(a*d)
```

Maple [A]
time = 0.03, size = 27, normalized size = 1.59

method	result	size
derivativedivides	$\frac{-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{da}$	27
default	$\frac{-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{da}$	27
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{ad}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-ln(1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 17, normalized size = 1.00

$$-\frac{\log\left(\cos\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -log(cos(d*x + c) + 1)/(a*d)
```

Fricas [A]
time = 2.72, size = 19, normalized size = 1.12

$$-\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -log(1/2*cos(d*x + c) + 1/2)/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.
time = 2.14, size = 41, normalized size = 2.41

$$\begin{cases} \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2ad}-\frac{\log\left(\sec\left(c+dx\right)+1\right)}{ad} & \text{for } d \neq 0 \\ \frac{x\tan\left(c\right)}{a\sec\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*a*d) - log(sec(c + d*x) + 1)/(a*d),
Ne(d, 0)), (x*tan(c)/(a*sec(c) + a), True))
```

Giac [A]
time = 0.46, size = 31, normalized size = 1.82

$$\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(a*d)
```

Mupad [B]
time = 1.21, size = 21, normalized size = 1.24

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.61 $\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{1}{2ad(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(1+\cos(c+dx))}{4ad}$$

```
[Out] 1/2/a/d/(1+cos(d*x+c))+1/4*ln(1-cos(d*x+c))/a/d+3/4*ln(1+cos(d*x+c))/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{1}{2ad(\cos(c+dx)+1)}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(\cos(c+dx)+1)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] 1/(2*a*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(4*a*d) + (3*Log[1 + C
os[c + d*x]])/(4*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{a^2\text{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\text{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{1}{2ad(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(1+\cos(c+dx))}{4ad}$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 1.10

$$\frac{\left(1+2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec(c+dx)}{2ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] ((1 + 2*Cos[(c + d*x)/2]^2*(3*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]
))*Sec[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.11, size = 43, normalized size = 0.70

method	result	size
derivativedivides	$\frac{\frac{\ln(-1+\cos(dx+c))}{4}+\frac{1}{2+2\cos(dx+c)}+\frac{3\ln(1+\cos(dx+c))}{4}}{da}$	43
default	$\frac{\frac{\ln(-1+\cos(dx+c))}{4}+\frac{1}{2+2\cos(dx+c)}+\frac{3\ln(1+\cos(dx+c))}{4}}{da}$	43
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{e^{i(dx+c)}}{ad\left(e^{i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}+\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2ad}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/4*ln(-1+cos(d*x+c))+1/2/(1+cos(d*x+c))+3/4*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.77

$$\frac{\frac{3\log(\cos(dx+c)+1)}{a}+\frac{\log(\cos(dx+c)-1)}{a}+\frac{2}{a\cos(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(3*log(cos(d*x + c) + 1)/a + log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c
) + a))/d
```

Fricas [A]
time = 2.60, size = 60, normalized size = 0.98

$$\frac{3\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + 1)*
log(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 86, normalized size = 1.41

$$\frac{\frac{\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)}{a}-\frac{4\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{a}-\frac{\cos\left(dx+c\right)-1}{a\left(\cos\left(dx+c\right)+1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - 4*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - (cos(d*x + c) - 1)/(a*(cos(d*x +
 c) + 1)))/d
```

Mupad [B]
time = 1.25, size = 49, normalized size = 0.80

$$\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/2 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*x
)/2)^2/4)/(a*d)
```

3.62 $\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{8ad(1-\cos(c+dx))}+\frac{1}{8ad(1+\cos(c+dx))^2}-\frac{3}{4ad(1+\cos(c+dx))}-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(1+\cos(c+dx))}{16ad}$$

```
[Out] -1/8/a/d/(1-cos(d*x+c))+1/8/a/d/(1+cos(d*x+c))^2-3/4/a/d/(1+cos(d*x+c))-5/1
6*ln(1-cos(d*x+c))/a/d-11/16*ln(1+cos(d*x+c))/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$-\frac{1}{8ad(1-\cos(c+dx))}-\frac{3}{4ad(\cos(c+dx)+1)}+\frac{1}{8ad(\cos(c+dx)+1)^2}-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(\cos(c+dx)+1)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*1/(a*d*(1 - Cos[c + d*x])) + 1/(8*a*d*(1 + Cos[c + d*x])^2) - 3/(4*a*d
*(1 + Cos[c + d*x])) - (5*Log[1 - Cos[c + d*x]])/(16*a*d) - (11*Log[1 + Cos
[c + d*x]])/(16*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{a^4\text{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^4\text{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2} + \frac{5}{16a^5(-1+x)} + \frac{1}{4a^5(1+x)^3} - \frac{3}{4a^5(1+x)^2} + \frac{11}{16a^5(1+x)}\right) dx, x, \text{c}\right.}{d}$$
$$= -\frac{1}{8ad(1-\cos(c+dx))} + \frac{1}{8ad(1+\cos(c+dx))^2} - \frac{3}{4ad(1+\cos(c+dx))} - \frac{5\log}{}$$

Mathematica [A]
time = 0.60, size = 107, normalized size = 1.04

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right) + 44\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 12\sec^2\left(\frac{1}{2}(c+dx)\right) - \sec^4\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{16ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
[Out] -1/16*(Cos[(c + d*x)/2]^2*(2*Csc[(c + d*x)/2]^2 + 44*Log[Cos[(c + d*x)/2]]
+ 20*Log[Sin[(c + d*x)/2]] + 12*Sec[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^4)*Se
c[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.12, size = 67, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{1}{-8+8\cos(dx+c)} - \frac{5\ln(-1+\cos(dx+c))}{16} + \frac{1}{8(1+\cos(dx+c))^2} - \frac{3}{4(1+\cos(dx+c))} - \frac{11\ln(1+\cos(dx+c))}{16}}{da}$
default	$\frac{\frac{1}{-8+8\cos(dx+c)} - \frac{5\ln(-1+\cos(dx+c))}{16} + \frac{1}{8(1+\cos(dx+c))^2} - \frac{3}{4(1+\cos(dx+c))} - \frac{11\ln(1+\cos(dx+c))}{16}}{da}$
risch	$\frac{ix}{a} + \frac{2ic}{ad} - \frac{5\,e^{5i(dx+c)} - 6\,e^{4i(dx+c)} - 14\,e^{3i(dx+c)} - 6\,e^{2i(dx+c)} + 5\,e^{i(dx+c)}}{4da\left(e^{i(dx+c)}+1\right)^4\left(e^{i(dx+c)}-1\right)^2} - \frac{5\ln\left(e^{i(dx+c)}-1\right)}{8ad} - \frac{11\ln\left(e^{i(dx+c)}+1\right.}{8ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/8/(-1+cos(d*x+c))-5/16*ln(-1+cos(d*x+c))+1/8/(1+cos(d*x+c))^2-3/4/
(1+cos(d*x+c))-11/16*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 0.88

$$-\frac{\frac{2\left(5\cos(dx+c)^2 - 3\cos(dx+c) - 6\right)}{a\cos(dx+c)^3 + a\cos(dx+c)^2 - a\cos(dx+c) - a} + \frac{11\log(\cos(dx+c)+1)}{a} + \frac{5\log(\cos(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*cos(d*x + c)^2 - 3*cos(d*x + c) - 6)/(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2 - a*cos(d*x + c) - a) + 11*log(cos(d*x + c) + 1)/a + 5*log(cos(d
*x + c) - 1)/a)/d
```

Fricas [A]
time = 2.54, size = 139, normalized size = 1.35

$$-\frac{10\cos\left(dx+c\right)^{2}+11\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+5\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-6\cos\left(dx+c\right)-12}{16\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}-ad\cos\left(dx+c\right)-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(d*x + c)^2 + 11*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x +
c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 -
cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c) - 12)/(a*d*
cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{3}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.48, size = 157, normalized size = 1.52

$$\frac{\frac{2\left(\frac{5(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{10\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{32\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{10a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{a^{2}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(2*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
*(cos(d*x + c) - 1)) - 10*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/a + 32*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (10*a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)/a^2)/d
```

Mupad [B]
time = 1.34, size = 76, normalized size = 0.74

$$-\frac{\frac{5\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] -((5*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 +
 (d*x)/2)^2/16 + (5*tan(c/2 + (d*x)/2)^2)/16 - tan(c/2 + (d*x)/2)^4/32)/(a*
d)
```

3.63 $\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{1}{32ad(1-\cos(c+dx))^2}+\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{24ad(1+\cos(c+dx))^3}-\frac{9}{32ad(1+\cos(c+dx))^2}+\frac{\ }{16ad(1\cdots}$$

```
[Out] -1/32/a/d/(1-cos(d*x+c))^2+1/4/a/d/(1-cos(d*x+c))+1/24/a/d/(1+cos(d*x+c))^3
-9/32/a/d/(1+cos(d*x+c))^2+15/16/a/d/(1+cos(d*x+c))+11/32*ln(1-cos(d*x+c))/
a/d+21/32*ln(1+cos(d*x+c))/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{1}{4ad(1-\cos(c+dx))}+\frac{15}{16ad(\cos(c+dx)+1)}-\frac{1}{32ad(1-\cos(c+dx))^2}-\frac{9}{32ad(\cos(c+dx)+1)^2}+\frac{1}{24ad(\cos(c+dx)+1)^3}+\frac{11\log(1-\cos(c+dx))}{32ad}+\frac{21\log(\cos(c+dx)+1)}{32ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/32*1/(a*d*(1 - Cos[c + d*x])^2) + 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(24*a
*d*(1 + Cos[c + d*x])^3) - 9/(32*a*d*(1 + Cos[c + d*x])^2) + 15/(16*a*d*(1
+ Cos[c + d*x])) + (11*Log[1 - Cos[c + d*x]])/(32*a*d) + (21*Log[1 + Cos[c
+ d*x]])/(32*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{a^6\text{Subst}\left(\int \frac{x^6}{(a-ax)^3(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{16a^7(-1+x)^3} - \frac{1}{4a^7(-1+x)^2} - \frac{11}{32a^7(-1+x)} + \frac{1}{8a^7(1+x)^4} - \frac{9}{16a^7(1+x)^3} + \frac{}{16\cdots}\right.\right.}{d}$$

$$= -\frac{1}{32ad(1-\cos(c+dx))^2} + \frac{1}{4ad(1-\cos(c+dx))} + \frac{1}{24ad(1+\cos(c+dx))^3} - \frac{}{3\cdots}$$

Mathematica [A]
time = 0.55, size = 135, normalized size = 0.93

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-48\csc^2\left(\frac{1}{2}(c+dx)\right)+3\csc^4\left(\frac{1}{2}(c+dx)\right)-504\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-264\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-180\sec^2\left(\frac{1}{2}(c+dx)\right)+27\sec^4\left(\frac{1}{2}(c+dx)\right)-2\sec^6\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{192ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/192*(Cos[(c + d*x)/2]^2*(-48*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 -
 504*Log[Cos[(c + d*x)/2]] - 264*Log[Sin[(c + d*x)/2]] - 180*Sec[(c + d*x)/
2]^2 + 27*Sec[(c + d*x)/2]^4 - 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1
+ Sec[c + d*x]))
```

Maple [A]
time = 0.14, size = 91, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{32(-1+\cos(dx+c))^2}-\frac{1}{4(-1+\cos(dx+c))}+\frac{11\ln(-1+\cos(dx+c))}{32}+\frac{1}{24(1+\cos(dx+c))^3}-\frac{9}{32(1+\cos(dx+c))^2}+\frac{15}{16(1+\cos(dx+c))}+\frac{2\cdots}{}}{da}$
default	$\frac{-\frac{1}{32(-1+\cos(dx+c))^2}-\frac{1}{4(-1+\cos(dx+c))}+\frac{11\ln(-1+\cos(dx+c))}{32}+\frac{1}{24(1+\cos(dx+c))^3}-\frac{9}{32(1+\cos(dx+c))^2}+\frac{15}{16(1+\cos(dx+c))}+\frac{2\cdots}{}}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{33\,e^{9i(dx+c)}-78\,e^{8i(dx+c)}-184\,e^{7i(dx+c)}-2\,e^{6i(dx+c)}+270\,e^{5i(dx+c)}-2\,e^{4i(dx+c)}-184\,e^{3i(dx+c)}-78\,e\cdots}{24da\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/32/(-1+cos(d*x+c))^2-1/4/(-1+cos(d*x+c))+11/32*ln(-1+cos(d*x+c))+
1/24/(1+cos(d*x+c))^3-9/32/(1+cos(d*x+c))^2+15/16/(1+cos(d*x+c))+21/32*ln(1
+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 130, normalized size = 0.90

$$\frac{\frac{2\left(33\cos(dx+c)^4-39\cos(dx+c)^3-79\cos(dx+c)^2+29\cos(dx+c)+44\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}+\frac{63\log(\cos(dx+c)+1)}{a}+\frac{33\log(\cos(dx+c)-1)}{a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*cos(d*x + c)^4 - 39*cos(d*x + c)^3 - 79*cos(d*x + c)^2 + 29*cos
(d*x + c) + 44)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 -
 2*a*cos(d*x + c)^2 + a*cos(d*x + c) + a) + 63*log(cos(d*x + c) + 1)/a + 33
*log(cos(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 3.10, size = 217, normalized size = 1.50

$$\frac{66\,\cos\left(dx+c\right)^{4}-78\,\cos\left(dx+c\right)^{3}-158\,\cos\left(dx+c\right)^{2}+63\,\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+33\,\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+58\,\cos\left(dx+c\right)+88}{96\,\left(ad\cos\left(dx+c\right)^{5}+ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{3}-2\,ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 78*cos(d*x + c)^3 - 158*cos(d*x + c)^2 + 63*(cos(
d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 33*(cos(d*x + c)^5 + cos(d*x + c)^
4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*
x + c) + 1/2) + 58*cos(d*x + c) + 88)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c
)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{5}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.52, size = 211, normalized size = 1.46

$$-\frac{\frac{3\left(\frac{14\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)(\cos(dx+c)+1)^{2}}{a(\cos(dx+c)-1)^{2}}-\frac{132\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{384\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{132\,a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21\,a^{2}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\,a^{2}(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{3}}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(3*(14*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 66*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2)
- 132*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 384*log(abs(-(c
```

```
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (132*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a^
2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

Mupad [B]
time = 1.31, size = 132, normalized size = 0.91

$$\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32\,a\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,a\,d}+\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{1}{4}\right)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^2)/(32*a*d) - (7*tan(c/2 + (d*x)/2)^4)/(128*a*d) + t
an(c/2 + (d*x)/2)^6/(192*a*d) + (11*log(tan(c/2 + (d*x)/2)))/(16*a*d) - log
(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) + (cot(c/2 + (d*x)/2)^4*((7*tan(c/2 + (d*x
)/2)^2)/2 - 1/4))/(32*a*d)
```

3.64 $\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{x}{a}-\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad}+\frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad}-\frac{(6-5\sec(c$$

```
[Out] x/a-5/16*arctanh(sin(d*x+c))/a/d-1/16*(16-5*sec(d*x+c))*tan(d*x+c)/a/d+1/24
*(8-5*sec(d*x+c))*tan(d*x+c)^3/a/d-1/30*(6-5*sec(d*x+c))*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3973, 3966, 3855}

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)(6-5\sec(c+dx))}{30ad}+\frac{\tan^3(c+dx)(8-5\sec(c+dx))}{24ad}-\frac{\tan(c+dx)(16-5\sec(c+dx))}{16ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - ((16 - 5*Sec[c + d*x])*Tan[c + d
*x])/(16*a*d) + ((8 - 5*Sec[c + d*x])*Tan[c + d*x]^3)/(24*a*d) - ((6 - 5*Se
c[c + d*x])*Tan[c + d*x]^5)/(30*a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\tan^6(c+dx)\,dx}{a^2}$$
$$= -\frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} - \frac{\int(-6a+5a\sec(c+dx))\tan^4(c+dx)\,dx}{6a^2}$$
$$= \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} + \frac{\int(-24a+$$
$$= -\frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\,\mathrm{s}$$
$$= \frac{x}{a} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-$$
$$= \frac{x}{a} - \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(105) = 210.
time = 0.87, size = 301, normalized size = 2.87

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2400\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec(c)\sec^6(c+dx)(2400dx\cos(c)+1800dx\cos(c+2dx)+1800dx\cos(3c+2dx)+720dx\cos(3c+4dx)+720dx\cos(5c+4dx)+120dx\cos(5c+6dx)+120dx\cos(7c+6dx)+3680\sin(c)+450\sin(dx)+450\sin(2c+dx)-3360\sin(c+2dx)+2160\sin(3c+2dx)-25\sin(2c+3dx)-25\sin(4c+3dx)-1488\sin(3c+4dx)+720\sin(5c+4dx)+165\sin(4c+5dx)+165\sin(6c+5dx)-368\sin(5c+6dx))\right)}{3840ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(2400*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]^6*(
2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d*x] + 720
*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c + 6*d*x]
 + 120*d*x*Cos[7*c + 6*d*x] + 3680*Sin[c] + 450*Sin[d*x] + 450*Sin[2*c + d*
x] - 3360*Sin[c + 2*d*x] + 2160*Sin[3*c + 2*d*x] - 25*Sin[2*c + 3*d*x] - 25
*Sin[4*c + 3*d*x] - 1488*Sin[3*c + 4*d*x] + 720*Sin[5*c + 4*d*x] + 165*Sin[
4*c + 5*d*x] + 165*Sin[6*c + 5*d*x] - 368*Sin[5*c + 6*d*x])))/(3840*a*d*(1
+ Sec[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(97) = 194.
time = 0.13, size = 230, normalized size = 2.19

method	result
risch	$\frac{x}{a} - \frac{i\left(165\,\mathrm{e}^{11i(dx+c)}+720\,\mathrm{e}^{10i(dx+c)}-25\,\mathrm{e}^{9i(dx+c)}+2160\,\mathrm{e}^{8i(dx+c)}+450\,\mathrm{e}^{7i(dx+c)}+3680\,\mathrm{e}^{6i(dx+c)}-450\,\mathrm{e}^{5i(dx+c)}+330}{120da\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{7}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{3}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{16\left(\tan\left(\right.\right.}}{}$

default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}+\frac{7}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{3}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{}{16\left(\tan\left(\frac{dx}{2}\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 256/d/a*(1/128*arctan(tan(1/2*d*x+1/2*c))+1/1536/(tan(1/2*d*x+1/2*c)-1)^6+7
/2560/(tan(1/2*d*x+1/2*c)-1)^5+3/1024/(tan(1/2*d*x+1/2*c)-1)^4-5/3072/(tan(
1/2*d*x+1/2*c)-1)^3-9/4096/(tan(1/2*d*x+1/2*c)-1)^2+21/4096/(tan(1/2*d*x+1/
2*c)-1)+5/4096*ln(tan(1/2*d*x+1/2*c)-1)-1/1536/(tan(1/2*d*x+1/2*c)+1)^6+7/2
560/(tan(1/2*d*x+1/2*c)+1)^5-3/1024/(tan(1/2*d*x+1/2*c)+1)^4-5/3072/(tan(1/
2*d*x+1/2*c)+1)^3+9/4096/(tan(1/2*d*x+1/2*c)+1)^2+21/4096/(tan(1/2*d*x+1/2*
c)+1)-5/4096*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(97) = 194$.
time = 0.48, size = 329, normalized size = 3.13

$$-\frac{\frac{2\left(\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{1095\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3138\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{5118\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{1945\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a-\frac{6a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{75\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{75\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(2*(165*sin(d*x + c)/(cos(d*x + c) + 1) - 1095*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3138*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5118*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 1945*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 315*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a - 6*a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*
x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)
- 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 75*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a - 75*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [A]
time = 3.95, size = 127, normalized size = 1.21

$$\frac{480\,dx\cos\left(dx+c\right)^{6}-75\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)+75\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(368\cos\left(dx+c\right)^{5}-165\cos\left(dx+c\right)^{4}-176\cos\left(dx+c\right)^{3}+130\cos\left(dx+c\right)^{2}+48\cos\left(dx+c\right)-40\right)\sin\left(dx+c\right)}{480\,ad\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 75*cos(d*x + c)^6*log(sin(d*x + c) + 1) + 7
5*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*cos(d*x + c)^5 - 165*cos(d
```

```
*x + c)^4 - 176*cos(d*x + c)^3 + 130*cos(d*x + c)^2 + 48*cos(d*x + c) - 40)
*sin(d*x + c))/(a*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 4.41, size = 149, normalized size = 1.42

$$\frac{\frac{240\,(dx+c)}{a} - \frac{75\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{75\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} + \frac{2\left(315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1945\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5118\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3138\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1095\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-165\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/240*(240*(d*x + c)/a - 75*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 75*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a + 2*(315*tan(1/2*d*x + 1/2*c)^11 - 1945*tan
(1/2*d*x + 1/2*c)^9 + 5118*tan(1/2*d*x + 1/2*c)^7 - 3138*tan(1/2*d*x + 1/2*
c)^5 + 1095*tan(1/2*d*x + 1/2*c)^3 - 165*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 - 1)^6*a))/d
```

Mupad [B]
time = 2.52, size = 193, normalized size = 1.84

$$\frac{x}{a} - \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d} - \frac{-\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{389\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-\frac{853\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{523\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-\frac{73\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-20\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+15\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-6\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x)),x)
[Out] x/a - (5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((11*tan(c/2 + (d*x)/2))/8 -
(73*tan(c/2 + (d*x)/2)^3)/8 + (523*tan(c/2 + (d*x)/2)^5)/20 - (853*tan(c/2
+ (d*x)/2)^7)/20 + (389*tan(c/2 + (d*x)/2)^9)/24 - (21*tan(c/2 + (d*x)/2)^1
1)/8)/(d*(a - 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 - 20*a*t
an(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8 - 6*a*tan(c/2 + (d*x)/2)^10
 + a*tan(c/2 + (d*x)/2)^12))
```

3.65 $\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=78

$$-\frac{x}{a}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad}-\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}$$

```
[Out] -x/a+3/8*arctanh(sin(d*x+c))/a/d+1/8*(8-3*sec(d*x+c))*tan(d*x+c)/a/d-1/12*(
4-3*sec(d*x+c))*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3973, 3966, 3855}

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)(4-3\sec(c+dx))}{12ad}+\frac{\tan(c+dx)(8-3\sec(c+dx))}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + ((8 - 3*Sec[c + d*x])*Tan[c +
d*x])/(8*a*d) - ((4 - 3*Sec[c + d*x])*Tan[c + d*x]^3)/(12*a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\tan^4(c+dx)\,dx}{a^2}$$
$$= -\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} - \frac{\int(-4a+3a\sec(c+dx))\tan^2(c+dx)\,dx}{4a^2}$$
$$= \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} + \frac{\int(-8a+3\ldots}{\ldots}$$
$$= -\frac{x}{a} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} + \frac{3\int\ldots}{\ldots}$$
$$= -\frac{x}{a} + \frac{3\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+\ldots}{1\ldots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 893 vs. 2(78) = 156.
time = 6.47, size = 893, normalized size = 11.45

$$-\frac{2x\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{a+a\sec(c+dx)} - \frac{3\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)}{4d(a+a\sec(c+dx))} + \frac{3\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)}{4d(a+a\sec(c+dx))} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{8d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4} - \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(-19\cos\left(\frac{c}{2}\right)+11\sin\left(\frac{c}{2}\right)\right)}{24d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} + \frac{8\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} - \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{8d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4} - \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(19\cos\left(\frac{c}{2}\right)+11\sin\left(\frac{c}{2}\right)\right)}{24d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} + \frac{8\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*x*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(a + a*Sec[c + d*x]) - (3*Cos[c/2
+ (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*
d*(a + a*Sec[c + d*x])) + (3*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*d*(a + a*Sec[c + d*x])) + (Cos[c/2 + (
d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2] - Sin
[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d
*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(-19*Cos[c/2] + 11*Sin[c
/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2
])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Si
n[c/2 + (d*x)/2])) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c
+ d*x])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^
2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Sin[c/2]
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(19*Cos[c/2] + 11*Sin[c/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*
x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Si
n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(72) = 144.
time = 0.11, size = 170, normalized size = 2.18

method	result
risch	$-\frac{x}{a}+\frac{i(15\,\mathrm{e}^{7i(dx+c)}+48\,\mathrm{e}^{6i(dx+c)}-9\,\mathrm{e}^{5i(dx+c)}+96\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}+80\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}+32)}{12da(\mathrm{e}^{2i(dx+c)}+1)^4}+\frac{3\ln(\mathrm{e}^{i(dx}}{8a}$
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{5}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{8}}{ad}$
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{5}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{8}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a*(-1/32*arctan(tan(1/2*d*x+1/2*c))+1/256/(tan(1/2*d*x+1/2*c)-1)^4+5/3
84/(tan(1/2*d*x+1/2*c)-1)^3+3/512/(tan(1/2*d*x+1/2*c)-1)^2-11/512/(tan(1/2*
d*x+1/2*c)-1)-3/512*ln(tan(1/2*d*x+1/2*c)-1)-1/256/(tan(1/2*d*x+1/2*c)+1)^4
+5/384/(tan(1/2*d*x+1/2*c)+1)^3-3/512/(tan(1/2*d*x+1/2*c)+1)^2-11/512/(tan(
1/2*d*x+1/2*c)+1)+3/512*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(72) = 144$.
time = 0.48, size = 247, normalized size = 3.17

$$\frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{71\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{137\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{33\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 71*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 137*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 48*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - 9*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [A]
time = 5.21, size = 107, normalized size = 1.37

$$-\frac{48\,dx\cos(dx+c)^4-9\cos(dx+c)^4\log(\sin(dx+c)+1)+9\cos(dx+c)^4\log(-\sin(dx+c)+1)-2\left(32\cos(dx+c)^3-15\cos(dx+c)^2-8\cos(dx+c)+6\right)\sin(dx+c)}{48\,ad\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 9*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 9*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 - 15*cos(d*x +
c)^2 - 8*cos(d*x + c) + 6)*sin(d*x + c))/(a*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 2.09, size = 123, normalized size = 1.58

$$-\frac{\frac{24\,(dx+c)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-137\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+71\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -1/24*(24*(d*x + c)/a - 9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 9*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/a + 2*(33*tan(1/2*d*x + 1/2*c)^7 - 137*tan(1/2*d
*x + 1/2*c)^5 + 71*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^4*a))/d
```

Mupad [B]
time = 2.00, size = 139, normalized size = 1.78

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}-\frac{x}{a}+\frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{137\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{71\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x)),x)
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) - x/a + ((5*tan(c/2 + (d*x)/2))/4 - (
71*tan(c/2 + (d*x)/2)^3)/12 + (137*tan(c/2 + (d*x)/2)^5)/12 - (11*tan(c/2 +
 (d*x)/2)^7)/4)/(d*(a - 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4
 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8))
```

3.66 $\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{x}{a}-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}$$

```
[Out] x/a-1/2*arctanh(sin(d*x+c))/a/d-1/2*(2-sec(d*x+c))*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3966, 3855}

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)(2-\sec(c+dx))}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - ArcTanh[Sin[c + d*x]]/(2*a*d) - ((2 - Sec[c + d*x])*Tan[c + d*x])/(2*
a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int(-a+a\sec(c+dx))\tan^2(c+dx)\,dx}{a^2}$$
$$= -\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad} - \frac{\int(-2a+a\sec(c+dx))\,dx}{2a^2}$$
$$= \frac{x}{a} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad} - \frac{\int\sec(c+dx)\,dx}{2a}$$
$$= \frac{x}{a} - \frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(49) = 98$.
time = 0.95, size = 241, normalized size = 4.92

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4x+\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{4\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{2a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(4*x + (2*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]])/d - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) - (4*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(2
*a*(1 + Sec[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(45) = 90$.
time = 0.09, size = 110, normalized size = 2.24

method	result
risch	$\frac{x}{a}-\frac{i\left(e^{3i(dx+c)}+2\,e^{2i(dx+c)}-e^{i(dx+c)}+2\right)}{da\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{2ad}$
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\right.\right.}}{ad}$
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\right.\right.}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*(1/8*arctan(tan(1/2*d*x+1/2*c))+1/32/(tan(1/2*d*x+1/2*c)-1)^2+3/32/(
tan(1/2*d*x+1/2*c)-1)+1/32*ln(tan(1/2*d*x+1/2*c)-1)-1/32/(tan(1/2*d*x+1/2*c
)+1)^2+3/32/(tan(1/2*d*x+1/2*c)+1)-1/32*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(43) = 86$.
time = 0.48, size = 163, normalized size = 3.33

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a)/d
```

Fricas [A]
time = 3.63, size = 86, normalized size = 1.76

$$\frac{4\,dx\cos\left(dx+c\right)^2-\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)}{4\,ad\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - cos(d*x + c)^2*log(sin(d*x + c) + 1) + cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) - 1)*sin(d*x + c))/(a*d*c
os(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.
time = 1.01, size = 96, normalized size = 1.96

$$\frac{\frac{2\,(dx+c)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} + \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

Mupad [B]
time = 1.29, size = 83, normalized size = 1.69

$$\frac{x}{a} - \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - atanh(tan(c/2 + (d*x)/2))/(a*d) - (tan(c/2 + (d*x)/2) - 3*tan(c/2 + (
d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4))
```

3.67 $\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=21

$$-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}$$

[Out] -x/a+arctanh(sin(d*x+c))/a/d

Rubi [A]
time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3973, 3855}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]

[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(a*d)

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int(-a+a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{x}{a}+\frac{\int\sec(c+dx)\,dx}{a}\\ &= -\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(21) = 42.
time = 0.10, size = 60, normalized size = 2.86

$$-\frac{dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((d*x + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.
time = 0.07, size = 50, normalized size = 2.38

method	result	size
risch	$-\frac{x}{a}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}$	49
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	50
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(-1/2*arctan(tan(1/2*d*x+1/2*c))-1/4*ln(tan(1/2*d*x+1/2*c)-1)+1/4*ln(
tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(21) = 42.
time = 0.49, size = 78, normalized size = 3.71

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [A]
time = 3.67, size = 35, normalized size = 1.67

$$-\frac{2\,dx-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(-\sin\left(dx+c\right)+1\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(sin(d*x + c) + 1) + log(-sin(d*x + c) + 1))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.
time = 0.59, size = 50, normalized size = 2.38

$$-\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a)/d
```

Mupad [B]
time = 1.11, size = 25, normalized size = 1.19

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) - x/a
```

3.68 $\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{x}{a}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}+\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}$$

```
[Out] -x/a-1/3*cot(d*x+c)*(3-2*sec(d*x+c))/a/d+1/3*cot(d*x+c)^3*(1-sec(d*x+c))/a/
d
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3973, 3967, 8}

$$\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) - (Cot[c + d*x]*(3 - 2*Sec[c + d*x]))/(3*a*d) + (Cot[c + d*x]^3*(1 -
 Sec[c + d*x]))/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}\\ &= \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}+\frac{\int \cot^2(c+dx)(3a-2a\sec(c+dx))\,dx}{3a^2}\\ &= -\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}+\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}+\frac{\int -3a\,dx}{3a^2}\\ &= -\frac{x}{a}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}+\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.88, size = 100, normalized size = 1.64

$$\frac{\sec(c+dx)\left(-12dx\cos^2\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(3\cot\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)+13\sec\left(\frac{c}{2}\right)\right)\sin\left(\frac{dx}{2}\right)-\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(-12*d*x*Cos[(c + d*x)/2]^2 + Cos[(c + d*x)/2]*(3*Cot[(c + d*
x)/2]*Csc[c/2] + 13*Sec[c/2])*Sin[(d*x)/2] - Tan[(c + d*x)/2]))/(6*a*d*(1 +
 Sec[c + d*x]))
```

Maple [A]
time = 0.09, size = 59, normalized size = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	59
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	59
risch	$-\frac{x}{a}+\frac{2i\left(3\,e^{3i(dx+c)}-5\,e^{i(dx+c)}-4\right)}{3da\left(e^{i(dx+c)}+1\right)^3\left(e^{i(dx+c)}-1\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(-1/3*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)-8*arctan(tan(1/2*d*
x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.48, size = 93, normalized size = 1.52

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3(\cos(dx+c)+1)}{a\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*(cos(d*x + c) +
1)/(a*sin(d*x + c)))/d
```

Fricas [A]
time = 3.09, size = 64, normalized size = 1.05

$$-\frac{4\cos\left(dx+c\right)^{2}+3\left(dx\cos\left(dx+c\right)+dx\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)-2}{3\left(ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(4*cos(d*x + c)^2 + 3*(d*x*cos(d*x + c) + d*x)*sin(d*x + c) + cos(d*x
+ c) - 2)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{2}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.50, size = 66, normalized size = 1.08

$$-\frac{\frac{12\,(dx+c)}{a}+\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{3}}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*tan(1/2*d*x +
1/2*c))/a^3 + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 1.29, size = 65, normalized size = 1.07

$$-\frac{x}{a}-\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}-\frac{7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{6}+\frac{1}{12}}{a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] - x/a - ((4*cos(c/2 + (d*x)/2)^4)/3 - (7*cos(c/2 + (d*x)/2)^2)/6 + 1/12)/(a
*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))
```

3.69 $\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{x}{a}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}$$

```
[Out] x/a+1/15*cot(d*x+c)*(15-8*sec(d*x+c))/a/d-1/15*cot(d*x+c)^3*(5-4*sec(d*x+c)
)/a/d+1/5*cot(d*x+c)^5*(1-sec(d*x+c))/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3967, 8}

$$\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a + (Cot[c + d*x]*(15 - 8*Sec[c + d*x]))/(15*a*d) - (Cot[c + d*x]^3*(5 -
4*Sec[c + d*x]))/(15*a*d) + (Cot[c + d*x]^5*(1 - Sec[c + d*x]))/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))\,dx}{a^2} \\
&= \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad} + \frac{\int \cot^4(c+dx)(5a-4a\sec(c+dx))\,dx}{5a^2} \\
&= -\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad} + \frac{\int \cot^2(c+d
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad} - \frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+dx)
\end{aligned}
$$

$$
= \frac{x}{a} + \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad} - \frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 254 vs. 2(88) = 176.
time = 0.86, size = 254, normalized size = 2.89

$$
\frac{\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)(360dx\cos(dx)-360dx\cos(2c+dx)+120dx\cos(c+2dx)-120dx\cos(3c+2dx)-120dx\cos(2c+3dx)+120dx\cos(4c+3dx)-60dx\cos(3c+4dx)+60dx\cos(5c+4dx)-200\sin(c)-584\sin(dx)+534\sin(c+dx)+178\sin(2(c+dx))-178\sin(3(c+dx))-89\sin(4(c+dx))-520\sin(2c+dx)-248\sin(c+2dx)-120\sin(3c+2dx)+248\sin(2c+3dx)+120\sin(4c+3dx)+184\sin(3c+4dx))}{1920ad(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]*(360*d*x*Cos[d*x] - 360*d*x*
Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*d*x] - 120*d*
x*Cos[2*c + 3*d*x] + 120*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos[3*c + 4*d*x] + 6
0*d*x*Cos[5*c + 4*d*x] - 200*Sin[c] - 584*Sin[d*x] + 534*Sin[c + d*x] + 178
*Sin[2*(c + d*x)] - 178*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)] - 520*Sin[2*
c + d*x] - 248*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 248*Sin[2*c + 3*d*x]
 + 120*Sin[4*c + 3*d*x] + 184*Sin[3*c + 4*d*x]))/(1920*a*d*(1 + Sec[c + d*x
]))
```

Maple [A]
time = 0.12, size = 85, normalized size = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$	85
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$	85
risch	$\frac{x}{a}-\frac{2i\left(15\,e^{7i(dx+c)}-15\,e^{6i(dx+c)}-65\,e^{5i(dx+c)}-25\,e^{4i(dx+c)}+73\,e^{3i(dx+c)}+31\,e^{2i(dx+c)}-31\,e^{i(dx+c)}-23\right)}{15da\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)^3}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a*(-1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3-16*tan(1/2*d*x+1
/2*c)+32*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+6/tan(1/2*d*x+
1/2*c))
```

Maxima [A]
time = 0.48, size = 137, normalized size = 1.56

$$-\frac{\frac{3\left(\frac{80 \sin(dx+c)}{\cos(dx+c)+1}-\frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a}-\frac{480 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{5\left(\frac{18 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a \sin(dx+c)^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(3*(80*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 480*arctan(sin(d*x +
 c)/(cos(d*x + c) + 1))/a - 5*(18*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*
(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [A]
time = 2.20, size = 134, normalized size = 1.52

$$\frac{23 \cos(dx+c)^4+8 \cos(dx+c)^3-27 \cos(dx+c)^2+15\left(dx\cos(dx+c)^3+dx\cos(dx+c)^2-dx\cos(dx+c)-dx\right)\sin(dx+c)-7 \cos(dx+c)+8}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 27*cos(d*x + c)^2 + 15*(d*x*co
s(d*x + c)^3 + d*x*cos(d*x + c)^2 - d*x*cos(d*x + c) - d*x)*sin(d*x + c) -
7*cos(d*x + c) + 8)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x
 + c) - a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.48, size = 98, normalized size = 1.11

$$\frac{\frac{240\,(dx+c)}{a}+\frac{5\left(18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{3\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)/a + 5*(18*tan(1/2*d*x + 1/2*c)^2 - 1)/(a*tan(1/2*d*x +
 1/2*c)^3) - 3*(a^4*tan(1/2*d*x + 1/2*c)^5 - 10*a^4*tan(1/2*d*x + 1/2*c)^3
+ 80*a^4*tan(1/2*d*x + 1/2*c))/a^5)/d
```

Mupad [B]
time = 1.43, size = 158, normalized size = 1.80

$$-\frac{5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-30\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-90\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-240\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(c+d\,x)}{240\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^8 + 3*sin(c/2 + (d*x)/2)^8 - 30*cos(c/2 + (d*x)/2)^2
*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 - 90*
cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 - 240*cos(c/2 + (d*x)/2)^5*sin(c/
2 + (d*x)/2)^3*(c + d*x))/(240*a*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^
3)
```

3.70 $\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{x}{a}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx)}{35ad}$$

```
[Out] -x/a+1/105*cot(d*x+c)^3*(35-24*sec(d*x+c))/a/d-1/35*cot(d*x+c)*(35-16*sec(d
*x+c))/a/d-1/35*cot(d*x+c)^5*(7-6*sec(d*x+c))/a/d+1/7*cot(d*x+c)^7*(1-sec(d
*x+c))/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3973, 3967, 8}

$$\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (Cot[c + d*x]^3*(35 - 24*Sec[c + d*x]))/(105*a*d) - (Cot[c + d*x]*
(35 - 16*Sec[c + d*x]))/(35*a*d) - (Cot[c + d*x]^5*(7 - 6*Sec[c + d*x]))/(3
5*a*d) + (Cot[c + d*x]^7*(1 - Sec[c + d*x]))/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}$$
$$= \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad} + \frac{\int \cot^6(c+dx)(7a-6a\sec(c+dx))\,dx}{7a^2}$$
$$= -\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad} + \frac{\int \cot^4(c+d}{}$$
$$= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+d}{}$$
$$= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \frac{\cot^5(c+}{}$$
$$= -\frac{x}{a} + \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \frac{\mathrm{co}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(117) = 234.
time = 1.14, size = 359, normalized size = 3.07

$$\frac{\csc\left(\frac{c}{2}\right)\csc^5(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)(-16800dx\cos(dx)+16800dx\cos(2c+dx)-4200dx\cos(c+2dx)+4200dx\cos(3c+2dx)+8400dx\cos(2c+3dx)-8400dx\cos(4c+3dx)+3360dx\cos(3c+4dx)-3360dx\cos(5c+4dx)-1680dx\cos(4c+5dx)+1680dx\cos(6c+5dx)-840dx\cos(5c+6dx)+840dx\cos(7c+6dx)+3136\sin(c)+30112\sin(dx)-22860\sin(c+dx)-5715\sin(2(c+dx))+11430\sin(3(c+dx))+4572\sin(4(c+dx))-2286\sin(5(c+dx))-1143\sin(6(c+dx))+26208\sin(2c+dx)+14080\sin(c+2dx)-16400\sin(2c+3dx)-11760\sin(4c+3dx)-7904\sin(3c+4dx)-3360\sin(5c+4dx)+3952\sin(4c+5dx)+1680\sin(6c+5dx)+2816\sin(5c+6dx))}{107520ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^5*Sec[c/2]*Sec[c + d*x]*(-16800*d*x*Cos[d*x] + 16800
*d*x*Cos[2*c + d*x] - 4200*d*x*Cos[c + 2*d*x] + 4200*d*x*Cos[3*c + 2*d*x] +
 8400*d*x*Cos[2*c + 3*d*x] - 8400*d*x*Cos[4*c + 3*d*x] + 3360*d*x*Cos[3*c +
 4*d*x] - 3360*d*x*Cos[5*c + 4*d*x] - 1680*d*x*Cos[4*c + 5*d*x] + 1680*d*x*
Cos[6*c + 5*d*x] - 840*d*x*Cos[5*c + 6*d*x] + 840*d*x*Cos[7*c + 6*d*x] + 31
36*Sin[c] + 30112*Sin[d*x] - 22860*Sin[c + d*x] - 5715*Sin[2*(c + d*x)] + 1
1430*Sin[3*(c + d*x)] + 4572*Sin[4*(c + d*x)] - 2286*Sin[5*(c + d*x)] - 114
3*Sin[6*(c + d*x)] + 26208*Sin[2*c + d*x] + 14080*Sin[c + 2*d*x] - 16400*Si
n[2*c + 3*d*x] - 11760*Sin[4*c + 3*d*x] - 7904*Sin[3*c + 4*d*x] - 3360*Sin[
5*c + 4*d*x] + 3952*Sin[4*c + 5*d*x] + 1680*Sin[6*c + 5*d*x] + 2816*Sin[5*c
 + 6*d*x]))/(107520*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.11, size = 111, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{29\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{}{3\,\mathrm{ta}}}{64da}$

default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{29\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+}{64da}$
risch	$-\frac{x}{a}+\frac{2i\left(105\,\mathrm{e}^{11i(dx+c)}-210\,\mathrm{e}^{10i(dx+c)}-735\,\mathrm{e}^{9i(dx+c)}+1638\,\mathrm{e}^{7i(dx+c)}+196\,\mathrm{e}^{6i(dx+c)}-1882\,\mathrm{e}^{5i(dx+c)}-880\,\mathrm{e}^{4i(dx+c)}\right.}{105da\left(\mathrm{e}^{i(dx+c)}+1\right)^7\left(\mathrm{e}^{i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a*(-1/7*tan(1/2*d*x+1/2*c)^7+8/5*tan(1/2*d*x+1/2*c)^5-29/3*tan(1/2*d
*x+1/2*c)^3+64*tan(1/2*d*x+1/2*c)-128*arctan(tan(1/2*d*x+1/2*c))-1/5/tan(1/
2*d*x+1/2*c)^5+8/3/tan(1/2*d*x+1/2*c)^3-29/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.47, size = 177, normalized size = 1.51

$$\frac{\frac{\frac{6720\sin(dx+c)}{\cos(dx+c)+1}-\frac{1015\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{168\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{13440\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{7\left(\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{435\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*((6720*sin(d*x + c)/(cos(d*x + c) + 1) - 1015*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 168*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a - 13440*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 7*(40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 435*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

Fricas [A]
time = 3.46, size = 198, normalized size = 1.69

$$-\frac{176\cos(dx+c)^6+71\cos(dx+c)^5-335\cos(dx+c)^4-125\cos(dx+c)^3+225\cos(dx+c)^2+105\left(dx\cos(dx+c)^5+dx\cos(dx+c)^4-2dx\cos(dx+c)^3-2dx\cos(dx+c)^2+dx\cos(dx+c)+dx\right)\sin(dx+c)+57\cos(dx+c)-48}{105\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2ad\cos(dx+c)^3-2ad\cos(dx+c)^2+ad\cos(dx+c)+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/105*(176*cos(d*x + c)^6 + 71*cos(d*x + c)^5 - 335*cos(d*x + c)^4 - 125*c
os(d*x + c)^3 + 225*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^5 + d*x*cos(d*x
+ c)^4 - 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 + d*x*cos(d*x + c) + d
*x)*sin(d*x + c) + 57*cos(d*x + c) - 48)/((a*d*cos(d*x + c)^5 + a*d*cos(d*x
 + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) +
a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.52, size = 127, normalized size = 1.09

$$-\frac{\frac{6720\,(dx+c)}{a}+\frac{7\left(435\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-168\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1015\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^7}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(6720*(d*x + c)/a + 7*(435*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x
+ 1/2*c)^2 + 3)/(a*tan(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7
 - 168*a^6*tan(1/2*d*x + 1/2*c)^5 + 1015*a^6*tan(1/2*d*x + 1/2*c)^3 - 6720*
a^6*tan(1/2*d*x + 1/2*c))/a^7)/d
```

Mupad [B]
time = 2.04, size = 206, normalized size = 1.76

$$-\frac{21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-168\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+1015\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-6720\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3045\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-280\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6720\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(c+d\,x)}{6720\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 - 168*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^10 + 1015*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 3045*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^4 - 280*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 6720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5*(c + d*x))/(6720*a*d*cos(c/
2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

3.71 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\sec^2(c+dx)}{2a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^4(c+dx)}{4a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\sec^6(c+dx)}{6a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)^2/a^2/d+4/3*sec(d*x
+c)^3/a^2/d-1/4*sec(d*x+c)^4/a^2/d-2/5*sec(d*x+c)^5/a^2/d+1/6*sec(d*x+c)^6/
a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{\sec^6(c+dx)}{6a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^4(c+dx)}{4a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) - Sec[c + d*x]^2/(2
*a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^4/(4*a^2*d) - (2*Sec[
c + d*x]^5)/(5*a^2*d) + Sec[c + d*x]^6/(6*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^2}{x^7}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7} - \frac{2a^6}{x^6} - \frac{a^6}{x^5} + \frac{4a^6}{x^4} - \frac{a^6}{x^3} - \frac{2a^6}{x^2} + \frac{a^6}{x}\right)dx, x, \cos(c+dx)\right)}{a^8 d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2 d} - \frac{2\sec(c+dx)}{a^2 d} - \frac{\sec^2(c+dx)}{2a^2 d} + \frac{4\sec^3(c+dx)}{3a^2 d} - \frac{\sec^4(c+}{4a^2 d}$$

Mathematica [A]
time = 0.52, size = 125, normalized size = 1.04

$$-\frac{(312\cos(c+dx)+5(14+28\cos(3(c+dx))+6\cos(4(c+dx))+12\cos(5(c+dx))+30\log(\cos(c+dx))+18\cos(4(c+dx))\log(\cos(c+dx))+3\cos(6(c+dx))\log(\cos(c+dx))+9\cos(2(c+dx))(4+5\log(\cos(c+dx)))))\sec^6(c+dx)}{480a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/480*((312*Cos[c + d*x] + 5*(14 + 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)
] + 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Co
s[c + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4
+ 5*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(a^2*d)
```

Maple [A]
time = 0.15, size = 74, normalized size = 0.62

method	result
derivativedivides	$\frac{\frac{(\sec^6(dx+c))}{6}-\frac{2(\sec^5(dx+c))}{5}-\frac{(\sec^4(dx+c))}{4}+\frac{4(\sec^3(dx+c))}{3}-\frac{(\sec^2(dx+c))}{2}-2\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^2}$
default	$\frac{\frac{(\sec^6(dx+c))}{6}-\frac{2(\sec^5(dx+c))}{5}-\frac{(\sec^4(dx+c))}{4}+\frac{4(\sec^3(dx+c))}{3}-\frac{(\sec^2(dx+c))}{2}-2\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^2}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2 d}-\frac{2\left(30\,e^{11i(dx+c)}+15\,e^{10i(dx+c)}+70\,e^{9i(dx+c)}+90\,e^{8i(dx+c)}+156\,e^{7i(dx+c)}+70\,e^{6i(dx+c)}+156\,e^{5i(dx+c)}+90\right.}{15d\,a^2\left(e^{2i(dx+c)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/6*sec(d*x+c)^6-2/5*sec(d*x+c)^5-1/4*sec(d*x+c)^4+4/3*sec(d*x+c)^
3-1/2*sec(d*x+c)^2-2*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 80, normalized size = 0.67

$$-\frac{\frac{60\,\log(\cos(dx+c))}{a^2}+\frac{120\,\cos(dx+c)^5+30\,\cos(dx+c)^4-80\,\cos(dx+c)^3+15\,\cos(dx+c)^2+24\,\cos(dx+c)-10}{a^2\cos(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^2 + (120*cos(d*x + c)^5 + 30*cos(d*x + c)^4 -
 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(a^2*cos(d*x
 + c)^6))/d
```

Fricas [**A**]
time = 2.63, size = 85, normalized size = 0.71

$$-\frac{60\cos\left(dx+c\right)^{6}\log\left(-\cos\left(dx+c\right)\right)+120\cos\left(dx+c\right)^{5}+30\cos\left(dx+c\right)^{4}-80\cos\left(dx+c\right)^{3}+15\cos\left(dx+c\right)^{2}+24\cos\left(dx+c\right)-10}{60\,a^{2}d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^6*log(-cos(d*x + c)) + 120*cos(d*x + c)^5 + 30*cos(d
*x + c)^4 - 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(
a^2*d*cos(d*x + c)^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{9}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.
time = 5.85, size = 223, normalized size = 1.86

$$\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^{2}}+\frac{\frac{234\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1005\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2220\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2925\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1002\,(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{147\,(\cos(dx+c)-1)^{6}}{(\cos(dx+c)+1)^{6}}+19}{a^{2}\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 + (234*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1005*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
220*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 1002*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 147*
```

```
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 19)/(a^2*((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 1)^6))/d
```

Mupad [B]
time = 5.18, size = 193, normalized size = 1.61

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d}+\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{54\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{32}{15}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d) + ((54*tan(c/2 + (d*x)/2)^2)/5 - 20
*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 12*tan(c/2 + (d*x)/2)^8 -
 2*tan(c/2 + (d*x)/2)^10 - 32/15)/(d*(15*a^2*tan(c/2 + (d*x)/2)^4 - 6*a^2*t
an(c/2 + (d*x)/2)^2 - 20*a^2*tan(c/2 + (d*x)/2)^6 + 15*a^2*tan(c/2 + (d*x)/
2)^8 - 6*a^2*tan(c/2 + (d*x)/2)^10 + a^2*tan(c/2 + (d*x)/2)^12 + a^2))
```

3.72 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{\log(\cos(c+dx))}{a^2d}+\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{\sec^4(c+dx)}{4a^2d}$$

```
[Out] ln(cos(d*x+c))/a^2/d+2*sec(d*x+c)/a^2/d-2/3*sec(d*x+c)^3/a^2/d+1/4*sec(d*x+
c)^4/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 76}

$$\frac{\sec^4(c+dx)}{4a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{2\sec(c+dx)}{a^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(
3*a^2*d) + Sec[c + d*x]^4/(4*a^2*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)}{x^5}\,dx, x, \cos(c+dx)\right)}{a^6d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}-\frac{2a^4}{x^4}+\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^6d}$$
$$= \frac{\log(\cos(c+dx))}{a^2d}+\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{\sec^4(c+dx)}{4a^2d}$$

Mathematica [A]
time = 0.20, size = 83, normalized size = 1.28

$$\frac{(20\cos(c+dx)+3(2+4\cos(3(c+dx))+3\log(\cos(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c+dx))))\sec^4(c+dx)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
[Out] ((20*Cos[c + d*x] + 3*(2 + 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] + 4*Cos
[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))*Sec[
c + d*x]^4)/(24*a^2*d)
```

Maple [A]
time = 0.11, size = 45, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{-\frac{\left(\sec^4(dx+c)\right)}{4}+\frac{2\left(\sec^3(dx+c)\right)}{3}-2\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^2}$	45
default	$-\frac{-\frac{\left(\sec^4(dx+c)\right)}{4}+\frac{2\left(\sec^3(dx+c)\right)}{3}-2\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^2}$	45
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{4\,e^{7i(dx+c)}+\frac{20\,e^{5i(dx+c)}}{3}+4\,e^{4i(dx+c)}+\frac{20\,e^{3i(dx+c)}}{3}+4\,e^{i(dx+c)}}{d\,a^2\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/d/a^2*(-1/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3-2*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 50, normalized size = 0.77

$$\frac{\frac{12\,\log(\cos(dx+c))}{a^2}+\frac{24\,\cos(dx+c)^3-8\,\cos(dx+c)+3}{a^2\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*log(cos(d*x + c))/a^2 + (24*cos(d*x + c)^3 - 8*cos(d*x + c) + 3)/(
a^2*cos(d*x + c)^4))/d
```

Fricas [A]
time = 4.51, size = 55, normalized size = 0.85

$$\frac{12 \cos(dx+c)^4 \log(-\cos(dx+c)) + 24 \cos(dx+c)^3 - 8 \cos(dx+c) + 3}{12 a^2 d \cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*cos(d*x + c)^4*log(-cos(d*x + c)) + 24*cos(d*x + c)^3 - 8*cos(d*x
+ c) + 3)/(a^2*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(61) = 122.
time = 3.69, size = 180, normalized size = 2.77

$$-\frac{\frac{12 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{12 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2} - \frac{\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{54(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{124(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{25(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+7}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (4*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 54*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 124*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 25*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

Mupad [B]
time = 3.87, size = 135, normalized size = 2.08

$$\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{8}{3}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (8*tan(c/2 + (d*x)/2)^4 - (26*tan(c/2 + (d*x)/2)^2)/3 + 2*tan(c/2 + (d*x)/2
)^6 + 8/3)/(d*(6*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d*x)/2)^2 - 4*
a^2*tan(c/2 + (d*x)/2)^6 + a^2*tan(c/2 + (d*x)/2)^8 + a^2)) - (2*atanh(tan(
c/2 + (d*x)/2)^2))/(a^2*d)
```

3.73 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=48

$$-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^2(c+dx)}{2a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d+1/2*sec(d*x+c)^2/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 45}

$$\frac{\sec^2(c+dx)}{2a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^2/(2
*a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^3}\,dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}-\frac{2a^2}{x^2}+\frac{a^2}{x}\right)dx, x, \cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{2\sec(c+dx)}{a^2d} + \frac{\sec^2(c+dx)}{2a^2d}$$

Mathematica [A]
time = 0.12, size = 51, normalized size = 1.06

$$-\frac{(-1+4\cos(c+dx)+\log(\cos(c+dx))+\cos(2(c+dx))\log(\cos(c+dx)))\sec^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((-1 + 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[Cos[c
 + d*x]])*Sec[c + d*x]^2)/(a^2*d)
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^2(dx+c)\right)}{2}-2\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^2}$	34
default	$\frac{\frac{\left(\sec^2(dx+c)\right)}{2}-2\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^2}$	34
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(2\,e^{3i(dx+c)}-e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/2*sec(d*x+c)^2-2*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.83

$$-\frac{\frac{2\,\log(\cos(dx+c))}{a^2}+\frac{4\,\cos(dx+c)-1}{a^2\,\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(cos(d*x + c))/a^2 + (4*cos(d*x + c) - 1)/(a^2*cos(d*x + c)^2))/
d
```

Fricas [A]
time = 3.82, size = 45, normalized size = 0.94

$$-\frac{2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+4\cos\left(dx+c\right)-1}{2\,a^2d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^2*log(-cos(d*x + c)) + 4*cos(d*x + c) - 1)/(a^2*d*cos(
d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^5\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(46) = 92.
time = 1.82, size = 136, normalized size = 2.83

$$\frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2}-\frac{\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 2*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5)/(a^2*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2))/d
```

Mupad [B]
time = 1.35, size = 77, normalized size = 1.60

$$\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}+\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (6*tan(c/2 + (d*x)/2)^2 - 4)/(d*(a^2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 +
 (d*x)/2)^2 + a^2)) + (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d)
```

3.74 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=33

$$-\frac{\log(\cos(c+dx))}{a^2d}+\frac{2\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+2*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3964, 78}

$$\frac{2\log(\cos(c+dx)+1)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a-ax}{x(a+ax)}\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{1+x}\right)\,dx, x, \cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\log(\cos(c+dx))}{a^2d} + \frac{2\log(1+\cos(c+dx))}{a^2d}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 0.91

$$\frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
[Out] (4*Log[Cos[(c + d*x)/2]] - Log[Cos[c + d*x]])/(a^2*d)
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{-2\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^2}$	28
default	$-\frac{-2\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^2}$	28
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/d/a^2*(-2*ln(1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.94

$$\frac{\frac{2\,\log(\cos(dx+c)+1)}{a^2} - \frac{\log(\cos(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c))/a^2)/d
```

Fricas [A]
time = 3.10, size = 31, normalized size = 0.94

$$-\frac{\log\left(-\cos\left(dx+c\right)\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(log(-cos(d*x + c)) - 2*log(1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^3\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.77, size = 33, normalized size = 1.00

$$-\frac{\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right|\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -log(abs((cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1))/(a^2*d)
```

Mupad [B]
time = 1.39, size = 22, normalized size = 0.67

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
[Out] -log(tan(c/2 + (d*x)/2)^4 - 1)/(a^2*d)
```

3.75 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=36

$$-\frac{1}{a^2d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] -1/a^2/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 45}

$$-\frac{1}{a^2d(\cos(c+dx)+1)}-\frac{\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(1/(a^2*d*(1 + Cos[c + d*x]))) - Log[1 + Cos[c + d*x]]/(a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(1+x)^2}+\frac{1}{a^2(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{1}{a^2d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 56, normalized size = 1.56

$$-\frac{\left(1+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((1 + 2*Log[Cos[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[Cos[(c + d*x)/2]])*
Sec[(c + d*x)/2]^2)/(a^2*d)
```

Maple [A]
time = 0.04, size = 37, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^2}$	37
default	$\frac{\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^2}$	37
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\,e^{i(dx+c)}}{a^2d\left(e^{i(dx+c)}+1\right)^2}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/(1+sec(d*x+c))-ln(1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.97

$$-\frac{\frac{1}{a^2\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(1/(a^2*cos(d*x + c) + a^2) + log(cos(d*x + c) + 1)/a^2)/d
```

Fricas [A]
time = 2.89, size = 43, normalized size = 1.19

$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^2d\cos(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 1)/(a^2*d*cos(d*x + c) +
 a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(31) = 62$.
time = 11.71, size = 177, normalized size = 4.92

$$\begin{cases} \frac{\log(\tan^2(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d} + \frac{\log(\tan^2(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d} - \frac{2\log(\sec(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d} - \frac{2\log(\sec(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d} + \frac{2}{2a^2d\sec(c+dx)+2a^2d} & \text{for } d \neq 0 \\ \frac{x\tan(c)}{(a\sec(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2
*a**2*d) + log(tan(c + d*x)**2 + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*
log(sec(c + d*x) + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*l
og(sec(c + d*x) + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) + 2/(2*a**2*d*sec(c
 + d*x) + 2*a**2*d), Ne(d, 0)), (x*tan(c)/(a*sec(c) + a)**2, True))
```

Giac [A]
time = 0.51, size = 57, normalized size = 1.58

$$\frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} + \frac{\cos(dx+c)-1}{a^2(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (cos(d*x
+ c) - 1)/(a^2*(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 1.13, size = 35, normalized size = 0.97

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2/2)/(a^2*d)
```

3.76 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{4a^2d(1+\cos(c+dx))^2}+\frac{5}{4a^2d(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(1+\cos(c+dx))}{8a^2d}$$

```
[Out] -1/4/a^2/d/(1+cos(d*x+c))^2+5/4/a^2/d/(1+cos(d*x+c))+1/8*ln(1-cos(d*x+c))/a
^2/d+7/8*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{5}{4a^2d(\cos(c+dx)+1)}-\frac{1}{4a^2d(\cos(c+dx)+1)^2}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(\cos(c+dx)+1)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*1/(a^2*d*(1 + Cos[c + d*x])^2) + 5/(4*a^2*d*(1 + Cos[c + d*x])) + Log[
1 - Cos[c + d*x]]/(8*a^2*d) + (7*Log[1 + Cos[c + d*x]])/(8*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a^2\text{Subst}\left(\int \frac{x^3}{(a-ax)(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\text{Subst}\left(\int \left(-\frac{1}{8a^4(-1+x)} - \frac{1}{2a^4(1+x)^3} + \frac{5}{4a^4(1+x)^2} - \frac{7}{8a^4(1+x)}\right) dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{1}{4a^2d(1+\cos(c+dx))^2} + \frac{5}{4a^2d(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{8a^2d} + \frac{7\,\text{l}}{}$$

Mathematica [A]
time = 0.19, size = 83, normalized size = 1.02

$$\frac{\left(-1+10\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^2(c+dx)}{4a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-1 + 10*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(7*Log[Cos[(c + d*x)/2]
] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(4*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.12, size = 55, normalized size = 0.68

method	result	size
derivativedivides	$\frac{\frac{\ln(-1+\cos(dx+c))}{8}-\frac{1}{4(1+\cos(dx+c))^2}+\frac{5}{4(1+\cos(dx+c))}+\frac{7\ln(1+\cos(dx+c))}{8}}{d\,a^2}$	55
default	$\frac{\frac{\ln(-1+\cos(dx+c))}{8}-\frac{1}{4(1+\cos(dx+c))^2}+\frac{5}{4(1+\cos(dx+c))}+\frac{7\ln(1+\cos(dx+c))}{8}}{d\,a^2}$	55
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{5\,e^{3i(dx+c)}+8\,e^{2i(dx+c)}+5\,e^{i(dx+c)}}{2d\,a^2\left(e^{i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{4a^2d}+\frac{7\ln\left(e^{i(dx+c)}+1\right)}{4a^2d}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/8*ln(-1+cos(d*x+c))-1/4/(1+cos(d*x+c))^2+5/4/(1+cos(d*x+c))+7/8*
ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 74, normalized size = 0.91

$$\frac{\frac{2\,(5\cos(dx+c)+4)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}+\frac{7\log(\cos(dx+c)+1)}{a^2}+\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(5*cos(d*x + c) + 4)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
 + 7*log(cos(d*x + c) + 1)/a^2 + log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 2.56, size = 106, normalized size = 1.31

$$\frac{7\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+10\,\cos\left(dx+c\right)+8}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 10*cos
(d*x + c) + 8)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.45, size = 117, normalized size = 1.44

$$\frac{\frac{2\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{2}}-\frac{16\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{\frac{8\,a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^{2}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{a^{4}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 - 16*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - (8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^4)/
d
```

Mupad [B]
time = 1.26, size = 62, normalized size = 0.77

$$\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{2}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{16}}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*x
)/2)^2/2 - tan(c/2 + (d*x)/2)^4/16)/(a^2*d)
```

3.77 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{1}{12a^2d(1+\cos(c+dx))^3}+\frac{1}{2a^2d(1+\cos(c+dx))^2}-\frac{23}{16a^2d(1+\cos(c+dx))}-3\,\mathrm{l}$$

```
[Out] -1/16/a^2/d/(1-cos(d*x+c))-1/12/a^2/d/(1+cos(d*x+c))^3+1/2/a^2/d/(1+cos(d*x
+c))^2-23/16/a^2/d/(1+cos(d*x+c))-3/16*ln(1-cos(d*x+c))/a^2/d-13/16*ln(1+co
s(d*x+c))/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$-\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{23}{16a^2d(\cos(c+dx)+1)}+\frac{1}{2a^2d(\cos(c+dx)+1)^2}-\frac{1}{12a^2d(\cos(c+dx)+1)^3}-\frac{3\log(1-\cos(c+dx))}{16a^2d}-\frac{13\log(\cos(c+dx)+1)}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/16*1/(a^2*d*(1 - Cos[c + d*x])) - 1/(12*a^2*d*(1 + Cos[c + d*x])^3) + 1/
(2*a^2*d*(1 + Cos[c + d*x])^2) - 23/(16*a^2*d*(1 + Cos[c + d*x])) - (3*Log[
1 - Cos[c + d*x]])/(16*a^2*d) - (13*Log[1 + Cos[c + d*x]])/(16*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a^4\text{Subst}\left(\int \frac{x^5}{(a-ax)^2(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^4\text{Subst}\left(\int \left(\frac{1}{16a^6(-1+x)^2} + \frac{3}{16a^6(-1+x)} - \frac{1}{4a^6(1+x)^4} + \frac{1}{a^6(1+x)^3} - \frac{23}{16a^6(1+x)^2} + \cdots\right.\right.}{d}$$

$$= -\frac{1}{16a^2d(1-\cos(c+dx))} - \frac{1}{12a^2d(1+\cos(c+dx))^3} + \frac{1}{2a^2d(1+\cos(c+dx))^2} \cdots$$

Mathematica [A]
time = 0.37, size = 121, normalized size = 0.98

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^2\left(\frac{1}{2}(c+dx)\right)+156\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+69\sec^2\left(\frac{1}{2}(c+dx)\right)-12\sec^4\left(\frac{1}{2}(c+dx)\right)+\sec^6\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)}{24a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/24*(Cos[(c + d*x)/2]^4*(3*Csc[(c + d*x)/2]^2 + 156*Log[Cos[(c + d*x)/2]]
 + 36*Log[Sin[(c + d*x)/2]] + 69*Sec[(c + d*x)/2]^2 - 12*Sec[(c + d*x)/2]^4
 + Sec[(c + d*x)/2]^6)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.13, size = 79, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{1}{-16+16\cos(dx+c)}-\frac{3\ln(-1+\cos(dx+c))}{16}-\frac{1}{12(1+\cos(dx+c))^3}+\frac{1}{2(1+\cos(dx+c))^2}-\frac{23}{16(1+\cos(dx+c))}-\frac{13\ln(1+\cos(dx+c))}{16}}{d\,a^2}$
default	$\frac{\frac{1}{-16+16\cos(dx+c)}-\frac{3\ln(-1+\cos(dx+c))}{16}-\frac{1}{12(1+\cos(dx+c))^3}+\frac{1}{2(1+\cos(dx+c))^2}-\frac{23}{16(1+\cos(dx+c))}-\frac{13\ln(1+\cos(dx+c))}{16}}{d\,a^2}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{33\,e^{7i(dx+c)}+36\,e^{6i(dx+c)}-49\,e^{5i(dx+c)}-136\,e^{4i(dx+c)}-49\,e^{3i(dx+c)}+36\,e^{2i(dx+c)}+33\,e^{i(dx+c)}}{12d\,a^2\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^2}-\frac{13\ln(\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/16/(-1+cos(d*x+c))-3/16*ln(-1+cos(d*x+c))-1/12/(1+cos(d*x+c))^3+
1/2/(1+cos(d*x+c))^2-23/16/(1+cos(d*x+c))-13/16*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.26, size = 110, normalized size = 0.89

$$-\frac{\frac{2\left(33\cos(dx+c)^3+18\cos(dx+c)^2-37\cos(dx+c)-26\right)}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2}+\frac{39\log(\cos(dx+c)+1)}{a^2}+\frac{9\log(\cos(dx+c)-1)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(33*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 37*cos(d*x + c) - 26)/(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c) - a^2) + 39*lo
g(cos(d*x + c) + 1)/a^2 + 9*log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 2.38, size = 162, normalized size = 1.32

$$-\frac{66\cos(dx+c)^3+36\cos(dx+c)^2+39\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+9\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-74\cos(dx+c)-52}{48\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(66*cos(d*x + c)^3 + 36*cos(d*x + c)^2 + 39*(cos(d*x + c)^4 + 2*cos(d
*x + c)^3 - 2*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x +
c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2)
- 74*cos(d*x + c) - 52)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*
a^2*d*cos(d*x + c) - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.54, size = 186, normalized size = 1.51

$$\frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{18\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{96\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\frac{48a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{a^4(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^6}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^2*(cos(d*x + c) - 1)) - 18*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^2 + 96*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (48*
a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^6)/d
```

Mupad [B]
time = 1.37, size = 89, normalized size = 0.72

$$-\frac{\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{32}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}-\frac{3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{32}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{96}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
[Out] -((3*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 +
 (d*x)/2)^2/32 + tan(c/2 + (d*x)/2)^2/2 - (3*tan(c/2 + (d*x)/2)^4)/32 + tan
(c/2 + (d*x)/2)^6/96)/(a^2*d)
```

3.78 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$-\frac{1}{64a^2d(1-\cos(c+dx))^2}+\frac{9}{64a^2d(1-\cos(c+dx))}-\frac{1}{32a^2d(1+\cos(c+dx))^4}+\frac{11}{48a^2d(1+\cos(c+dx))^3}-$$

```
[Out] -1/64/a^2/d/(1-cos(d*x+c))^2+9/64/a^2/d/(1-cos(d*x+c))-1/32/a^2/d/(1+cos(d*
x+c))^4+11/48/a^2/d/(1+cos(d*x+c))^3-3/4/a^2/d/(1+cos(d*x+c))^2+51/32/a^2/d
/(1+cos(d*x+c))+29/128*ln(1-cos(d*x+c))/a^2/d+99/128*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A]

time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{9}{64a^2d(1-\cos(c+dx))}+\frac{51}{32a^2d(\cos(c+dx)+1)}-\frac{1}{64a^2d(1-\cos(c+dx))^2}-\frac{3}{4a^2d(\cos(c+dx)+1)^2}+\frac{11}{48a^2d(\cos(c+dx)+1)^3}-\frac{1}{32a^2d(\cos(c+dx)+1)^4}+\frac{29\log(1-\cos(c+dx))}{128a^2d}+\frac{99\log(\cos(c+dx)+1)}{128a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*1/(a^2*d*(1 - Cos[c + d*x])^2) + 9/(64*a^2*d*(1 - Cos[c + d*x])) - 1/
(32*a^2*d*(1 + Cos[c + d*x])^4) + 11/(48*a^2*d*(1 + Cos[c + d*x])^3) - 3/(4
*a^2*d*(1 + Cos[c + d*x])^2) + 51/(32*a^2*d*(1 + Cos[c + d*x])) + (29*Log[1
 - Cos[c + d*x]])/(128*a^2*d) + (99*Log[1 + Cos[c + d*x]])/(128*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a^6\text{Subst}\left(\int \frac{x^7}{(a-ax)^3(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{32a^8(-1+x)^3} - \frac{9}{64a^8(-1+x)^2} - \frac{29}{128a^8(-1+x)} - \frac{1}{8a^8(1+x)^5} + \frac{11}{16a^8(1+x)^4} -\right.\right.}{d}$$

$$= -\frac{1}{64a^2d(1-\cos(c+dx))^2} + \frac{9}{64a^2d(1-\cos(c+dx))} - \frac{1}{32a^2d(1+\cos(c+dx))}$$

Mathematica [A]

time = 0.82, size = 154, normalized size = 0.93

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-108\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)-24\left(99\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+29\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-1224\sec^2\left(\frac{1}{2}(c+dx)\right)+288\sec^4\left(\frac{1}{2}(c+dx)\right)-44\sec^6\left(\frac{1}{2}(c+dx)\right)+3\sec^8\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)}{384a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(Cos[(c + d*x)/2]^4*(-108*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4
- 24*(99*Log[Cos[(c + d*x)/2]] + 29*Log[Sin[(c + d*x)/2]]) - 1224*Sec[(c +
d*x)/2]^2 + 288*Sec[(c + d*x)/2]^4 - 44*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x
)/2]^8)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]

time = 0.14, size = 103, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{64(-1+\cos(dx+c))^2}-\frac{9}{64(-1+\cos(dx+c))}+\frac{29\ln(-1+\cos(dx+c))}{128}-\frac{1}{32(1+\cos(dx+c))^4}+\frac{11}{48(1+\cos(dx+c))^3}-\frac{3}{4(1+\cos(dx+c))^2}+\frac{32}{}}{d\,a^2}$
default	$\frac{-\frac{1}{64(-1+\cos(dx+c))^2}-\frac{9}{64(-1+\cos(dx+c))}+\frac{29\ln(-1+\cos(dx+c))}{128}-\frac{1}{32(1+\cos(dx+c))^4}+\frac{11}{48(1+\cos(dx+c))^3}-\frac{3}{4(1+\cos(dx+c))^2}+\frac{32}{}}{d\,a^2}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{279\,e^{11i(dx+c)}+156\,e^{10i(dx+c)}-1141\,e^{9i(dx+c)}-2080\,e^{8i(dx+c)}+670\,e^{7i(dx+c)}+2696\,e^{6i(dx+c)}+670\,e^{5i(}}{96d\,a^2\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/64/(-1+cos(d*x+c))^2-9/64/(-1+cos(d*x+c))+29/128*ln(-1+cos(d*x+
c))-1/32/(1+cos(d*x+c))^4+11/48/(1+cos(d*x+c))^3-3/4/(1+cos(d*x+c))^2+51/32
/(1+cos(d*x+c))+99/128*ln(1+cos(d*x+c)))
```

Maxima [A]

time = 0.27, size = 167, normalized size = 1.01

$$\frac{\frac{2\left(279\cos(dx+c)^5+78\cos(dx+c)^4-634\cos(dx+c)^3-338\cos(dx+c)^2+343\cos(dx+c)+224\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}+\frac{297\log(\cos(dx+c)+1)}{a^2}+\frac{87\log(\cos(dx+c)-1)}{a^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(2*(279*cos(d*x + c)^5 + 78*cos(d*x + c)^4 - 634*cos(d*x + c)^3 - 338
*cos(d*x + c)^2 + 343*cos(d*x + c) + 224)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d
*x + c)^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2
+ 2*a^2*cos(d*x + c) + a^2) + 297*log(cos(d*x + c) + 1)/a^2 + 87*log(cos(d*
x + c) - 1)/a^2)/d
```

Fricas [A]
time = 2.28, size = 283, normalized size = 1.72

$$\frac{558 \cos(dx+c)^5 + 156 \cos(dx+c)^4 - 1268 \cos(dx+c)^3 - 676 \cos(dx+c)^2 + 297 \left(\cos(dx+c)^6 + 2 \cos(dx+c)^5 - \cos(dx+c)^4 - 4 \cos(dx+c)^3 - \cos(dx+c)^2 + 2 \cos(dx+c) + 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 87 \left(\cos(dx+c)^6 + 2 \cos(dx+c)^5 - \cos(dx+c)^4 - 4 \cos(dx+c)^3 - \cos(dx+c)^2 + 2 \cos(dx+c) + 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 686 \cos(dx+c) + 448}{384 \left(a^2 d \cos(dx+c)^6 + 2 a^2 d \cos(dx+c)^5 - a^2 d \cos(dx+c)^4 - 4 a^2 d \cos(dx+c)^3 - a^2 d \cos(dx+c)^2 + 2 a^2 d \cos(dx+c) + a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(558*cos(d*x + c)^5 + 156*cos(d*x + c)^4 - 1268*cos(d*x + c)^3 - 676*
cos(d*x + c)^2 + 297*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 -
4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c
) + 1/2) + 87*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d
*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 686*cos(d*x + c) + 448)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5
 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.56, size = 236, normalized size = 1.43

$$-\frac{\frac{6\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2} - \frac{348\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{1536\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} + \frac{\frac{768a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{174a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{32a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3a^6(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{a^8}}{1536\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1536*(6*(16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^
2) - 348*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 1536*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (768*a^6*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 174*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 32*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

Mupad [B]
time = 1.24, size = 151, normalized size = 0.92

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,a^2\,d}-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{48\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{512\,a^2\,d}+\frac{29\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{1}{4}\right)}{64\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(2*a^2*d) - (29*tan(c/2 + (d*x)/2)^4)/(256*a^2*d) + ta
n(c/2 + (d*x)/2)^6/(48*a^2*d) - tan(c/2 + (d*x)/2)^8/(512*a^2*d) + (29*log(
tan(c/2 + (d*x)/2)))/(64*a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) + (
cot(c/2 + (d*x)/2)^4*(4*tan(c/2 + (d*x)/2)^2 - 1/4))/(64*a^2*d)
```

3.79 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{x}{a^2}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c+d}{2a^2d}$$

```
[Out] x/a^2-3/4*arctanh(sin(d*x+c))/a^2/d-tan(d*x+c)/a^2/d+3/4*sec(d*x+c)*tan(d*x
+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d-1/2*sec(d*x+c)*tan(d*x+c)^3/a^2/d+1/5*tan(
d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan^3(c+dx)\sec(c+dx)}{2a^2d}+\frac{3\tan(c+dx)\sec(c+dx)}{4a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (3*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - Tan[c + d*x]/(a^2*d) + (3*Sec
[c + d*x]*Tan[c + d*x])/(4*a^2*d) + Tan[c + d*x]^3/(3*a^2*d) - (Sec[c + d*x
]*Tan[c + d*x]^3)/(2*a^2*d) + Tan[c + d*x]^5/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int(-a+a\sec(c+dx))^2\tan^4(c+dx)\,dx}{a^4}\\
&= \frac{\int\left(a^2\tan^4(c+dx)-2a^2\sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx)\tan^4(c+dx\right.}{a^4}\\
&= \frac{\int\tan^4(c+dx)\,dx}{a^2}+\frac{\int\sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}-\frac{2\int\sec(c+dx)\tan^4(c+}{a^2}\\
&= \frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d}-\frac{\int\tan^2(c+dx)\,dx}{a^2}+\frac{3\int\sec(c+dx}{2}\\
&= -\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c}{2a^2d}\\
&= \frac{x}{a^2}-\frac{3\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}+\frac{\tan^3(}{3}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 495 vs. $2(119) = 238$.

time = 5.88, size = 495, normalized size = 4.16

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(240x+\frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{\left(293\cos\left(\frac{dx}{2}\right)+333\cos\left(2c+\frac{3dx}{2}\right)+287\cos\left(2c+\frac{5dx}{2}\right)+67\cos\left(4c+\frac{7dx}{2}\right)+68\cos\left(4c+\frac{9dx}{2}\right)\right)\sec(c)\sec^5(c+dx)\sin\left(\frac{dx}{2}\right)}{2d}+\frac{36\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{151\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{36\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{151\sin\left(\frac{c}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{\cos\left(\frac{c}{2}\right)\sec(c)\sec^4(c+dx)\left(308\sin\left(\frac{c}{2}\right)-43\sin\left(\frac{c}{2}+dx\right)-43\sin\left(\frac{3c}{2}+dx\right)-346\sin\left(\frac{3c}{2}+2dx\right)+346\sin\left(\frac{5c}{2}+2dx\right)+149\sin\left(\frac{5c}{2}+3dx\right)+149\sin\left(\frac{7c}{2}+3dx\right)\right)}{4d}\right)}{60a^2(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(240*x + (180*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d - (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d - ((29
3*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)/2] + 67*C
os[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x]^5*Sin[(d
*x)/2])/(2*d) + (36*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2) + (36*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*Sec[c]*Sec[c + d*x]
^4*(308*Sin[c/2] - 43*Sin[c/2 + d*x] - 43*Sin[(3*c)/2 + d*x] - 346*Sin[(3*c
)/2 + 2*d*x] + 346*Sin[(5*c)/2 + 2*d*x] + 149*Sin[(5*c)/2 + 3*d*x] + 149*Si
n[(7*c)/2 + 3*d*x]))/(4*d)))/(60*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A]

time = 0.13, size = 200, normalized size = 1.68

method	result
risch	$\frac{x}{a^2} - \frac{i\left(75\,e^{9i(dx+c)}+60\,e^{8i(dx+c)}+30\,e^{7i(dx+c)}+360\,e^{6i(dx+c)}+320\,e^{4i(dx+c)}-30\,e^{3i(dx+c)}+280\,e^{2i(dx+c)}-75\,e^{i(dx+c)}\ldots}{30d\,a^2\left(e^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{4\left(\tan\left(\frac{dx}{2}\ldots\right.\right.}}{}$
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{4\left(\tan\left(\frac{dx}{2}\ldots\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a^2*(1/64*arctan(tan(1/2*d*x+1/2*c))-1/640/(tan(1/2*d*x+1/2*c)-1)^5-1
/128/(tan(1/2*d*x+1/2*c)-1)^4-19/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(
1/2*d*x+1/2*c)-1)^2+7/512/(tan(1/2*d*x+1/2*c)-1)+3/512*ln(tan(1/2*d*x+1/2*c
)-1)-1/640/(tan(1/2*d*x+1/2*c)+1)^5+1/128/(tan(1/2*d*x+1/2*c)+1)^4-19/1536/
(tan(1/2*d*x+1/2*c)+1)^3+1/1024/(tan(1/2*d*x+1/2*c)+1)^2+7/512/(tan(1/2*d*x
+1/2*c)+1)-3/512*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(109) = 218$.

time = 0.48, size = 301, normalized size = 2.53

$$-\frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{110\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{328\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{530\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^2-\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{45\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}-\frac{45\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 110*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 328*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 530*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7 + 105*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^2 - 5*a^
2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 45*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^2 - 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A]

time = 3.02, size = 117, normalized size = 0.98

$$\frac{120\,dx\cos(dx+c)^5-45\cos(dx+c)^5\log(\sin(dx+c)+1)+45\cos(dx+c)^5\log(-\sin(dx+c)+1)-2\left(68\cos(dx+c)^4-75\cos(dx+c)^3+4\cos(dx+c)^2+30\cos(dx+c)-12\right)\sin(dx+c)}{120\,a^2d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(120*d*x*cos(d*x + c)^5 - 45*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 4
5*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*cos(d*x + c)^4 - 75*cos(d*x
 + c)^3 + 4*cos(d*x + c)^2 + 30*cos(d*x + c) - 12)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^5)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^8(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]

time = 4.82, size = 136, normalized size = 1.14

$$\frac{\frac{60(dx+c)}{a^2}-\frac{45\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{45\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-530\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+328\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^2 - 45*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 45*log
(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^9 - 530*t
an(1/2*d*x + 1/2*c)^7 + 328*tan(1/2*d*x + 1/2*c)^5 - 110*tan(1/2*d*x + 1/2*
c)^3 + 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^2))/d
```

Mupad [B]
time = 2.26, size = 179, normalized size = 1.50

$$\frac{x}{a^2}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}+\frac{\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}-\frac{53\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{164\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

```
[Out] x/a^2 - (3*atanh(tan(c/2 + (d*x)/2)))/(2*a^2*d) + (tan(c/2 + (d*x)/2)/2 - (
11*tan(c/2 + (d*x)/2)^3)/3 + (164*tan(c/2 + (d*x)/2)^5)/15 - (53*tan(c/2 +
(d*x)/2)^7)/3 + (7*tan(c/2 + (d*x)/2)^9)/2)/(d*(5*a^2*tan(c/2 + (d*x)/2)^2
- 10*a^2*tan(c/2 + (d*x)/2)^4 + 10*a^2*tan(c/2 + (d*x)/2)^6 - 5*a^2*tan(c/2
 + (d*x)/2)^8 + a^2*tan(c/2 + (d*x)/2)^10 - a^2))
```

3.80 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{x}{a^2}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] -x/a^2+arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d-sec(d*x+c)*tan(d*x+c)/a^2
/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + ArcTanh[Sin[c + d*x]]/(a^2*d) + Tan[c + d*x]/(a^2*d) - (Sec[c +
d*x]*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int(-a+a\sec(c+dx))^2\tan^2(c+dx)\,dx}{a^4}$$

$$= \frac{\int\left(a^2\tan^2(c+dx)-2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+dx)\tan^2(c+d\right.}{a^4}$$

$$= \frac{\int\tan^2(c+dx)\,dx}{a^2}+\frac{\int\sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2}-\frac{2\int\sec(c+dx)\tan^2(c-}{a^2}$$

$$= \frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}+\frac{\int\sec(c+dx)\,dx}{a^2}+\frac{\text{Subst}(}{}$$

$$= -\frac{x}{a^2}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}+\frac{\tan^3(}{3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 767 vs. 2(72) = 144.
time = 6.33, size = 767, normalized size = 10.65

$$-\frac{4x\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{(a+a\sec(c+dx))^2}-\frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)}{d(a+a\sec(c+dx))^2}+\frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)}{d(a+a\sec(c+dx))^2}+\frac{2\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{8\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{2\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}+\frac{8\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 - (4*Cos[
c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^
2)/(d*(a + a*Sec[c + d*x])^2) + (4*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2)/(d*(a + a*Sec[c + d*x])^2) + (2*C
os[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])^
2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-5*Cos[c/2] + 7*Sin[c/2]))/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])) + (2*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[
c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])
^3) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(5*Cos[c/2] + 7*Sin[c/2]))/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2]))
```

Maple [A]
time = 0.11, size = 140, normalized size = 1.94

method	result
risch	$-\frac{x}{a^2}+\frac{2i\left(3\,\mathrm{e}^{5i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+2\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^2 d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2 d}$
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^2 d}$
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^2 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*(-1/16*arctan(tan(1/2*d*x+1/2*c))-1/96/(tan(1/2*d*x+1/2*c)-1)^3-3/
64/(tan(1/2*d*x+1/2*c)-1)^2-1/16/(tan(1/2*d*x+1/2*c)-1)-1/32*ln(tan(1/2*d*x
+1/2*c)-1)-1/96/(tan(1/2*d*x+1/2*c)+1)^3+3/64/(tan(1/2*d*x+1/2*c)+1)^2-1/16
/(tan(1/2*d*x+1/2*c)+1)+1/32*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(70) = 140.

time = 0.47, size = 196, normalized size = 2.72

$$-\frac{\frac{4\left(\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A]
time = 3.19, size = 97, normalized size = 1.35

$$-\frac{6\,dx\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)+3\,\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(2\,\cos\left(dx+c\right)^2-3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{6\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*d*x*cos(d*x + c)^3 - 3*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*cos
(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^2 - 3*cos(d*x + c) +
 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 2.32, size = 99, normalized size = 1.38

$$-\frac{\frac{3\,(dx+c)}{a^2}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}+\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{4\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(3*tan(1/2*d*x + 1/2*c)^5 - tan(1/2*d*
x + 1/2*c)^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2))/d
```

Mupad [B]
time = 1.39, size = 111, normalized size = 1.54

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{x}{a^2}+\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - x/a^2 + ((4*tan(c/2 + (d*x)/2)^3)/3
 - 4*tan(c/2 + (d*x)/2)^5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 +
 (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2))
```

3.81 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$\frac{x}{a^2} - \frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{a^2d}$$

```
[Out] x/a^2-2*arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3973, 3858, 3855, 3852, 8}

$$\frac{\tan(c+dx)}{a^2d} - \frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int (-a+a\sec(c+dx))^2\,dx}{a^4}\\ &= \frac{x}{a^2}+\frac{\int \sec^2(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\,dx}{a^2}\\ &= \frac{x}{a^2}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\operatorname{Subst}(\int 1\,dx, x, -\tan(c+dx))}{a^2d}\\ &= \frac{x}{a^2}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(34) = 68$.
time = 0.56, size = 177, normalized size = 5.21

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(dx+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c
/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.
time = 0.09, size = 80, normalized size = 2.35

method	result	size
risch	$\frac{x}{a^2}+\frac{2i}{d\,a^2\left(e^{2i(dx+c)}+1\right)}+\frac{2\ln\left(e^{i(dx+c)}-i\right)}{a^2d}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^2d}$	71
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}$	80
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*(1/4*arctan(tan(1/2*d*x+1/2*c))-1/8/(tan(1/2*d*x+1/2*c)-1)+1/4*ln(t
an(1/2*d*x+1/2*c)-1)-1/8/(tan(1/2*d*x+1/2*c)+1)-1/4*ln(tan(1/2*d*x+1/2*c)+1
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(34) = 68$.
time = 0.49, size = 123, normalized size = 3.62

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + sin(d*x
 + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))
/d
```

Fricas [A]
time = 3.83, size = 66, normalized size = 1.94

$$\frac{dx\cos(dx+c)-\cos(dx+c)\log(\sin(dx+c)+1)+\cos(dx+c)\log(-\sin(dx+c)+1)+\sin(dx+c)}{a^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) - cos(d*x + c)*log(sin(d*x + c) + 1) + cos(d*x + c)*log(-
sin(d*x + c) + 1) + sin(d*x + c))/(a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.
time = 1.13, size = 79, normalized size = 2.32

$$\frac{\frac{dx+c}{a^2}-\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 2*log(abs(tan(1
/2*d*x + 1/2*c) - 1))/a^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^2))/d
```

Mupad [B]
time = 1.19, size = 61, normalized size = 1.79

$$\frac{x}{a^2}-\frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] x/a^2 - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2*tan(c/2 + (d*x)/2))/(d*(
a^2*tan(c/2 + (d*x)/2)^2 - a^2))
```

3.82 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=33

$$-\frac{x}{a^2}+\frac{2\tan(c+dx)}{ad(a+a\sec(c+dx))}$$

```
[Out] -x/a^2+2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3973, 3971, 3554, 8, 2686, 3852}

$$-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cot[c + d*x])/(a^2*d) + (2*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2\cot^2(c+dx)-2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx)\right)\,dx}{a^4}\\ &= \frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot(c+dx)\csc(c+dx)\,dx}{a^2}\\ &= -\frac{\cot(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}+\frac{2\text{Subst}(\int 1\,dx,x,\csc(c+dx))}{a^2d}\\ &= -\frac{x}{a^2}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.27

$$\frac{-\frac{2\text{ArcTan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*ArcTan[Tan[c/2 + (d*x)/2]])/d + (2*Tan[c/2 + (d*x)/2])/d)/a^2
```

Maple [A]
time = 0.10, size = 31, normalized size = 0.94

method	result	size
risch	$-\frac{x}{a^2}+\frac{4i}{a^2d\left(e^{i(dx+c)}+1\right)}$	30

derivativedivides	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	31
default	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(tan(1/2*d*x+1/2*c)-arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.47, size = 49, normalized size = 1.48

$$-\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*
x + c) + 1)))/d
```

Fricas [A]
time = 2.99, size = 42, normalized size = 1.27

$$-\frac{dx\cos\left(dx+c\right)+dx-2\sin\left(dx+c\right)}{a^2d\cos\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - 2*sin(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^2\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.63, size = 29, normalized size = 0.88

$$-\frac{\frac{dx+c}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -((d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/a^2)/d
```

Mupad [B]
time = 1.10, size = 22, normalized size = 0.67

$$\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
[Out] (2*(tan(c/2 + (d*x)/2) - (d*x)/2))/(a^2*d)
```

3.83 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}$$

```
[Out] -x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d+2*csc
(d*x+c)/a^2/d-4/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x
]^5)/(5*a^2*d) + (2*Csc[c + d*x])/(a^2*d) - (4*Csc[c + d*x]^3)/(3*a^2*d) +
(2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^6(c+dx)-2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c+dx)\right.}{a^4} \\
&= \frac{\int \cot^6(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^5(c+dx)\csc(c+}{a^2} \\
&= -\frac{\cot^5(c+dx)}{5a^2d}-\frac{\int \cot^4(c+dx)\,dx}{a^2}+\frac{\operatorname{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{2\mathrm{Sub}}{} \\
&= \frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{2\operatorname{Subst}\left(\int (1-2x^2+x^4)\right.}{a^2d} \\
&= -\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+dx}{3a^2d} \\
&= -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c}{3a}
\end{aligned}
$$

Mathematica [A]

time = 1.40, size = 149, normalized size = 1.39

$$\frac{\sec^2(c+dx)\left(-120dx\cos^4\left(\frac{1}{2}(c+dx)\right)-31\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos^3\left(\frac{1}{2}(c+dx)\right)\left(15\cot\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)+193\sec\left(\frac{c}{2}\right)\right)\sin\left(\frac{dx}{2}\right)-31\cos^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)+3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{30a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(-120*d*x*Cos[(c + d*x)/2]^4 - 31*Cos[(c + d*x)/2]*Sec[c/2]
*Sin[(d*x)/2] + Cos[(c + d*x)/2]^3*(15*Cot[(c + d*x)/2]*Csc[c/2] + 193*Sec[
c/2])*Sin[(d*x)/2] - 31*Cos[(c + d*x)/2]^2*Tan[c/2] + 3*Tan[(c + d*x)/2]))/
(30*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.11, size = 72, normalized size = 0.67

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$	72
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$	72
risch	$-\frac{x}{a^2}+\frac{4i\left(15\,e^{5i(dx+c)}+30\,e^{4i(dx+c)}+10\,e^{3i(dx+c)}-35\,e^{2i(dx+c)}-37\,e^{i(dx+c)}-13\right)}{15d\,a^2\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3+11*tan(1/2*d*x
+1/2*c)-16*arctan(tan(1/2*d*x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.48, size = 113, normalized size = 1.06

$$\frac{\frac{\frac{165\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{240\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{15\,(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 240*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2 - 15*(cos(d*x + c) + 1)/(a^2*sin(d*x + c)))/d
```

Fricas [A]
time = 2.70, size = 106, normalized size = 0.99

$$-\frac{26\cos(dx+c)^3+22\cos(dx+c)^2+15\left(dx\cos(dx+c)^2+2\,dx\cos(dx+c)+dx\right)\sin(dx+c)-17\cos(dx+c)-16}{15\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*(d*x*cos(d*x + c)^2 + 2*d
*x*cos(d*x + c) + d*x)*sin(d*x + c) - 17*cos(d*x + c) - 16)/((a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.49, size = 84, normalized size = 0.79

$$-\frac{\frac{120\,(dx+c)}{a^2}+\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-25\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+165\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a^2 + 15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 25*a^8*tan(1/2*d*x + 1/2*c)^3 + 165*a^8*tan(1/2*d*x + 1/2*
c))/a^10)/d
```

Mupad [B]
time = 1.40, size = 78, normalized size = 0.73

$$-\frac{x}{a^2}-\frac{\frac{26\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}-\frac{28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{17\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{60}-\frac{1}{40}}{a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] - x/a^2 - ((17*cos(c/2 + (d*x)/2)^2)/60 - (28*cos(c/2 + (d*x)/2)^4)/15 + (2
6*cos(c/2 + (d*x)/2)^6)/15 - 1/40)/(a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d
*x)/2))
```

3.84 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=139

$$\frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{6\csc^5(c+d}{5a^2d}$$

```
[Out] x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+1/5*cot(d*x+c)^5/a^2/d-2/7*co
t(d*x+c)^7/a^2/d-2*csc(d*x+c)/a^2/d+2*csc(d*x+c)^3/a^2/d-6/5*csc(d*x+c)^5/a
^2/d+2/7*csc(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + Cot[c + d*x]^5/(5
*a^2*d) - (2*Cot[c + d*x]^7)/(7*a^2*d) - (2*Csc[c + d*x])/(a^2*d) + (2*Csc[
c + d*x]^3)/(a^2*d) - (6*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*
a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^8(c+dx) - 2a^2\cot^7(c+dx)\csc(c+dx) + a^2\cot^6(c+dx)\csc^2(c+dx\right.}{a^4}$$
$$= \frac{\int \cot^8(c+dx)\,dx}{a^2} + \frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^7(c+dx)\csc(c+}{a^2}$$
$$= -\frac{\cot^7(c+dx)}{7a^2d} - \frac{\int \cot^6(c+dx)\,dx}{a^2} + \frac{\text{Subst}\left(\int x^6\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\text{Su}}{}$$
$$= \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} + \frac{\int \cot^4(c+dx)\,dx}{a^2} + \frac{2\text{Subst}\left(\int (-1+3x^2-\right.}{}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc(c+dx)}{a^2d} + \frac{2\csc^3(c+}{a^2d}$$
$$= \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc(c+dx)}{a^2d}$$
$$= \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc(c+}{a^2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(139) = 278.
time = 1.02, size = 314, normalized size = 2.26

$$\frac{\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^2(c+dx)(5880dx\cos(dx) - 5880dx\cos(2c+dx) + 3360dx\cos(c+2dx) - 3360dx\cos(3c+2dx) - 1260dx\cos(2c+3dx) + 1260dx\cos(4c+3dx) - 1680dx\cos(3c+4dx) + 1680dx\cos(5c+4dx) - 420dx\cos(4c+5dx) + 420dx\cos(6c+5dx) - 4032\sin(c) - 9632\sin(dx) + 16002\sin(c+dx) + 9144\sin(2(c+dx)) - 3429\sin(3(c+dx)) - 4572\sin(4(c+dx)) - 1143\sin(5(c+dx)) - 11760\sin(2c+dx) - 8864\sin(c+2dx) - 3360\sin(3c+2dx) + 2064\sin(2c+3dx) + 2520\sin(4c+3dx) + 4432\sin(3c+4dx) + 1680\sin(5c+4dx) + 1528\sin(4c+5dx))}{26880a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]^2*(5880*d*x*Cos[d*x] - 5880*
d*x*Cos[2*c + d*x] + 3360*d*x*Cos[c + 2*d*x] - 3360*d*x*Cos[3*c + 2*d*x] -
1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] - 1680*d*x*Cos[3*c +
4*d*x] + 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] + 420*d*x*Cos
[6*c + 5*d*x] - 4032*Sin[c] - 9632*Sin[d*x] + 16002*Sin[c + d*x] + 9144*Sin
[2*(c + d*x)] - 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] - 1143*Sin[5*
(c + d*x)] - 11760*Sin[2*c + d*x] - 8864*Sin[c + 2*d*x] - 3360*Sin[3*c + 2*
d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 4432*Sin[3*c + 4*d*x
] + 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(26880*a^2*d*(1 + Sec[c
 + d*x])^2)
```

Maple [**A**]
time = 0.12, size = 98, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{22\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{}{\tan\left(\frac{d}{}\right.}}{32d\,a^2}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{22\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{}{\tan\left(\frac{d}{}\right.}}{32d\,a^2}$
risch	$\frac{x}{a^2}-\frac{2i\left(210\,e^{9i(dx+c)}+315\,e^{8i(dx+c)}-420\,e^{7i(dx+c)}-1470\,e^{6i(dx+c)}-504\,e^{5i(dx+c)}+1204\,e^{4i(dx+c)}+1108\,e^{3i(dx+c)}-25\right.}{105d\,a^2\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7-7/5*tan(1/2*d*x+1/2*c)^5+22/3*tan(1/2*
d*x+1/2*c)^3-42*tan(1/2*d*x+1/2*c)+64*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(1/
2*d*x+1/2*c)^3+7/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.48, size = 157, normalized size = 1.13

$$-\frac{\frac{\frac{4410\sin(dx+c)}{\cos(dx+c)+1}-\frac{770\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{35\left(\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((4410*sin(d*x + c)/(cos(d*x + c) + 1) - 770*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^2 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^2 - 35*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(
a^2*sin(d*x + c)^3))/d
```

Fricas [A]
time = 3.05, size = 154, normalized size = 1.11

$$\frac{191\cos(dx+c)^5+172\cos(dx+c)^4-253\cos(dx+c)^3-258\cos(dx+c)^2+105\left(dx\cos(dx+c)^4+2\,dx\cos(dx+c)^3-2\,dx\cos(dx+c)-dx\right)\sin(dx+c)+87\cos(dx+c)+96}{105\left(a^2d\cos(dx+c)^4+2\,a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*cos(d*x + c)^5 + 172*cos(d*x + c)^4 - 253*cos(d*x + c)^3 - 258*c
os(d*x + c)^2 + 105*(d*x*cos(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(
d*x + c) - d*x)*sin(d*x + c) + 87*cos(d*x + c) + 96)/((a^2*d*cos(d*x + c)^4
 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.52, size = 114, normalized size = 0.82

$$\frac{\frac{3360\,(dx+c)}{a^2}+\frac{35\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{15\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+770\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4410\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{14}}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(d*x + c)/a^2 + 35*(21*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/
2*d*x + 1/2*c)^3) + (15*a^12*tan(1/2*d*x + 1/2*c)^7 - 147*a^12*tan(1/2*d*x
+ 1/2*c)^5 + 770*a^12*tan(1/2*d*x + 1/2*c)^3 - 4410*a^12*tan(1/2*d*x + 1/2*
c))/a^14)/d
```

Mupad [B]
time = 1.65, size = 182, normalized size = 1.31

$$\frac{15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-147\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+770\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4410\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+735\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3360\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(c+d\,x)}{3360\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (15*sin(c/2 + (d*x)/2)^10 - 35*cos(c/2 + (d*x)/2)^10 - 147*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^8 + 770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 -
 4410*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 735*cos(c/2 + (d*x)/2)^8*
sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3*(c +
d*x))/(3360*a^2*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)
```

3.85 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+dx}{3a^2d}$$

```
[Out] -x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a^2/d+1/7*c
ot(d*x+c)^7/a^2/d-2/9*cot(d*x+c)^9/a^2/d+2*csc(d*x+c)/a^2/d-8/3*csc(d*x+c)^
3/a^2/d+12/5*csc(d*x+c)^5/a^2/d-8/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2
/d
```

Rubi [A]
time = 0.16, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5
/(5*a^2*d) + Cot[c + d*x]^7/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) + (2*C
sc[c + d*x])/(a^2*d) - (8*Csc[c + d*x]^3)/(3*a^2*d) + (12*Csc[c + d*x]^5)/(
5*a^2*d) - (8*Csc[c + d*x]^7)/(7*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^{10}(c+dx) - 2a^2\cot^9(c+dx)\csc(c+dx) + a^2\cot^8(c+dx)\csc^2(c+dx\right.}{a^4}$$

$$= \frac{\int \cot^{10}(c+dx)\,dx}{a^2} + \frac{\int \cot^8(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^9(c+dx)\csc(c+}{a^2}$$

$$= -\frac{\cot^9(c+dx)}{9a^2d} - \frac{\int \cot^8(c+dx)\,dx}{a^2} + \frac{\text{Subst}\left(\int x^8\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\text{Sub}}{}$$

$$= \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + \frac{\int \cot^6(c+dx)\,dx}{a^2} + \frac{2\text{Subst}\left(\int (1-4x^2+6x^4\right.}{}$$

$$= -\frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + \frac{2\csc(c+dx)}{a^2d} - \frac{8\csc^3(c+d}{3a^2d}$$

$$= \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} + \frac{2\csc(c+dx)}{a^2d}$$

$$= -\frac{\cot(c+dx)}{a^2d} + \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d}$$

$$= -\frac{x}{a^2} - \frac{\cot(c+dx)}{a^2d} + \frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5a^2d} + \frac{\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+}{9a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 802 vs. 2(179) = 358.
time = 6.60, size = 802, normalized size = 4.48

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 + (17*Cos
[c/2 + (d*x)/2]^2*Cot[c/2]*Cot[c/2 + (d*x)/2]^2*Sec[c + d*x]^2)/(160*d*(a +
 a*Sec[c + d*x])^2) - (Cot[c/2]*Cot[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(160*d
*(a + a*Sec[c + d*x])^2) + (201*Cos[c/2 + (d*x)/2]^3*Cot[c/2 + (d*x)/2]*Csc
[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(160*d*(a + a*Sec[c + d*x])^2) - (17*Cos
[c/2 + (d*x)/2]*Cot[c/2 + (d*x)/2]^3*Csc[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/
(160*d*(a + a*Sec[c + d*x])^2) + (Cot[c/2 + (d*x)/2]^4*Csc[c/2]*Csc[c/2 + (
d*x)/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(160*d*(a + a*Sec[c + d*x])^2) - (7891
*Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c + d*x]^2*Sin[(d*x)/2])/(5040*d*(a + a*Se
c[c + d*x])^2) + (63881*Cos[c/2 + (d*x)/2]^3*Sec[c/2]*Sec[c + d*x]^2*Sin[(d
*x)/2])/(10080*d*(a + a*Sec[c + d*x])^2) + (313*Sec[c/2]*Sec[c/2 + (d*x)/2]
*Sec[c + d*x]^2*Sin[(d*x)/2])/(840*d*(a + a*Sec[c + d*x])^2) - (109*Sec[c/2
```

```
]*Sec[c/2 + (d*x)/2]^3*Sec[c + d*x]^2*Sin[(d*x)/2])/(2016*d*(a + a*Sec[c +
d*x])^2) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sec[c + d*x]^2*Sin[(d*x)/2])/(288
*d*(a + a*Sec[c + d*x])^2) + (313*Sec[c + d*x]^2*Tan[c/2])/(840*d*(a + a*Se
c[c + d*x])^2) - (7891*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]^2*Tan[c/2])/(5040*
d*(a + a*Sec[c + d*x])^2) - (109*Sec[c/2 + (d*x)/2]^2*Sec[c + d*x]^2*Tan[c/
2])/(2016*d*(a + a*Sec[c + d*x])^2) + (Sec[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*
Tan[c/2])/(288*d*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 0.13, size = 124, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{9\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{37\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-31\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{128d\,a^{2}}$
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{9\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{37\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-31\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{128d\,a^{2}}$
risch	$-\frac{x}{a^{2}}+\frac{4i\left(315\,e^{13i(dx+c)}+315\,e^{12i(dx+c)}-1470\,e^{11i(dx+c)}-3360\,e^{10i(dx+c)}+1113\,e^{9i(dx+c)}+6447\,e^{8i(dx+c)}+2028\,e^{7}\right.}{315d\,a^{2}\left(e^{i(dx+c)}+1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9-9/7*tan(1/2*d*x+1/2*c)^7+37/5*tan(1/2
*d*x+1/2*c)^5-31*tan(1/2*d*x+1/2*c)^3+163*tan(1/2*d*x+1/2*c)-256*arctan(tan
(1/2*d*x+1/2*c))-1/5/tan(1/2*d*x+1/2*c)^5+3/tan(1/2*d*x+1/2*c)^3-37/tan(1/2
*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 197, normalized size = 1.10

$$\frac{\frac{\frac{51345\sin(dx+c)}{\cos(dx+c)+1}-\frac{9765\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2331\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{405\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{2}}-\frac{80640\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}+\frac{63\left(\frac{15\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{185\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-1\right)(\cos(dx+c)+1)^{5}}{a^{2}\sin(dx+c)^{5}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/40320*((51345*sin(d*x + c)/(cos(d*x + c) + 1) - 9765*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 2331*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 405*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 - 8
0640*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 63*(15*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 185*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x
+ c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Fricas [A]
time = 2.45, size = 250, normalized size = 1.40

$$-\frac{598\cos(dx+c)^{7}+566\cos(dx+c)^{6}-1212\cos(dx+c)^{5}-1310\cos(dx+c)^{4}+860\cos(dx+c)^{3}+1014\cos(dx+c)^{2}+315\left(dx\cos(dx+c)^{6}+2\,dx\cos(dx+c)^{5}-dx\cos(dx+c)^{4}-4\,dx\cos(dx+c)^{3}-dx\cos(dx+c)^{2}+2\,dx\cos(dx+c)+dx\right)\sin(dx+c)-197\cos(dx+c)-256}{315\left(a^{2}d\cos(dx+c)^{6}+2\,a^{2}d\cos(dx+c)^{5}-a^{2}d\cos(dx+c)^{4}-4\,a^{2}d\cos(dx+c)^{3}-a^{2}d\cos(dx+c)^{2}+2\,a^{2}d\cos(dx+c)+a^{2}d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*cos(d*x + c)^7 + 566*cos(d*x + c)^6 - 1212*cos(d*x + c)^5 - 131
0*cos(d*x + c)^4 + 860*cos(d*x + c)^3 + 1014*cos(d*x + c)^2 + 315*(d*x*cos(
d*x + c)^6 + 2*d*x*cos(d*x + c)^5 - d*x*cos(d*x + c)^4 - 4*d*x*cos(d*x + c)
^3 - d*x*cos(d*x + c)^2 + 2*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 197*cos(
d*x + c) - 256)/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos
(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.53, size = 144, normalized size = 0.80

$$-\frac{\frac{40320\,(dx+c)}{a^2}+\frac{63\,\left(185\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{35\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-405\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2331\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9765\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+51345\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{18}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(40320*(d*x + c)/a^2 + 63*(185*tan(1/2*d*x + 1/2*c)^4 - 15*tan(1/2
*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x +
1/2*c)^9 - 405*a^16*tan(1/2*d*x + 1/2*c)^7 + 2331*a^16*tan(1/2*d*x + 1/2*c)
^5 - 9765*a^16*tan(1/2*d*x + 1/2*c)^3 + 51345*a^16*tan(1/2*d*x + 1/2*c))/a^
18)/d
```

Mupad [B]
time = 2.51, size = 230, normalized size = 1.28

$$-\frac{63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+405\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-2331\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+9765\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-51345\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+11655\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-945\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+40320\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(c+d\,x)}{40320\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(63*cos(c/2 + (d*x)/2)^14 - 35*sin(c/2 + (d*x)/2)^14 + 405*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^12 - 2331*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
```

```
10 + 9765*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 51345*cos(c/2 + (d*x)
/2)^8*sin(c/2 + (d*x)/2)^6 + 11655*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^4 - 945*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5*(c + d*x))/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin
(c/2 + (d*x)/2)^5)
```

3.86 $\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=137

$$\frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^2(c+dx)}{2a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}+\frac{\sec^5(c+dx)}{5a^3d}-\frac{\sec^6(c+dx)}{2a^3d}$$

```
[Out] ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-1/2*sec(d*x+c)^2/a^3/d-5/3*sec(d*x+
c)^3/a^3/d+5/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d-1/2*sec(d*x+c)^6/a
^3/d+1/7*sec(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{\sec^7(c+dx)}{7a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}-\frac{\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - Sec[c + d*x]^2/(2*a^
3*d) - (5*Sec[c + d*x]^3)/(3*a^3*d) + (5*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
+ d*x]^5/(5*a^3*d) - Sec[c + d*x]^6/(2*a^3*d) + Sec[c + d*x]^7/(7*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^5(a+ax)^2}{x^8}\,dx, x, \cos(c+dx)\right)}{a^{10}d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{3a^7}{x^7}+\frac{a^7}{x^6}+\frac{5a^7}{x^5}-\frac{5a^7}{x^4}-\frac{a^7}{x^3}+\frac{3a^7}{x^2}-\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^{10}d}$$
$$= \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^2(c+dx)}{2a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}+\frac{5\sec^4(c-}{4a^3c}$$

Mathematica [A]
time = 0.31, size = 140, normalized size = 1.02

$$\frac{(3732+4522\cos(2(c+dx))+1050\cos(3(c+dx))+2380\cos(4(c+dx))-210\cos(5(c+dx))+630\cos(6(c+dx))+2205\cos(3(c+dx))\log(\cos(c+dx))+735\cos(5(c+dx))\log(\cos(c+dx))+105\cos(7(c+dx))\log(\cos(c+dx))+105\cos(c+dx)(8+35\log(\cos(c+dx))))\sec^7(c+dx)}{6720a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((3732 + 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] + 2380*Cos[4*(c + d*
x)] - 210*Cos[5*(c + d*x)] + 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d*x)]*L
og[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7*(c +
d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]]))*Sec[
c + d*x]^7)/(6720*a^3*d)
```

Maple [A]
time = 0.17, size = 85, normalized size = 0.62

method	result
derivativedivides	$-\frac{-\frac{\left(\sec^7(dx+c)\right)}{7}+\frac{\left(\sec^6(dx+c)\right)}{2}-\frac{\left(\sec^5(dx+c)\right)}{5}-\frac{5\left(\sec^4(dx+c)\right)}{4}+\frac{5\left(\sec^3(dx+c)\right)}{3}+\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\ln(\sec(dx}{d\,a^3}$
default	$-\frac{-\frac{\left(\sec^7(dx+c)\right)}{7}+\frac{\left(\sec^6(dx+c)\right)}{2}-\frac{\left(\sec^5(dx+c)\right)}{5}-\frac{5\left(\sec^4(dx+c)\right)}{4}+\frac{5\left(\sec^3(dx+c)\right)}{3}+\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\ln(\sec(dx}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{6\,e^{13i(dx+c)}-2\,e^{12i(dx+c)}+\frac{68\,e^{11i(dx+c)}}{3}+10\,e^{10i(dx+c)}+\frac{646\,e^{9i(dx+c)}}{15}+8\,e^{8i(dx+c)}+\frac{2488\,e^{7i(dx+c)}}{35}+}{d\,a^3\left(e^{2i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^3*(-1/7*sec(d*x+c)^7+1/2*sec(d*x+c)^6-1/5*sec(d*x+c)^5-5/4*sec(d*x+c
)^4+5/3*sec(d*x+c)^3+1/2*sec(d*x+c)^2-3*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 90, normalized size = 0.66

$$\frac{\frac{420\log(\cos(dx+c))}{a^3}+\frac{1260\cos(dx+c)^6-210\cos(dx+c)^5-700\cos(dx+c)^4+525\cos(dx+c)^3+84\cos(dx+c)^2-210\cos(dx+c)+60}{a^3\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(420*log(cos(d*x + c))/a^3 + (1260*cos(d*x + c)^6 - 210*cos(d*x + c)^
5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2 - 210*cos(d
*x + c) + 60)/(a^3*cos(d*x + c)^7))/d
```

Fricas [A]
time = 2.61, size = 95, normalized size = 0.69

$$\frac{420 \cos(dx+c)^7 \log(-\cos(dx+c)) + 1260 \cos(dx+c)^6 - 210 \cos(dx+c)^5 - 700 \cos(dx+c)^4 + 525 \cos(dx+c)^3 + 84 \cos(dx+c)^2 - 210 \cos(dx+c) + 60}{420\, a^3 d \cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 1260*cos(d*x + c)^6 - 210*co
s(d*x + c)^5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2
- 210*cos(d*x + c) + 60)/(a^3*d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{11}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**11/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Giac [A]
time = 18.01, size = 246, normalized size = 1.80

$$-\frac{\frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{1393(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{819(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{6755(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{20195(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \frac{28749(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{8463(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} - \frac{1089(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7} + 319}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^7}}{420\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 420*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (1393*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 819*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 6755*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 20195*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*(cos(d*x + c) - 1)^
```

```
7/(cos(d*x + c) + 1)^7 + 319)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^7))/d
```

Mupad [B]

time = 5.29, size = 225, normalized size = 1.64

$$-\frac{2\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}-\frac{-2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+14\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-\frac{224\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{282\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}-\frac{322\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{352}{105}}{d\,\left(a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+35\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+7\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((282*tan(c/2 + (d*x)/2)^4)/5 -
 (322*tan(c/2 + (d*x)/2)^2)/15 - (224*tan(c/2 + (d*x)/2)^6)/3 + (128*tan(c/
2 + (d*x)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 35
2/105)/(d*(7*a^3*tan(c/2 + (d*x)/2)^2 - 21*a^3*tan(c/2 + (d*x)/2)^4 + 35*a^
3*tan(c/2 + (d*x)/2)^6 - 35*a^3*tan(c/2 + (d*x)/2)^8 + 21*a^3*tan(c/2 + (d*
x)/2)^10 - 7*a^3*tan(c/2 + (d*x)/2)^12 + a^3*tan(c/2 + (d*x)/2)^14 - a^3))
```

3.87 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3} \, dx$

Optimal. Leaf size=99

$$-\frac{\log(\cos(c+dx))}{a^3d}-\frac{3\sec(c+dx)}{a^3d}+\frac{\sec^2(c+dx)}{a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}-\frac{3\sec^4(c+dx)}{4a^3d}+\frac{\sec^5(c+dx)}{5a^3d}$$

```
[Out] -ln(cos(d*x+c))/a^3/d-3*sec(d*x+c)/a^3/d+sec(d*x+c)^2/a^3/d+2/3*sec(d*x+c)^
3/a^3/d-3/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 76}

$$\frac{\sec^5(c+dx)}{5a^3d}-\frac{3\sec^4(c+dx)}{4a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}+\frac{\sec^2(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^3*d)) - (3*Sec[c + d*x])/(a^3*d) + Sec[c + d*x]^2/(a
^3*d) + (2*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
 + d*x]^5/(5*a^3*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)}{x^6}\,dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6} - \frac{3a^5}{x^5} + \frac{2a^5}{x^4} + \frac{2a^5}{x^3} - \frac{3a^5}{x^2} + \frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{\log(\cos(c+dx))}{a^3 d} - \frac{3\sec(c+dx)}{a^3 d} + \frac{\sec^2(c+dx)}{a^3 d} + \frac{2\sec^3(c+dx)}{3a^3 d} - \frac{3\sec^4(}{4a}$$

Mathematica [**A**]
time = 0.36, size = 93, normalized size = 0.94

$$-\frac{(142+280\cos(2(c+dx))+90\cos(4(c+dx))+150\cos(c+dx)\log(\cos(c+dx))+15\cos(5(c+dx))\log(\cos(c+dx))+15\cos(3(c+dx))(-4+5\log(\cos(c+dx))))\sec^5(c+dx)}{240a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/240*((142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + 150*Cos[c + d*x
]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[3*(c +
 d*x)]*(-4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(a^3*d)
```

Maple [**A**]
time = 0.12, size = 62, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{(\sec^5(dx+c))}{5} - \frac{3(\sec^4(dx+c))}{4} + \frac{2(\sec^3(dx+c))}{3} + \sec^2(dx+c) - 3\sec(dx+c) + \ln(\sec(dx+c))}{d\,a^3}$
default	$\frac{\frac{(\sec^5(dx+c))}{5} - \frac{3(\sec^4(dx+c))}{4} + \frac{2(\sec^3(dx+c))}{3} + \sec^2(dx+c) - 3\sec(dx+c) + \ln(\sec(dx+c))}{d\,a^3}$
risch	$\frac{ix}{a^3} + \frac{2ic}{a^3 d} - \frac{2(45\,e^{9i(dx+c)} - 30\,e^{8i(dx+c)} + 140\,e^{7i(dx+c)} + 142\,e^{5i(dx+c)} + 140\,e^{3i(dx+c)} - 30\,e^{2i(dx+c)} + 45\,e^{i(dx+c)})}{15d\,a^3(e^{2i(dx+c)}+1)^5} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/5*sec(d*x+c)^5-3/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3+sec(d*x+c)^2-3*
sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [**A**]
time = 0.27, size = 70, normalized size = 0.71

$$-\frac{\frac{60\,\log(\cos(dx+c))}{a^3} + \frac{180\,\cos(dx+c)^4 - 60\,\cos(dx+c)^3 - 40\,\cos(dx+c)^2 + 45\,\cos(dx+c) - 12}{a^3\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^3 + (180*cos(d*x + c)^4 - 60*cos(d*x + c)^3 -
 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*cos(d*x + c)^5))/d
```

Fricas [A]
time = 2.53, size = 75, normalized size = 0.76

$$-\frac{60\cos(dx+c)^5\log(-\cos(dx+c))+180\cos(dx+c)^4-60\cos(dx+c)^3-40\cos(dx+c)^2+45\cos(dx+c)-12}{60\,a^3d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 180*cos(d*x + c)^4 - 60*cos(d
*x + c)^3 - 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*d*cos(d*x + c)^5
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^9(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(93) = 186.
time = 5.89, size = 202, normalized size = 2.04

$$\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{475(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{590(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{50(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{805(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+119}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (475*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 590*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 50
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*(cos(d*x + c) - 1)^4/(cos(
```

```
d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 119)/(a^3
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

Mupad [B]
time = 5.92, size = 167, normalized size = 1.69

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{98\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{58\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{64}{15}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((58*tan(c/2 + (d*x)/2)^2)/3 - (9
8*tan(c/2 + (d*x)/2)^4)/3 + 22*tan(c/2 + (d*x)/2)^6 + 2*tan(c/2 + (d*x)/2)^
8 - 64/15)/(d*(5*a^3*tan(c/2 + (d*x)/2)^2 - 10*a^3*tan(c/2 + (d*x)/2)^4 + 1
0*a^3*tan(c/2 + (d*x)/2)^6 - 5*a^3*tan(c/2 + (d*x)/2)^8 + a^3*tan(c/2 + (d*
x)/2)^10 - a^3))
```

3.88 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\, dx$

Optimal. Leaf size=65

$$\frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-3/2*sec(d*x+c)^2/a^3/d+1/3*sec(d*x+
c)^3/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$\frac{\sec^3(c+dx)}{3a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - (3*Sec[c + d*x]^2)/(
2*a^3*d) + Sec[c + d*x]^3/(3*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(a-ax)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4} - \frac{3a^3}{x^3} + \frac{3a^3}{x^2} - \frac{a^3}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\cos(c+dx))}{a^3 d} + \frac{3\sec(c+dx)}{a^3 d} - \frac{3\sec^2(c+dx)}{2a^3 d} + \frac{\sec^3(c+dx)}{3a^3 d}$$

Mathematica [A]
time = 0.18, size = 64, normalized size = 0.98

$$\frac{(22+18\cos(2(c+dx))+9\cos(c+dx)(-2+\log(\cos(c+dx)))+3\cos(3(c+dx))\log(\cos(c+dx)))\sec^3(c+dx)}{12a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
[Out] ((22 + 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*x]]) + 3*Co
s[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/(12*a^3*d)
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{-\frac{\left(\sec^3(dx+c)\right)}{3}+\frac{3\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^3}$	45
default	$-\frac{-\frac{\left(\sec^3(dx+c)\right)}{3}+\frac{3\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\ln(\sec(dx+c))}{d\,a^3}$	45
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3 d}+\frac{6\,e^{5i(dx+c)}-6\,e^{4i(dx+c)}+\frac{44\,e^{3i(dx+c)}}{3}-6\,e^{2i(dx+c)}+6\,e^{i(dx+c)}}{d\,a^3\left(e^{2i(dx+c)}+1\right)^3}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^3 d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/d/a^3*(-1/3*sec(d*x+c)^3+3/2*sec(d*x+c)^2-3*sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.77

$$\frac{\frac{6\,\log(\cos(dx+c))}{a^3}+\frac{18\,\cos(dx+c)^2-9\,\cos(dx+c)+2}{a^3\,\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(cos(d*x + c))/a^3 + (18*cos(d*x + c)^2 - 9*cos(d*x + c) + 2)/(a^
3*cos(d*x + c)^3))/d
```

Fricas [A]
time = 3.67, size = 55, normalized size = 0.85

$$\frac{6\cos(dx+c)^3\log(-\cos(dx+c)) + 18\cos(dx+c)^2 - 9\cos(dx+c) + 2}{6a^3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*cos(d*x + c)^2 - 9*cos(d*x +
c) + 2)/(a^3*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(61) = 122.
time = 3.72, size = 158, normalized size = 2.43

$$-\frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{75(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{51(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + 29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 6*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (75*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 51*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 11*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 29)/(a^3*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^3))/d
```

Mupad [B]
time = 2.03, size = 109, normalized size = 1.68

$$-\frac{14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{20}{3}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - (14*tan(c/2 + (d*x)/2)^4 - 18*tan(c/2 + (d*x)/2)^2 + 20/3)/(d*(3*a^3*tan(
c/2 + (d*x)/2)^2 - 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2 + (d*x)/2)^6 -
a^3)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d)
```

3.89 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=46

$$\frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(1+\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}$$

```
[Out] 3*ln(cos(d*x+c))/a^3/d-4*ln(1+cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{\sec(c+dx)}{a^3d}+\frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Log[Cos[c + d*x]])/(a^3*d) - (4*Log[1 + Cos[c + d*x]])/(a^3*d) + Sec[c +
 d*x]/(a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^2(a+ax)}\,dx,x,\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3a}{x}+\frac{4a}{1+x}\right)\,dx,x,\cos(c+dx)\right)}{a^4d}\\ &= \frac{3\log(\cos(c+dx))}{a^3d}-\frac{4\log(1+\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 36, normalized size = 0.78

$$\frac{-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log(\cos(c+dx))+\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-8*Log[Cos[(c + d*x)/2]] + 3*Log[Cos[c + d*x]] + Sec[c + d*x])/(a^3*d)
```

Maple [A]
time = 0.09, size = 33, normalized size = 0.72

method	result	size
derivativedivides	$\frac{\sec(dx+c)-4\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^3}$	33
default	$\frac{\sec(dx+c)-4\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^3}$	33
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}+\frac{2\,e^{i(dx+c)}}{d\,a^3\left(e^{2i(dx+c)}+1\right)}-\frac{8\ln\left(e^{i(dx+c)}+1\right)}{a^3d}+\frac{3\ln\left(e^{2i(dx+c)}+1\right)}{a^3d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(sec(d*x+c)-4*ln(1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 45, normalized size = 0.98

$$-\frac{\frac{4\log(\cos(dx+c)+1)}{a^3}-\frac{3\log(\cos(dx+c))}{a^3}-\frac{1}{a^3\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(cos(d*x + c) + 1)/a^3 - 3*log(cos(d*x + c))/a^3 - 1/(a^3*cos(d*x +
c)))/d
```

Fricas [A]
time = 3.73, size = 53, normalized size = 1.15

$$\frac{3\cos(dx+c)\log(-\cos(dx+c))-4\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (3*cos(d*x + c)*log(-cos(d*x + c)) - 4*cos(d*x + c)*log(1/2*cos(d*x + c) +
1/2) + 1)/(a^3*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(46) = 92.
time = 1.81, size = 112, normalized size = 2.43

$$\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1}+1}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + 3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 1.27, size = 72, normalized size = 1.57

$$\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{a^3\,d}-\frac{2}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^3\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*log(tan(c/2 + (d*x)/2)^2 - 1))/(a^3*d) - 2/(d*(a^3*tan(c/2 + (d*x)/2)^2
- a^3)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d)
```

3.90 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=35

$$\frac{2}{a^3d(1+\cos(c+dx))}+\frac{\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] 2/a^3/d/(1+cos(d*x+c))+ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3964, 45}

$$\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 2/(a^3*d*(1 + Cos[c + d*x])) + Log[1 + Cos[c + d*x]]/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{a-ax}{(a+ax)^2}\,dx,x,\cos(c+dx)\right)}{a^2d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{2}{a(1+x)^2}-\frac{1}{a(1+x)}\right)\,dx,x,\cos(c+dx)\right)}{a^2d}\\
&= \frac{2}{a^3d(1+\cos(c+dx))}+\frac{\log(1+\cos(c+dx))}{a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 33, normalized size = 0.94

$$\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\tan^2\left(\frac{1}{2}(c+dx)\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
[Out] (2*Log[Cos[(c + d*x)/2]] + Tan[(c + d*x)/2]^2)/(a^3*d)
```

Maple [A]
time = 0.11, size = 40, normalized size = 1.14

method	result	size
derivativedivides	$-\frac{\frac{2}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^3}$	40
default	$-\frac{\frac{2}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^3}$	40
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{4\,e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^2}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/d/a^3*(2/(1+sec(d*x+c))-ln(1+sec(d*x+c))+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.03

$$\frac{\frac{2}{a^3\cos(dx+c)+a^3}+\frac{\log(\cos(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] (2/(a^3*cos(d*x + c) + a^3) + log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 3.01, size = 42, normalized size = 1.20

$$\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{a^3d\cos(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 2)/(a^3*d*cos(d*x + c) +
a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(29) = 58$.
time = 11.44, size = 457, normalized size = 13.06

$$\begin{cases} -\frac{\log(\tan^2(c+dx)+1)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2\log(\tan^2(c+dx)+1)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{\log(\tan^2(c+dx)+1)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{2\log(\sec(c+dx)+1)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{4\log(\sec(c+dx)+1)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{2\log(\sec(c+dx)+1)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{\tan^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} & \text{for } d \neq 0 \\ \frac{x\tan^3(c)}{(a\sec(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)
**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(tan(c + d*x)**2 + 1)*sec(c
+ d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - log(
tan(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*
a**3*d) + 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 4*log(sec(c + d*x) + 1)*sec(c + d*x)
/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(sec(
c + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
 + tan(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a*
*3*d) - 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) +
2*a**3*d) - 2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*tan(c)**3/(a*sec(c) + a)**3, True))
```

Giac [A]
time = 0.90, size = 56, normalized size = 1.60

$$-\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\cos(dx+c)-1}{a^3(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (cos(d*x + c)
- 1)/(a^3*(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 1.17, size = 36, normalized size = 1.03

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2)/(a^3*d)
```

3.91 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=56

$$\frac{1}{2a^3d(1+\cos(c+dx))^2}-\frac{2}{a^3d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] 1/2/a^3/d/(1+cos(d*x+c))^2-2/a^3/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 45}

$$-\frac{2}{a^3d(\cos(c+dx)+1)}+\frac{1}{2a^3d(\cos(c+dx)+1)^2}-\frac{\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(2*a^3*d*(1 + Cos[c + d*x])^2) - 2/(a^3*d*(1 + Cos[c + d*x])) - Log[1 + C
os[c + d*x]]/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3(1+x)^3} - \frac{2}{a^3(1+x)^2} + \frac{1}{a^3(1+x)}\right)\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{1}{2a^3d(1+\cos(c+dx))^2} - \frac{2}{a^3d(1+\cos(c+dx))} - \frac{\log(1+\cos(c+dx))}{a^3d}$$

Mathematica [A]
time = 0.19, size = 77, normalized size = 1.38

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 8\sec^2\left(\frac{1}{2}(c+dx)\right) + \sec^4\left(\frac{1}{2}(c+dx)\right)\right)\sec^3(c+dx)}{a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-16*Log[Cos[(c + d*x)/2]] - 8*Sec[(c + d*x)/2]^2 + Sec
[(c + d*x)/2]^4)*Sec[c + d*x]^3)/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.05, size = 49, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{1}{2(1+\sec(dx+c))^2}+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^3}$	49
default	$\frac{\frac{1}{2(1+\sec(dx+c))^2}+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{d\,a^3}$	49
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{2\left(2\,\mathrm{e}^{3i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^4}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/2/(1+sec(d*x+c))^2+1/(1+sec(d*x+c))-ln(1+sec(d*x+c))+ln(sec(d*x+
c)))
```

Maxima [A]
time = 0.28, size = 60, normalized size = 1.07

$$-\frac{\frac{4\,\cos(dx+c)+3}{a^3\cos(dx+c)^2+2\,a^3\cos(dx+c)+a^3} + \frac{2\,\log(\cos(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*cos(d*x + c) + 3)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3)
+ 2*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 2.63, size = 76, normalized size = 1.36

$$-\frac{2\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+4\cos\left(dx+c\right)+3}{2\left(a^{3}d\cos\left(dx+c\right)^{2}+2a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
 4*cos(d*x + c) + 3)/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(48) = 96.
time = 11.13, size = 411, normalized size = 7.34

$$\begin{cases} \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)\sec^{2}\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}+\frac{2\log\left(\tan^{2}\left(c+dx\right)+1\right)\sec\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}+\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}-\frac{2\log\left(\sec\left(c+dx\right)+1\right)\sec^{2}\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}-\frac{4\log\left(\sec\left(c+dx\right)+1\right)\sec\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}-\frac{2\log\left(\sec\left(c+dx\right)+1\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}+\frac{2\sec\left(c+dx\right)}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d}+\frac{3}{2a^{3}d\sec^{2}\left(c+dx\right)+4a^{3}d\sec\left(c+dx\right)+2a^{3}d} & \text{for}\: d \neq 0 \\ \frac{x\tan\left(c\right)}{\left(a\sec\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)*
*2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(tan(c + d*x)**2 + 1)*sec(c +
 d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + log(t
an(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a
**3*d) - 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
+ 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 4*log(sec(c + d*x) + 1)*sec(c + d*x)/
(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(sec(c
 + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
+ 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3
*d) + 3/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d), Ne(d
, 0)), (x*tan(c)/(a*sec(c) + a)**3, True))
```

Giac [A]
time = 0.58, size = 87, normalized size = 1.55

$$\frac{\frac{8\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{a^{3}}+\frac{\frac{6a^{3}\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{a^{3}\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}}{a^{6}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (6*a^3*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2)/a^6)/d
```

Mupad [B]
time = 1.15, size = 48, normalized size = 0.86

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1) - (3*tan(c/2 + (d*x)/2)^2)/4 + tan(c/2 + (d*
x)/2)^4/8)/(a^3*d)
```

3.92 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=101

$$\frac{1}{6a^3d(1+\cos(c+dx))^3}-\frac{7}{8a^3d(1+\cos(c+dx))^2}+\frac{17}{8a^3d(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(1+\cos(c+dx))}{16a^3d}$$

```
[Out] 1/6/a^3/d/(1+cos(d*x+c))^3-7/8/a^3/d/(1+cos(d*x+c))^2+17/8/a^3/d/(1+cos(d*x
+c))+1/16*ln(1-cos(d*x+c))/a^3/d+15/16*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\frac{17}{8a^3d(\cos(c+dx)+1)}-\frac{7}{8a^3d(\cos(c+dx)+1)^2}+\frac{1}{6a^3d(\cos(c+dx)+1)^3}+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(\cos(c+dx)+1)}{16a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(6*a^3*d*(1 + Cos[c + d*x])^3) - 7/(8*a^3*d*(1 + Cos[c + d*x])^2) + 17/(8
*a^3*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(16*a^3*d) + (15*Log[1 +
 Cos[c + d*x]])/(16*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{a^2\text{Subst}\left(\int \frac{x^4}{(a-ax)(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\text{Subst}\left(\int \left(-\frac{1}{16a^5(-1+x)}+\frac{1}{2a^5(1+x)^4}-\frac{7}{4a^5(1+x)^3}+\frac{17}{8a^5(1+x)^2}-\frac{15}{16a^5(1+x)}\right)dx, \right.}{d}$$
$$= \frac{1}{6a^3d(1+\cos(c+dx))^3}-\frac{7}{8a^3d(1+\cos(c+dx))^2}+\frac{17}{8a^3d(1+\cos(c+dx))}+$$

Mathematica [A]
time = 0.33, size = 97, normalized size = 0.96

$$\frac{\left(2-21\cos^2\left(\frac{1}{2}(c+dx)\right)+102\cos^4\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^3(c+dx)}{12a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((2 - 21*Cos[(c + d*x)/2]^2 + 102*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x)/2]^
6*(15*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(12*a
^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 67, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(dx+c))}{16}+\frac{1}{6(1+\cos(dx+c))^3}-\frac{7}{8(1+\cos(dx+c))^2}+\frac{17}{8(1+\cos(dx+c))}+\frac{15\ln(1+\cos(dx+c))}{16}}{d\,a^3}$
default	$\frac{\frac{\ln(-1+\cos(dx+c))}{16}+\frac{1}{6(1+\cos(dx+c))^3}-\frac{7}{8(1+\cos(dx+c))^2}+\frac{17}{8(1+\cos(dx+c))}+\frac{15\ln(1+\cos(dx+c))}{16}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{51\,e^{5i(dx+c)}+162\,e^{4i(dx+c)}+238\,e^{3i(dx+c)}+162\,e^{2i(dx+c)}+51\,e^{i(dx+c)}}{12d\,a^3\left(e^{i(dx+c)}+1\right)^6}+\frac{15\ln\left(e^{i(dx+c)}+1\right)}{8a^3d}+\frac{\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/16*ln(-1+cos(d*x+c))+1/6/(1+cos(d*x+c))^3-7/8/(1+cos(d*x+c))^2+1
7/8/(1+cos(d*x+c))+15/16*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.26, size = 98, normalized size = 0.97

$$\frac{\frac{2\left(51\cos(dx+c)^2+81\cos(dx+c)+34\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{45\log(\cos(dx+c)+1)}{a^3}+\frac{3\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(51*cos(d*x + c)^2 + 81*cos(d*x + c) + 34)/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 45*log(cos(d*x + c) + 1)/a
^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 2.41, size = 151, normalized size = 1.50

$$\frac{102 \cos\left(dx+c\right)^{2}+45 \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+3 \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+162 \cos\left(dx+c\right)+68}{48 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*cos(d*x + c)^2 + 45*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 162*cos(d*x + c) +
 68)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 0.54, size = 143, normalized size = 1.42

$$\frac{\frac{6 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{3}}-\frac{96 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{3}}-\frac{\frac{66 a^{6}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15 a^{6}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2 a^{6}(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{9}}}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 - 96*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (66*a^6*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

Mupad [B]
time = 1.24, size = 75, normalized size = 0.74

$$\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{11\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{16}-\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{32}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{48}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + (11*tan(c/2 +
(d*x)/2)^2)/16 - (5*tan(c/2 + (d*x)/2)^4)/32 + tan(c/2 + (d*x)/2)^6/48)/(a^
3*d)
```

3.93 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{32a^3d(1-\cos(c+dx))}+\frac{1}{16a^3d(1+\cos(c+dx))^4}-\frac{5}{12a^3d(1+\cos(c+dx))^3}+\frac{39}{32a^3d(1+\cos(c+dx))^2}-$$

```
[Out] -1/32/a^3/d/(1-cos(d*x+c))+1/16/a^3/d/(1+cos(d*x+c))^4-5/12/a^3/d/(1+cos(d*
x+c))^3+39/32/a^3/d/(1+cos(d*x+c))^2-9/4/a^3/d/(1+cos(d*x+c))-7/64*ln(1-cos
(d*x+c))/a^3/d-57/64*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$-\frac{1}{32a^3d(1-\cos(c+dx))}-\frac{9}{4a^3d(\cos(c+dx)+1)}+\frac{39}{32a^3d(\cos(c+dx)+1)^2}-\frac{5}{12a^3d(\cos(c+dx)+1)^3}+\frac{1}{16a^3d(\cos(c+dx)+1)^4}-\frac{7\log(1-\cos(c+dx))}{64a^3d}-\frac{57\log(\cos(c+dx)+1)}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/32*1/(a^3*d*(1 - Cos[c + d*x])) + 1/(16*a^3*d*(1 + Cos[c + d*x])^4) - 5/
(12*a^3*d*(1 + Cos[c + d*x])^3) + 39/(32*a^3*d*(1 + Cos[c + d*x])^2) - 9/(4
*a^3*d*(1 + Cos[c + d*x])) - (7*Log[1 - Cos[c + d*x]])/(64*a^3*d) - (57*Log
[1 + Cos[c + d*x]])/(64*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{a^4\text{Subst}\left(\int \frac{x^6}{(a-ax)^2(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^4\text{Subst}\Big(\int \Big(\frac{1}{32a^7(-1+x)^2} + \frac{7}{64a^7(-1+x)} + \frac{1}{4a^7(1+x)^5} - \frac{5}{4a^7(1+x)^4} + \frac{39}{16a^7(1+x)^3} - \frac{}{4a}$$

$$= -\frac{1}{32a^3d(1-\cos(c+dx))} + \frac{1}{16a^3d(1+\cos(c+dx))^4} - \frac{5}{12a^3d(1+\cos(c+dx)}$$

Mathematica [A]
time = 0.63, size = 140, normalized size = 0.98

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+24\left(57\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+864\sec^2\left(\frac{1}{2}(c+dx)\right)-234\sec^4\left(\frac{1}{2}(c+dx)\right)+40\sec^6\left(\frac{1}{2}(c+dx)\right)-3\sec^8\left(\frac{1}{2}(c+dx)\right)\right)\sec^3(c+dx)}{96a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/96*(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(57*Log[Cos[(c + d*x)
/2]] + 7*Log[Sin[(c + d*x)/2]]) + 864*Sec[(c + d*x)/2]^2 - 234*Sec[(c + d*x
)/2]^4 + 40*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(a^3
*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 91, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{1}{-32+32\cos(dx+c)}-\frac{7\ln(-1+\cos(dx+c))}{64}+\frac{1}{16(1+\cos(dx+c))^4}-\frac{5}{12(1+\cos(dx+c))^3}+\frac{39}{32(1+\cos(dx+c))^2}-\frac{9}{4(1+\cos(dx+c))}-\frac{57\ln(1}{}}{d\,a^3}$
default	$\frac{\frac{1}{-32+32\cos(dx+c)}-\frac{7\ln(-1+\cos(dx+c))}{64}+\frac{1}{16(1+\cos(dx+c))^4}-\frac{5}{12(1+\cos(dx+c))^3}+\frac{39}{32(1+\cos(dx+c))^2}-\frac{9}{4(1+\cos(dx+c))}-\frac{57\ln(1}{}}{d\,a^3}$
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{213\,e^{9i(dx+c)}+606\,e^{8i(dx+c)}+472\,e^{7i(dx+c)}-846\,e^{6i(dx+c)}-1658\,e^{5i(dx+c)}-846\,e^{4i(dx+c)}+472\,e^{3i(dx+c}}{48d\,a^3\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/32/(-1+cos(d*x+c))-7/64*ln(-1+cos(d*x+c))+1/16/(1+cos(d*x+c))^4-
5/12/(1+cos(d*x+c))^3+39/32/(1+cos(d*x+c))^2-9/4/(1+cos(d*x+c))-57/64*ln(1+
cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 146, normalized size = 1.02

$$-\frac{\frac{2\left(213\cos(dx+c)^4+303\cos(dx+c)^3-95\cos(dx+c)^2-333\cos(dx+c)-136\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}+\frac{171\log(\cos(dx+c)+1)}{a^3}+\frac{21\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(213*cos(d*x + c)^4 + 303*cos(d*x + c)^3 - 95*cos(d*x + c)^2 - 33
3*cos(d*x + c) - 136)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*co
s(d*x + c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) + 171*log(c
os(d*x + c) + 1)/a^3 + 21*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 3.23, size = 240, normalized size = 1.68

$$-\frac{426 \cos \left(dx+c\right)^{4}+606 \cos \left(dx+c\right)^{3}-190 \cos \left(dx+c\right)^{2}+171 \left(\cos \left(dx+c\right)^{5}+3 \cos \left(dx+c\right)^{4}+2 \cos \left(dx+c\right)^{3}-2 \cos \left(dx+c\right)^{2}-3 \cos \left(dx+c\right)-1\right) \log \left(\frac{1}{2} \cos \left(dx+c\right)+\frac{1}{2}\right)+21 \left(\cos \left(dx+c\right)^{5}+3 \cos \left(dx+c\right)^{4}+2 \cos \left(dx+c\right)^{3}-2 \cos \left(dx+c\right)^{2}-3 \cos \left(dx+c\right)-1\right) \log \left(-\frac{1}{2} \cos \left(dx+c\right)+\frac{1}{2}\right)-666 \cos \left(dx+c\right)-272}{192 \left(a^{3} d \cos \left(dx+c\right)^{5}+3 a^{3} d \cos \left(dx+c\right)^{4}+2 a^{3} d \cos \left(dx+c\right)^{3}-2 a^{3} d \cos \left(dx+c\right)^{2}-3 a^{3} d \cos \left(dx+c\right)-a^{3} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*cos(d*x + c)^4 + 606*cos(d*x + c)^3 - 190*cos(d*x + c)^2 + 171*
(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 -
3*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^5 + 3*co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*lo
g(-1/2*cos(d*x + c) + 1/2) - 666*cos(d*x + c) - 272)/(a^3*d*cos(d*x + c)^5
+ 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2
- 3*a^3*d*cos(d*x + c) - a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{3}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.57, size = 212, normalized size = 1.48

$$\frac{\frac{12\left(\frac{7\left(\cos(dx+c)-1\right)}{\cos(dx+c)+1}+1\right)\left(\cos(dx+c)+1\right)}{a^{3}\left(\cos(dx+c)-1\right)}-\frac{84\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)}{a^{3}}+\frac{768\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{3}}+\frac{\frac{504a^{9}\left(\cos(dx+c)-1\right)}{\cos(dx+c)+1}+\frac{132a^{9}\left(\cos(dx+c)-1\right)^{2}}{\left(\cos(dx+c)+1\right)^{2}}+\frac{28a^{9}\left(\cos(dx+c)-1\right)^{3}}{\left(\cos(dx+c)+1\right)^{3}}+\frac{3a^{9}\left(\cos(dx+c)-1\right)^{4}}{\left(\cos(dx+c)+1\right)^{4}}}{a^{12}}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/
(a^3*(cos(d*x + c) - 1)) - 84*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
```

```
1))/a^3 + 768*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (
504*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 132*a^9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 28*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

Mupad [B]
time = 1.40, size = 102, normalized size = 0.71

$$-\frac{\frac{7\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64}+\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{256}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -((7*log(tan(c/2 + (d*x)/2)))/32 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/64 + (21*tan(c/2 + (d*x)/2)^2)/32 - (11*tan(c/2 + (d*x)/2)^4)/
64 + (7*tan(c/2 + (d*x)/2)^6)/192 - tan(c/2 + (d*x)/2)^8/256)/(a^3*d)
```

3.94 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=185

$$-\frac{1}{128a^3d(1-\cos(c+dx))^2}+\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{1}{40a^3d(1+\cos(c+dx))^5}-\frac{13}{64a^3d(1+\cos(c+dx))^4}+$$

```
[Out] -1/128/a^3/d/(1-cos(d*x+c))^2+5/64/a^3/d/(1-cos(d*x+c))+1/40/a^3/d/(1+cos(d
*x+c))^5-13/64/a^3/d/(1+cos(d*x+c))^4+35/48/a^3/d/(1+cos(d*x+c))^3-99/64/a^
3/d/(1+cos(d*x+c))^2+303/128/a^3/d/(1+cos(d*x+c))+37/256*ln(1-cos(d*x+c))/a
^3/d+219/256*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 90}

$$\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{303}{128a^3d(\cos(c+dx)+1)}-\frac{1}{128a^3d(1-\cos(c+dx))^2}-\frac{99}{64a^3d(\cos(c+dx)+1)^2}+\frac{35}{48a^3d(\cos(c+dx)+1)^3}-\frac{13}{64a^3d(\cos(c+dx)+1)^4}+\frac{1}{40a^3d(\cos(c+dx)+1)^5}+\frac{37\log(1-\cos(c+dx))}{256a^3d}+\frac{219\log(\cos(c+dx)+1)}{256a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/128*1/(a^3*d*(1 - Cos[c + d*x])^2) + 5/(64*a^3*d*(1 - Cos[c + d*x])) + 1
/(40*a^3*d*(1 + Cos[c + d*x])^5) - 13/(64*a^3*d*(1 + Cos[c + d*x])^4) + 35/
(48*a^3*d*(1 + Cos[c + d*x])^3) - 99/(64*a^3*d*(1 + Cos[c + d*x])^2) + 303/
(128*a^3*d*(1 + Cos[c + d*x])) + (37*Log[1 - Cos[c + d*x]])/(256*a^3*d) + (
219*Log[1 + Cos[c + d*x]])/(256*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{a^6\text{Subst}\left(\int \frac{x^8}{(a-ax)^3(a+ax)^6}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{64a^9(-1+x)^3} - \frac{5}{64a^9(-1+x)^2} - \frac{37}{256a^9(-1+x)} + \frac{1}{8a^9(1+x)^6} - \frac{13}{16a^9(1+x)^5}\right.\right.}{d}$$
$$= -\frac{1}{128a^3d(1-\cos(c+dx))^2} + \frac{5}{64a^3d(1-\cos(c+dx))} + \frac{1}{40a^3d(1+\cos(c+dx$$

Mathematica [A]

time = 1.18, size = 169, normalized size = 0.91

$$\frac{\left(12-195\cos^2\left(\frac{1}{2}(c+dx)\right)+1400\cos^4\left(\frac{1}{2}(c+dx)\right)+60\cos^8\left(\frac{1}{2}(c+dx)\right)\left(303+10\cot^2\left(\frac{1}{2}(c+dx)\right)\right)-30\cos^6\left(\frac{1}{2}(c+dx)\right)\left(198+\cot^4\left(\frac{1}{2}(c+dx)\right)\right)+120\cos^{10}\left(\frac{1}{2}(c+dx)\right)\left(219\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+37\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)}{1920a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((12 - 195*Cos[(c + d*x)/2]^2 + 1400*Cos[(c + d*x)/2]^4 + 60*Cos[(c + d*x)/
2]^8*(303 + 10*Cot[(c + d*x)/2]^2) - 30*Cos[(c + d*x)/2]^6*(198 + Cot[(c +
d*x)/2]^4) + 120*Cos[(c + d*x)/2]^10*(219*Log[Cos[(c + d*x)/2]] + 37*Log[Si
n[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(1920*a^3*d*(1 + Sec[c
 + d*x])^3)
```

Maple [A]

time = 0.15, size = 115, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{128(-1+\cos(dx+c))^2}-\frac{5}{64(-1+\cos(dx+c))}+\frac{37\ln(-1+\cos(dx+c))}{256}+\frac{1}{40(1+\cos(dx+c))^5}-\frac{13}{64(1+\cos(dx+c))^4}+\frac{35}{48(1+\cos(dx+c))^3}}{d\,a^3}$
default	$\frac{-\frac{1}{128(-1+\cos(dx+c))^2}-\frac{5}{64(-1+\cos(dx+c))}+\frac{37\ln(-1+\cos(dx+c))}{256}+\frac{1}{40(1+\cos(dx+c))^5}-\frac{13}{64(1+\cos(dx+c))^4}+\frac{35}{48(1+\cos(dx+c))^3}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{4395\,e^{13i(dx+c)}+11010\,e^{12i(dx+c)}-1390\,e^{11i(dx+c)}-47190\,e^{10i(dx+c)}-50987\,e^{9i(dx+c)}+25428\,e^{8i(dx}}{960d\,a^3(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/128/(-1+cos(d*x+c))^2-5/64/(-1+cos(d*x+c))+37/256*ln(-1+cos(d*x
+c))+1/40/(1+cos(d*x+c))^5-13/64/(1+cos(d*x+c))^4+35/48/(1+cos(d*x+c))^3-99
/64/(1+cos(d*x+c))^2+303/128/(1+cos(d*x+c))+219/256*ln(1+cos(d*x+c)))
```

Maxima [A]

time = 0.27, size = 188, normalized size = 1.02

$$\frac{\frac{2\left(4395\cos(dx+c)^6+5505\cos(dx+c)^5-6940\cos(dx+c)^4-12780\cos(dx+c)^3-367\cos(dx+c)^2+6939\cos(dx+c)+2768\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{3285\log(\cos(dx+c)+1)}{a^3}+\frac{555\log(\cos(dx+c)-1)}{a^3}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(4395*cos(d*x + c)^6 + 5505*cos(d*x + c)^5 - 6940*cos(d*x + c)^4
- 12780*cos(d*x + c)^3 - 367*cos(d*x + c)^2 + 6939*cos(d*x + c) + 2768)/(a^
3*cos(d*x + c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*
x + c)^4 - 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
 a^3) + 3285*log(cos(d*x + c) + 1)/a^3 + 555*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 2.57, size = 317, normalized size = 1.71

$$\frac{8790 \cos\left(dx+c\right)^{6}+11010 \cos\left(dx+c\right)^{5}-13880 \cos\left(dx+c\right)^{4}-25560 \cos\left(dx+c\right)^{3}-734 \cos\left(dx+c\right)^{2}+3285 \left(\cos\left(dx+c\right)^{7}+3 \cos\left(dx+c\right)^{6}+\cos\left(dx+c\right)^{5}-5 \cos\left(dx+c\right)^{4}-5 \cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+555 \left(\cos\left(dx+c\right)^{7}+3 \cos\left(dx+c\right)^{6}+\cos\left(dx+c\right)^{5}-5 \cos\left(dx+c\right)^{4}-5 \cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+13878 \cos\left(dx+c\right)+5536}{3840 \left(a^{3} d \cos\left(dx+c\right)^{7}+3 a^{3} d \cos\left(dx+c\right)^{6}+a^{3} d \cos\left(dx+c\right)^{5}-5 a^{3} d \cos\left(dx+c\right)^{4}-5 a^{3} d \cos\left(dx+c\right)^{3}+a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3840*(8790*cos(d*x + c)^6 + 11010*cos(d*x + c)^5 - 13880*cos(d*x + c)^4 -
 25560*cos(d*x + c)^3 - 734*cos(d*x + c)^2 + 3285*(cos(d*x + c)^7 + 3*cos(d
*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 555*(cos(d*x + c
)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)
^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 13
878*cos(d*x + c) + 5536)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a
^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{5}{\left(c+dx \right)}}{\sec^{3}{\left(c+dx \right)}+3 \sec^{2}{\left(c+dx \right)}+3 \sec{\left(c+dx \right)}+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.65, size = 261, normalized size = 1.41

$$-\frac{\frac{30 \left(\frac{18 (\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{111 (\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)(\cos(dx+c)+1)^{2}}{a^{3}(\cos(dx+c)-1)^{2}}-\frac{2220 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{3}}+\frac{15360 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{3}}+\frac{\frac{9780 a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2790 a^{12}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{740 a^{12}(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{135 a^{12}(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{12 a^{12}(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{15}}}{15360 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] -1/15360*(30*(18*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 111*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2) - 2220*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + 15360*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (9780*a^12*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2790*a^12*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 740*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 135*a^12*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 12*a^12*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/a^15)/d

Mupad [B]
time = 1.33, size = 170, normalized size = 0.92

$$\frac{163\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{256\,a^3\,d}-\frac{93\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,a^3\,d}+\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{768\,a^3\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{1024\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{1280\,a^3\,d}+\frac{37\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a^3\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}-\frac{1}{4}\right)}{128\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^3,x)

[Out] (163*tan(c/2 + (d*x)/2)^2)/(256*a^3*d) - (93*tan(c/2 + (d*x)/2)^4)/(512*a^3*d) + (37*tan(c/2 + (d*x)/2)^6)/(768*a^3*d) - (9*tan(c/2 + (d*x)/2)^8)/(1024*a^3*d) + tan(c/2 + (d*x)/2)^10/(1280*a^3*d) + (37*log(tan(c/2 + (d*x)/2)))/(128*a^3*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) + (cot(c/2 + (d*x)/2)^4*((9*tan(c/2 + (d*x)/2)^2)/2 - 1/4))/(128*a^3*d)

3.95 $\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$\frac{x}{a^3}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d}+\frac{\tan^3(\cdots}{3\cdots}$$

```
[Out] x/a^3-125/128*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+115/128*sec(d*x+c)
*tan(d*x+c)/a^3/d+5/64*sec(d*x+c)^3*tan(d*x+c)/a^3/d+1/3*tan(d*x+c)^3/a^3/d
-5/8*sec(d*x+c)*tan(d*x+c)^3/a^3/d-5/48*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-1/5
*tan(d*x+c)^5/a^3/d+1/2*sec(d*x+c)*tan(d*x+c)^5/a^3/d+1/8*sec(d*x+c)^3*tan(
d*x+c)^5/a^3/d-3/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$-\frac{3\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8a^3d}-\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48a^3d}+\frac{5\tan(c+dx)\sec^3(c+dx)}{64a^3d}+\frac{\tan^5(c+dx)\sec(c+dx)}{2a^3d}-\frac{5\tan^3(c+dx)\sec(c+dx)}{8a^3d}+\frac{115\tan(c+dx)\sec(c+dx)}{128a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (125*ArcTanh[Sin[c + d*x]])/(128*a^3*d) - Tan[c + d*x]/(a^3*d) + (1
15*Sec[c + d*x]*Tan[c + d*x])/(128*a^3*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])
/(64*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - (5*Sec[c + d*x]*Tan[c + d*x]^3)/(8
*a^3*d) - (5*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*a^3*d) - Tan[c + d*x]^5/(5*
a^3*d) + (Sec[c + d*x]*Tan[c + d*x]^5)/(2*a^3*d) + (Sec[c + d*x]^3*Tan[c +
d*x]^5)/(8*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int(-a+a\sec(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$

$$= \frac{\int\left(-a^3\tan^6(c+dx)+3a^3\sec(c+dx)\tan^6(c+dx)-3a^3\sec^2(c+dx)\tan^6(c+\right.}{a^6}$$

$$= -\frac{\int\tan^6(c+dx)\,dx}{a^3}+\frac{\int\sec^3(c+dx)\tan^6(c+dx)\,dx}{a^3}+\frac{3\int\sec(c+dx)\tan^6(c\cdot}{a^3}$$

$$= -\frac{\tan^5(c+dx)}{5a^3d}+\frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d}-\frac{5\int s}{}$$

$$= \frac{\tan^3(c+dx)}{3a^3d}-\frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d}-\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d}-\frac{\tan}{}$$

$$= -\frac{\tan(c+dx)}{a^3d}+\frac{15\sec(c+dx)\tan(c+dx)}{16a^3d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d}+\frac{\tan}{}$$

$$= \frac{x}{a^3}-\frac{15\tanh^{-1}(\sin(c+dx))}{16a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+\frac{5}{}$$

$$= \frac{x}{a^3}-\frac{125\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{115\sec(c+dx)\tan(c+dx)}{128a^3d}+$$

Mathematica [A]
time = 1.33, size = 362, normalized size = 1.53

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(1680000*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x
]^8*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x] + 376320*d*x*Cos[3*c + 2
*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*Cos[5*c + 4*d*x] + 53760*d
*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] + 6720*d*x*Cos[7*c + 8*d*x
] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] + 133175*Sin[d*x] + 133175*Si
n[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[3*c + 2*d*x] + 63595*Sin[
2*c + 3*d*x] + 63595*Sin[4*c + 3*d*x] - 254464*Sin[3*c + 4*d*x] + 161280*Si
n[5*c + 4*d*x] + 65135*Sin[4*c + 5*d*x] + 65135*Sin[6*c + 5*d*x] - 118784*S
in[5*c + 6*d*x] + 27195*Sin[6*c + 7*d*x] + 27195*Sin[8*c + 7*d*x] - 14848*S
in[7*c + 8*d*x])))/(215040*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.18, size = 290, normalized size = 1.22

method	result

risch	$\frac{x}{a^3} - \frac{i(27195\, e^{15i(dx+c)} + 65135\, e^{13i(dx+c)} + 161280\, e^{12i(dx+c)} + 63595\, e^{11i(dx+c)} + 286720\, e^{10i(dx+c)} + 133175\, e^{9i(dx+c}}{}$
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8} + \frac{13}{14\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7} + \frac{65}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6} + \frac{143}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5} + \frac{}{64(\tan}}{}$
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8} + \frac{13}{14\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7} + \frac{65}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6} + \frac{143}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5} + \frac{}{64(\tan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1024/d/a^3*(1/512*arctan(tan(1/2*d*x+1/2*c))+1/8192/(tan(1/2*d*x+1/2*c)-1)^
8+13/14336/(tan(1/2*d*x+1/2*c)-1)^7+65/24576/(tan(1/2*d*x+1/2*c)-1)^6+143/4
0960/(tan(1/2*d*x+1/2*c)-1)^5+79/65536/(tan(1/2*d*x+1/2*c)-1)^4-49/32768/(t
an(1/2*d*x+1/2*c)-1)^3-29/131072/(tan(1/2*d*x+1/2*c)-1)^2+253/131072/(tan(1
/2*d*x+1/2*c)-1)+125/131072*ln(tan(1/2*d*x+1/2*c)-1)-1/8192/(tan(1/2*d*x+1/
2*c)+1)^8+13/14336/(tan(1/2*d*x+1/2*c)+1)^7-65/24576/(tan(1/2*d*x+1/2*c)+1)
^6+143/40960/(tan(1/2*d*x+1/2*c)+1)^5-79/65536/(tan(1/2*d*x+1/2*c)+1)^4-49/
32768/(tan(1/2*d*x+1/2*c)+1)^3+29/131072/(tan(1/2*d*x+1/2*c)+1)^2+253/13107
2/(tan(1/2*d*x+1/2*c)+1)-125/131072*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]

time = 0.49, size = 429, normalized size = 1.81

$$-\frac{\frac{2\left(\frac{315\sin(dx+c)}{\cos(dx+c)+1} - \frac{11375\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{79723\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{269879\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{550089\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{749973\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{212625\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{26565\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}\right)}{a^3 - \frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}} - \frac{26880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{13125\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} - \frac{13125\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/13440*(2*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 11375*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 79723*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 269879*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 550089*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 749973*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 212625*sin(d*x + c)^13/(
cos(d*x + c) + 1)^13 - 26565*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 -
8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 26880*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 13125*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^3 - 13125*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [A]

time = 3.37, size = 147, normalized size = 0.62

$$\frac{26880\,dx\cos(dx+c)^8 - 13125\cos(dx+c)^8\log(\sin(dx+c)+1) + 13125\cos(dx+c)^8\log(-\sin(dx+c)+1) - 2\left(14848\cos(dx+c)^7 - 27195\cos(dx+c)^6 + 7424\cos(dx+c)^5 + 17710\cos(dx+c)^4 - 14592\cos(dx+c)^3 - 1960\cos(dx+c)^2 + 5760\cos(dx+c) - 1680\right)\sin(dx+c)}{26880\,a^3d\cos(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*d*x*cos(d*x + c)^8 - 13125*cos(d*x + c)^8*log(sin(d*x + c) +
 1) + 13125*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*cos(d*x + c)^7
 - 27195*cos(d*x + c)^6 + 7424*cos(d*x + c)^5 + 17710*cos(d*x + c)^4 - 1459
2*cos(d*x + c)^3 - 1960*cos(d*x + c)^2 + 5760*cos(d*x + c) - 1680)*sin(d*x
+ c))/(a^3*d*cos(d*x + c)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{12}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**12/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**12/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Giac [A]
time = 20.48, size = 175, normalized size = 0.74

$$\frac{\frac{13440\,(dx+c)}{a^3}-\frac{13125\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}+\frac{13125\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{2\left(26565\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-212625\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+749973\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-550089\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+269879\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-79723\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+11375\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{8}a^3}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^3 - 13125*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3
 + 13125*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(26565*tan(1/2*d*x + 1/
2*c)^15 - 212625*tan(1/2*d*x + 1/2*c)^13 + 749973*tan(1/2*d*x + 1/2*c)^11 -
 550089*tan(1/2*d*x + 1/2*c)^9 + 269879*tan(1/2*d*x + 1/2*c)^7 - 79723*tan(
1/2*d*x + 1/2*c)^5 + 11375*tan(1/2*d*x + 1/2*c)^3 - 315*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^8*a^3))/d
```

Mupad [B]
time = 2.56, size = 265, normalized size = 1.12

$$\frac{x}{a^3}-\frac{125\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a^3\,d}-\frac{-\frac{253\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{2025\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}-\frac{35713\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{320}+\frac{183363\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{2240}-\frac{269879\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720}+\frac{11389\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{960}-\frac{325\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{192}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+70\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^12/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (125*atanh(tan(c/2 + (d*x)/2)))/(64*a^3*d) - ((3*tan(c/2 + (d*x)/2)
)/64 - (325*tan(c/2 + (d*x)/2)^3)/192 + (11389*tan(c/2 + (d*x)/2)^5)/960 -
```

```
(269879*tan(c/2 + (d*x)/2)^7)/6720 + (183363*tan(c/2 + (d*x)/2)^9)/2240 - (
35713*tan(c/2 + (d*x)/2)^11)/320 + (2025*tan(c/2 + (d*x)/2)^13)/64 - (253*t
an(c/2 + (d*x)/2)^15)/64)/(d*(28*a^3*tan(c/2 + (d*x)/2)^4 - 8*a^3*tan(c/2 +
 (d*x)/2)^2 - 56*a^3*tan(c/2 + (d*x)/2)^6 + 70*a^3*tan(c/2 + (d*x)/2)^8 - 5
6*a^3*tan(c/2 + (d*x)/2)^10 + 28*a^3*tan(c/2 + (d*x)/2)^12 - 8*a^3*tan(c/2
+ (d*x)/2)^14 + a^3*tan(c/2 + (d*x)/2)^16 + a^3))
```

3.96 $\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=169

$$-\frac{x}{a^3}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d}-\frac{\tan^3(c-}{3a^3}$$

```
[Out] -x/a^3+19/16*arctanh(sin(d*x+c))/a^3/d+tan(d*x+c)/a^3/d-17/16*sec(d*x+c)*ta
n(d*x+c)/a^3/d-1/8*sec(d*x+c)^3*tan(d*x+c)/a^3/d-1/3*tan(d*x+c)^3/a^3/d+3/4
*sec(d*x+c)*tan(d*x+c)^3/a^3/d+1/6*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-3/5*tan(
d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan^3(c+dx)\sec^3(c+dx)}{6a^3d}-\frac{\tan(c+dx)\sec^3(c+dx)}{8a^3d}+\frac{3\tan^3(c+dx)\sec(c+dx)}{4a^3d}-\frac{17\tan(c+dx)\sec(c+dx)}{16a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (19*ArcTanh[Sin[c + d*x]])/(16*a^3*d) + Tan[c + d*x]/(a^3*d) - (
17*Sec[c + d*x]*Tan[c + d*x])/(16*a^3*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(8
*a^3*d) - Tan[c + d*x]^3/(3*a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*a^3
*d) + (Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int(-a+a\sec(c+dx))^3\tan^4(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)-3a^3\sec^2(c+dx)\tan^4(c+\right.}{a^6}$$
$$= -\frac{\int\tan^4(c+dx)\,dx}{a^3}+\frac{\int\sec^3(c+dx)\tan^4(c+dx)\,dx}{a^3}+\frac{3\int\sec(c+dx)\tan^4(c}{a^3}$$
$$= -\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d}+\frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}-\frac{\int \mathrm{s}}{}$$
$$= \frac{\tan(c+dx)}{a^3d}-\frac{9\sec(c+dx)\tan(c+dx)}{8a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d}-\frac{\tan^3(c}{3a^3}$$
$$= -\frac{x}{a^3}+\frac{9\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\mathrm{se}$$
$$= -\frac{x}{a^3}+\frac{19\tanh^{-1}(\sin(c+dx))}{16a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\mathrm{s}$$

Mathematica [A]
time = 0.87, size = 303, normalized size = 1.79

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(9120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec(c)\sec^6(c+dx)(2400dx\cos(c)+1800dx\cos(c+2dx)+1800dx\cos(3c+2dx)+720dx\cos(3c+4dx)+720dx\cos(5c+4dx)+120dx\cos(5c+6dx)+120dx\cos(7c+6dx)+1760\sin(c)-210\sin(dx)-210\sin(2c+dx)-1440\sin(c+2dx)+1200\sin(3c+2dx)+865\sin(2c+3dx)+865\sin(4c+3dx)-1296\sin(3c+4dx)-240\sin(5c+4dx)+435\sin(4c+5dx)+435\sin(6c+5dx)-176\sin(5c+6dx))\right)}{960a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/960*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(9120*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c +
 d*x]^6*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d
*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c
 + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] - 210*Sin[d*x] - 210*Sin
[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x] + 865*Sin[2*c + 3
*d*x] + 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 240*Sin[5*c + 4*d*x]
 + 435*Sin[4*c + 5*d*x] + 435*Sin[6*c + 5*d*x] - 176*Sin[5*c + 6*d*x])))/(a
^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 230, normalized size = 1.36

method	result
risch	$-\frac{x}{a^3}+\frac{i\left(435\,e^{11i(dx+c)}-240\,e^{10i(dx+c)}+865\,e^{9i(dx+c)}+1200\,e^{8i(dx+c)}-210\,e^{7i(dx+c)}+1760\,e^{6i(dx+c)}+210\,e^{5i(dx+c)}+\right.}{120d\,a^3\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{11}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{16\left(\tan\left(\frac{d}{}\right.\right.}}{}$

default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}+\frac{11}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{}{16(\tan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 256/d/a^3*(-1/128*arctan(tan(1/2*d*x+1/2*c))+1/1536/(tan(1/2*d*x+1/2*c)-1)^
6+11/2560/(tan(1/2*d*x+1/2*c)-1)^5+11/1024/(tan(1/2*d*x+1/2*c)-1)^4+11/1024
/(tan(1/2*d*x+1/2*c)-1)^3-5/4096/(tan(1/2*d*x+1/2*c)-1)^2-35/4096/(tan(1/2*
d*x+1/2*c)-1)-19/4096*ln(tan(1/2*d*x+1/2*c)-1)-1/1536/(tan(1/2*d*x+1/2*c)+1
)^6+11/2560/(tan(1/2*d*x+1/2*c)+1)^5-11/1024/(tan(1/2*d*x+1/2*c)+1)^4+11/10
24/(tan(1/2*d*x+1/2*c)+1)^3+5/4096/(tan(1/2*d*x+1/2*c)+1)^2-35/4096/(tan(1/
2*d*x+1/2*c)+1)+19/4096*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(155) = 310$.
time = 0.50, size = 343, normalized size = 2.03

$$-\frac{\frac{2\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{95\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{366\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{1746\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3135\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{525\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^{3}-\frac{6a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{285\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}+\frac{285\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(2*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 95*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 366*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1746*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 3135*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 525*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 - 6*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12) + 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 285*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 285*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^3)/d
```

Fricas [A]
time = 3.00, size = 127, normalized size = 0.75

$$-\frac{480\,dx\cos\left(dx+c\right)^{6}-285\,\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)+285\,\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(176\,\cos\left(dx+c\right)^{5}-435\,\cos\left(dx+c\right)^{4}+208\,\cos\left(dx+c\right)^{3}+110\,\cos\left(dx+c\right)^{2}-144\,\cos\left(dx+c\right)+40\right)\sin\left(dx+c\right)}{480\,a^{3}d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(480*d*x*cos(d*x + c)^6 - 285*cos(d*x + c)^6*log(sin(d*x + c) + 1) +
 285*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*cos(d*x + c)^5 - 435*co
```

```
s(d*x + c)^4 + 208*cos(d*x + c)^3 + 110*cos(d*x + c)^2 - 144*cos(d*x + c) +
 40)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{10}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**10/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**10/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Giac [A]
time = 12.02, size = 149, normalized size = 0.88

$$-\frac{\frac{240\,(dx+c)}{a^3}-\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}+\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-3135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1746\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-366\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-95\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)/a^3 - 285*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 28
5*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(525*tan(1/2*d*x + 1/2*c)^11 -
 3135*tan(1/2*d*x + 1/2*c)^9 + 1746*tan(1/2*d*x + 1/2*c)^7 - 366*tan(1/2*d*
x + 1/2*c)^5 - 95*tan(1/2*d*x + 1/2*c)^3 + 45*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 - 1)^6*a^3))/d
```

Mupad [B]
time = 2.35, size = 208, normalized size = 1.23

$$\frac{19\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^3\,d}-\frac{x}{a^3}-\frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{209\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{291\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{61\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^10/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (19*atanh(tan(c/2 + (d*x)/2)))/(8*a^3*d) - x/a^3 - ((3*tan(c/2 + (d*x)/2))/
8 - (19*tan(c/2 + (d*x)/2)^3)/24 - (61*tan(c/2 + (d*x)/2)^5)/20 + (291*tan(
c/2 + (d*x)/2)^7)/20 - (209*tan(c/2 + (d*x)/2)^9)/8 + (35*tan(c/2 + (d*x)/2
)^11)/8)/(d*(15*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3*tan(c/2 + (d*x)/2)^2 - 20*
a^3*tan(c/2 + (d*x)/2)^6 + 15*a^3*tan(c/2 + (d*x)/2)^8 - 6*a^3*tan(c/2 + (d
*x)/2)^10 + a^3*tan(c/2 + (d*x)/2)^12 + a^3))
```

3.97 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$\frac{x}{a^3}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\tan^3(c+}{a^3d}$$

```
[Out] x/a^3-13/8*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+11/8*sec(d*x+c)*tan(d
*x+c)/a^3/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d-tan(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$-\frac{\tan^3(c+dx)}{a^3d}-\frac{\tan(c+dx)}{a^3d}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{11\tan(c+dx)\sec(c+dx)}{8a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (13*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - Tan[c + d*x]/(a^3*d) + (11*S
ec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*
d) - Tan[c + d*x]^3/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int(-a+a\sec(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)-3a^3\sec^2(c+dx)\tan^2(c+\right.}{a^6}$$
$$= -\frac{\int\tan^2(c+dx)\,dx}{a^3}+\frac{\int\sec^3(c+dx)\tan^2(c+dx)\,dx}{a^3}+\frac{3\int\sec(c+dx)\tan^2(c}{a^3}$$
$$= -\frac{\tan(c+dx)}{a^3d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\int\sec}{}$$
$$= \frac{x}{a^3}-\frac{3\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec}{}$$
$$= \frac{x}{a^3}-\frac{13\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\mathrm{s}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(99) = 198.
time = 0.81, size = 230, normalized size = 2.32

$$\frac{\sec^4(c+dx)\left(24dx+39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\cos(2(c+dx))\left(8dx+13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos(4(c+dx))\left(8dx+13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+38\sin(c+dx)-32\sin(2(c+dx))+22\sin(3(c+dx))\right)}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*(24*d*x + 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*C
os[2*(c + d*x)]*(8*d*x + 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x + 13*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) - 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 38*Sin[c + d*x] -
 32*Sin[2*(c + d*x)] + 22*Sin[3*(c + d*x)]))/(64*a^3*d)
```

Maple [A]
time = 0.12, size = 170, normalized size = 1.72

method	result
risch	$\frac{x}{a^3}-\frac{i\left(11\,\mathrm{e}^{7i(dx+c)}-16\,\mathrm{e}^{6i(dx+c)}+19\,\mathrm{e}^{5i(dx+c)}-19\,\mathrm{e}^{3i(dx+c)}+16\,\mathrm{e}^{2i(dx+c)}-11\,\mathrm{e}^{i(dx+c)}\right)}{4d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{13\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^3d}+\frac{13}{}$
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{27}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{13\ln\left(\tan\left(\frac{d}{}\right.\right.}{8}}{a^3d}$
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{27}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{13\ln\left(\tan\left(\frac{d}{}\right.\right.}{8}}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 64/d/a^3*(1/32*arctan(tan(1/2*d*x+1/2*c))+1/256/(tan(1/2*d*x+1/2*c)-1)^4+3/128/(tan(1/2*d*x+1/2*c)-1)^3+27/512/(tan(1/2*d*x+1/2*c)-1)^2+21/512/(tan(1/2*d*x+1/2*c)-1)+13/512*ln(tan(1/2*d*x+1/2*c)-1)-1/256/(tan(1/2*d*x+1/2*c)+1)^4+3/128/(tan(1/2*d*x+1/2*c)+1)^3-27/512/(tan(1/2*d*x+1/2*c)+1)^2+21/512/(tan(1/2*d*x+1/2*c)+1)-13/512*ln(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(93) = 186$.
time = 0.48, size = 257, normalized size = 2.60

$$\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^3-\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{13\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{13\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/8*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^3 - 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 13*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 13*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d

Fricas [A]
time = 3.63, size = 97, normalized size = 0.98

$$\frac{16\,dx\cos(dx+c)^4-13\cos(dx+c)^4\log(\sin(dx+c)+1)+13\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(11\cos(dx+c)^2-8\cos(dx+c)+2\right)\sin(dx+c)}{16\,a^3d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 13*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 13*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(11*cos(d*x + c)^2 - 8*cos(d*x + c) + 2)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^8(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 4.90, size = 123, normalized size = 1.24

$$\frac{\frac{8(dx+c)}{a^3}-\frac{13\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{13\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{2\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-13\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a^3 - 13*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 13*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 3*tan(1/
2*d*x + 1/2*c)^5 - 13*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^4*a^3))/d
```

Mupad [B]
time = 1.93, size = 148, normalized size = 1.49

$$\frac{x}{a^3}-\frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}+\frac{\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (13*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((5*tan(c/2 + (d*x)/2))/
4 - (13*tan(c/2 + (d*x)/2)^3)/4 + (3*tan(c/2 + (d*x)/2)^5)/4 + (21*tan(c/2
+ (d*x)/2)^7)/4)/(d*(6*a^3*tan(c/2 + (d*x)/2)^4 - 4*a^3*tan(c/2 + (d*x)/2)^
2 - 4*a^3*tan(c/2 + (d*x)/2)^6 + a^3*tan(c/2 + (d*x)/2)^8 + a^3))
```

3.98 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=66

$$-\frac{x}{a^3}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{5\tan(c+dx)}{2a^3d}-\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}$$

```
[Out] -x/a^3+7/2*arctanh(sin(d*x+c))/a^3/d-5/2*tan(d*x+c)/a^3/d-1/2*(1-sec(d*x+c)
)*tan(d*x+c)/a^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3973, 3860, 3999, 3852, 8, 3855}

$$-\frac{5\tan(c+dx)}{2a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)(1-\sec(c+dx))}{2a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (7*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (5*Tan[c + d*x])/(2*a^3*d)
 - ((1 - Sec[c + d*x])*Tan[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int (-a+a\sec(c+dx))^3\,dx}{a^6} \\
&= -\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} - \frac{\int (-a+a\sec(c+dx))(-2a+5a\sec(c+dx \\
&= -\frac{x}{a^3} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} - \frac{5\int \sec^2(c+dx)\,dx}{2a^3} + \frac{7\int \sec(c+dx)}{2a^3} \\
&= -\frac{x}{a^3} + \frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} + \frac{5\text{Subst}(\int 1\, d}{} \\
&= -\frac{x}{a^3} + \frac{7\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{5\tan(c+dx)}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx}{2a^3d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(66) = 132.

time = 1.00, size = 241, normalized size = 3.65

$$
\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-4x-\frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{12\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{a^3(1+\sec(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-4*x - (14*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2) - (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]))))/(a^3*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.10, size = 110, normalized size = 1.67

method	result
risch	$-\frac{x}{a^{3}}-\frac{i\left(\mathrm{e}^{3 i(d x+c)}+6 \,\mathrm{e}^{2 i(d x+c)}-\mathrm{e}^{i(d x+c)}+6\right)}{d \,a^{3} \left(\mathrm{e}^{2 i(d x+c)}+1\right)^{2}}-\frac{7 \ln\left(\mathrm{e}^{i(d x+c)}-i\right)}{2 a^{3} d}+\frac{7 \ln\left(\mathrm{e}^{i(d x+c)}+i\right)}{2 a^{3} d}$
derivativedivides	$\frac{-2 \arctan\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{7}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}-\frac{7 \ln\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{7}{2\left(\tan\left(\frac{d x}{2}\right.\right.}}{d \,a^{3}}$
default	$\frac{-2 \arctan\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{7}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}-\frac{7 \ln\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{7}{2\left(\tan\left(\frac{d x}{2}\right.\right.}}{d \,a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*(-1/8*arctan(tan(1/2*d*x+1/2*c))+1/32/(tan(1/2*d*x+1/2*c)-1)^2+7/3
2/(tan(1/2*d*x+1/2*c)-1)-7/32*ln(tan(1/2*d*x+1/2*c)-1)-1/32/(tan(1/2*d*x+1/
2*c)+1)^2+7/32/(tan(1/2*d*x+1/2*c)+1)+7/32*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(58) = 116.
time = 0.50, size = 171, normalized size = 2.59

$$-\frac{\frac{2\left(\frac{5 \sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{7 \sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}\right)}{a^{3}-\frac{2 a^{3} \sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{a^{3} \sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}}+\frac{4 \arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}-\frac{7 \log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^{3}}+\frac{7 \log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^{3}}}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 -
 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 7*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3)/d
```

Fricas [A]
time = 3.94, size = 87, normalized size = 1.32

$$-\frac{4 \, d x \cos\left(d x + c\right)^{2} - 7 \, \cos\left(d x + c\right)^{2} \log\left(\sin\left(d x + c\right) + 1\right) + 7 \, \cos\left(d x + c\right)^{2} \log\left(-\sin\left(d x + c\right) + 1\right) + 2 \, {\left(6 \, \cos\left(d x + c\right) - 1\right)} \sin\left(d x + c\right)}{4 \, a^{3} d \cos\left(d x + c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 7*cos(d*x + c)^2*log(sin(d*x + c) + 1) + 7*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*cos(d*x + c) - 1)*sin(d*x + c))/(
a^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 2.46, size = 97, normalized size = 1.47

$$-\frac{\frac{2(dx+c)}{a^3}-\frac{7\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{7\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{2\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/a^3 - 7*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 7*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B]
time = 1.31, size = 92, normalized size = 1.39

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{x}{a^3}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - x/a^3 - (5*tan(c/2 + (d*x)/2) - 7*t
an(c/2 + (d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2
)^2 + a^3))
```

3.99 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a^3}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{4\tan(c+dx)}{a^2d(a+a\sec(c+dx))}$$

```
[Out] x/a^3+arctanh(sin(d*x+c))/a^3/d-4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2686, 3852, 2701, 327, 213}

$$\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + ArcTanh[Sin[c + d*x]]/(a^3*d) + (4*Cot[c + d*x])/(a^3*d) - (4*Csc[c
 + d*x])/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int (-a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^2(\ldots}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^3}+\frac{3\int \cot(c+dx)\csc(c+\ldots}{a^3}$$
$$= \frac{\cot(c+dx)}{a^3d}+\frac{\int 1\,dx}{a^3}-\frac{\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d}+\frac{3\mathrm{Subst}(\int 1\,dx,x,\ldots}{a^3d}$$
$$= \frac{x}{a^3}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d}$$
$$= \frac{x}{a^3}+\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.
time = 0.28, size = 117, normalized size = 2.54

$$\frac{8\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(dx-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-4\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (8*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*(Cos[(c + d*x)/2]*(d*x - Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4
*Sec[c/2]*Sin[(d*x)/2]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.10, size = 61, normalized size = 1.33

method	result	size
derivativedivides	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}$	61
default	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}$	61
risch	$\frac{x}{a^3}-\frac{8i}{a^3d\left(e^{i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{a^3d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^3d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*(-tan(1/2*d*x+1/2*c)+1/2*arctan(tan(1/2*d*x+1/2*c))-1/4*ln(tan(1/2*
d*x+1/2*c)-1)+1/4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.
time = 0.49, size = 98, normalized size = 2.13

$$\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}-\frac{4\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^3 - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3 - 4*sin(d
*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 3.39, size = 83, normalized size = 1.80

$$\frac{2\,dx\cos\left(dx+c\right)+2\,dx+\left(\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+1\right)-8\,\sin\left(dx+c\right)}{2\left(a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*cos(d*x + c) + 2*d*x + (cos(d*x + c) + 1)*log(sin(d*x + c) + 1)
- (cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 8*sin(d*x + c))/(a^3*d*cos(d*
x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^4\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 1.17, size = 63, normalized size = 1.37

$$\frac{\frac{dx+c}{a^3}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^3 - 4*tan(1/2*d*x + 1/2*c)/a^3)/d
```

Mupad [B]
time = 1.16, size = 37, normalized size = 0.80

$$\frac{x}{a^3}+\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 + (2*atanh(tan(c/2 + (d*x)/2)) - 4*tan(c/2 + (d*x)/2))/(a^3*d)
```

3.100 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{x}{a^3}+\frac{2\tan(c+dx)}{a^2d(a+a\sec(c+dx))}-\frac{\tan^3(c+dx)}{3d(a+a\sec(c+dx))^3}$$

```
[Out] -x/a^3+2*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))-1/3*tan(d*x+c)^3/d/(a+a*sec(d*x+
c))^3
```

Rubi [A]
time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3973, 3971, 3554, 8, 2686, 2687, 30}

$$\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Csc[c +
 d*x])/(a^3*d) - (4*Csc[c + d*x]^3)/(3*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\csc^2(c+\right.}{a^6} \\
&= -\frac{\int \cot^4(c+dx)\,dx}{a^3}+\frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^3(c+dx)\csc(c+}{a^3} \\
&= \frac{\cot^3(c+dx)}{3a^3d}+\frac{\int \cot^2(c+dx)\,dx}{a^3}-\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\operatorname{Subst}(\int}{} \\
&= -\frac{\cot(c+dx)}{a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}-\frac{\int 1\,dx}{a^3} \\
&= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(60) = 120.
time = 0.40, size = 125, normalized size = 2.08

$$
-\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(180dx\cos\left(\frac{dx}{2}\right)+180dx\cos\left(c+\frac{dx}{2}\right)+60dx\cos\left(c+\frac{3dx}{2}\right)+60dx\cos\left(2c+\frac{3dx}{2}\right)-471\sin\left(\frac{dx}{2}\right)+351\sin\left(c+\frac{dx}{2}\right)-277\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)\right)}{480a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(Sec[c/2]*Sec[(c + d*x)/2]^3*(180*d*x*Cos[(d*x)/2] + 180*d*x*Cos[c +
 (d*x)/2] + 60*d*x*Cos[c + (3*d*x)/2] + 60*d*x*Cos[2*c + (3*d*x)/2] - 471*S
in[(d*x)/2] + 351*Sin[c + (d*x)/2] - 277*Sin[c + (3*d*x)/2] - 3*Sin[2*c + (
3*d*x)/2]))/(a^3*d)
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.75

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$	45
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$	45
risch	$-\frac{x}{a^3}+\frac{2i\left(9\,\mathrm{e}^{2i(dx+c)}+12\,\mathrm{e}^{i(dx+c)}+7\right)}{3d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/3*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1/2*c)-2*arctan(tan(1/2*d*
x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 1.20

$$\frac{\frac{\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{6\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((6*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/a^3 - 6*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 2.95, size = 80, normalized size = 1.33

$$-\frac{3\,dx\cos\left(dx+c\right)^2+6\,dx\cos\left(dx+c\right)+3\,dx-\left(7\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{3\,\left(a^3d\cos\left(dx+c\right)^2+2\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (7*cos(d*x + c) +
 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.72, size = 50, normalized size = 0.83

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^6*tan(1/2*d*x + 1
/2*c))/a^9)/d
```

Mupad [B]
time = 1.16, size = 35, normalized size = 0.58

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,d\,x}{3\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^3 - 6*tan(c/2 + (d*x)/2) + 3*d*x)/(3*a^3*d)
```

3.101 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+\frac{11\csc^5(c}{5a^3}$$

```
[Out] -x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+4/7*c
ot(d*x+c)^7/a^3/d+3*csc(d*x+c)/a^3/d-10/3*csc(d*x+c)^3/a^3/d+11/5*csc(d*x+c
)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{10\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (10*
Csc[c + d*x]^3)/(3*a^3*d) + (11*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]
^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)-3a^3\cot^6(c+dx)\csc^2(c+\right.}{a^6}$$
$$= -\frac{\int \cot^8(c+dx)\,dx}{a^3}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^7(c+dx)\csc(c}{a^3}$$
$$= \frac{\cot^7(c+dx)}{7a^3d}+\frac{\int \cot^6(c+dx)\,dx}{a^3}-\frac{\operatorname{Subst}\left(\int x^2(-1+x^2)^2\,dx,x,\csc(c+dx)\right)}{a^3d}$$
$$= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\int \cot^4(c+dx)\,dx}{a^3}-\frac{\operatorname{Subst}\left(\int (x^2-2x^4+\right.}{a^3}$$
$$= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\csc^3(c+a}{3a^3d}$$
$$= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx}{a^3d}$$
$$= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c}{a^3}$$

Mathematica [A]
time = 1.27, size = 252, normalized size = 1.76

$$\frac{\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-5880dx\cos(dx)+5880dx\cos(2c+dx)-5880dx\cos(c+2dx)+5880dx\cos(3c+2dx)-2520dx\cos(2c+3dx)+2520dx\cos(4c+3dx)-420dx\cos(3c+4dx)+420dx\cos(5c+4dx)+4200\sin(c)+11032\sin(dx)-23282\sin(c+dx)-23282\sin(2(c+dx))-9978\sin(3(c+dx))-1663\sin(4(c+dx))+13720\sin(2c+dx)+15512\sin(c+2dx)+9240\sin(3c+2dx)+8088\sin(2c+3dx)+2520\sin(4c+3dx)+1768\sin(3c+4dx)\right)}{215040a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[(c + d*x)/2]*Sec[c/2]*Sec[(c + d*x)/2]^7*(-5880*d*x*Cos[d*x]
+ 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c + 2*
d*x] - 2520*d*x*Cos[2*c + 3*d*x] + 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*Cos[
3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] + 11032*Sin[d*x] - 23
282*Sin[c + d*x] - 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] - 1663*Si
n[4*(c + d*x)] + 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[3*c
 + 2*d*x] + 8088*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c +
4*d*x]))/(215040*a^3*d)
```

Maple [A]
time = 0.11, size = 85, normalized size = 0.59

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^3}$

default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{16\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16 d a^{3}}$
risch	$-\frac{x}{a^{3}}+\frac{2i(315\,e^{7i(dx+c)}+1155\,e^{6i(dx+c)}+1715\,e^{5i(dx+c)}+525\,e^{4i(dx+c)}-1379\,e^{3i(dx+c)}-1939\,e^{2i(dx+c)}-1011\,e^{i(dx+c)}}{105 d a^{3}\left(e^{i(dx+c)}+1\right)^{7}\left(e^{i(dx+c)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/16/d/a^3*(-1/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-16/3*tan(1/2*d*x+1/2*c)^3+26*tan(1/2*d*x+1/2*c)-32*arctan(tan(1/2*d*x+1/2*c))-1/tan(1/2*d*x+1/2*c))

Maxima [A]
time = 0.49, size = 133, normalized size = 0.93

$$\frac{\frac{\frac{2730\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{560\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{126\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{3}}-\frac{3360\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{105\,(\cos(dx+c)+1)}{a^{3}\sin(dx+c)}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/1680*((2730*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 126*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 - 3360*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 105*(cos(d*x + c) + 1)/(a^3*sin(d*x + c)))/d

Fricas [A]
time = 2.82, size = 142, normalized size = 0.99

$$-\frac{221\,\cos(dx+c)^{4}+348\,\cos(dx+c)^{3}-25\,\cos(dx+c)^{2}+105\,\left(dx\cos(dx+c)^{3}+3\,dx\cos(dx+c)^{2}+3\,dx\cos(dx+c)+dx\right)\sin(dx+c)-303\,\cos(dx+c)-136}{105\,\left(a^{3}d\cos(dx+c)^{3}+3\,a^{3}d\cos(dx+c)^{2}+3\,a^{3}d\cos(dx+c)+a^{3}d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/105*(221*cos(d*x + c)^4 + 348*cos(d*x + c)^3 - 25*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^3 + 3*d*x*cos(d*x + c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 303*cos(d*x + c) - 136)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{2}(c+dx)}{\sec^{3}(c+dx)+3\sec^{2}(c+dx)+3\sec(c+dx)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.55, size = 99, normalized size = 0.69

$$-\frac{\frac{1680\,(dx+c)}{a^3}+\frac{105}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{15\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-126\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+560\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2730\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1680*(1680*(d*x + c)/a^3 + 105/(a^3*tan(1/2*d*x + 1/2*c)) + (15*a^18*tan
(1/2*d*x + 1/2*c)^7 - 126*a^18*tan(1/2*d*x + 1/2*c)^5 + 560*a^18*tan(1/2*d*
x + 1/2*c)^3 - 2730*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

Mupad [B]
time = 1.70, size = 91, normalized size = 0.64

$$-\frac{x}{a^3}-\frac{\frac{221\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{105}-\frac{268\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{105}+\frac{257\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420}-\frac{31\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{280}+\frac{1}{112}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - x/a^3 - ((257*cos(c/2 + (d*x)/2)^4)/420 - (31*cos(c/2 + (d*x)/2)^2)/280 -
 (268*cos(c/2 + (d*x)/2)^6)/105 + (221*cos(c/2 + (d*x)/2)^8)/105 + 1/112)/(
a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2))
```

3.102 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=177

$$\frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13\csc^3(c+dx)}{3a^3d}$$

```
[Out] x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d-1/7*co
t(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3*csc(d*x+c)/a^3/d+13/3*csc(d*x+c)^
3/a^3/d-21/5*csc(d*x+c)^5/a^3/d+15/7*csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^
3/d
```

Rubi [A]

time = 0.19, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{21\csc^5(c+dx)}{5a^3d}+\frac{13\csc^3(c+dx)}{3a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d) - Cot[c + d*x]^7/(7*a^3*d) + (4*Cot[c + d*x]^9)/(9*a^3*d) - (3*Csc[
c + d*x])/(a^3*d) + (13*Csc[c + d*x]^3)/(3*a^3*d) - (21*Csc[c + d*x]^5)/(5*
a^3*d) + (15*Csc[c + d*x]^7)/(7*a^3*d) - (4*Csc[c + d*x]^9)/(9*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^{10}(c+dx) + 3a^3\cot^9(c+dx)\csc(c+dx) - 3a^3\cot^8(c+dx)\csc^2(c+\right.}{a^6}$$

$$= -\frac{\int \cot^{10}(c+dx)\,dx}{a^3} + \frac{\int \cot^7(c+dx)\csc^3(c+dx)\,dx}{a^3} + \frac{3\int \cot^9(c+dx)\csc(c}{a^3}$$

$$= \frac{\cot^9(c+dx)}{9a^3d} + \frac{\int \cot^8(c+dx)\,dx}{a^3} - \frac{\text{Subst}\left(\int x^2(-1+x^2)^3\,dx, x, \csc(c+dx)\right)}{a^3d}$$

$$= -\frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{\int \cot^6(c+dx)\,dx}{a^3} - \frac{\text{Subst}\left(\int (-x^2+3x^4-\right.}{}$$

$$= \frac{\cot^5(c+dx)}{5a^3d} - \frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{3\csc(c+dx)}{a^3d} + \frac{13\csc^3(c+dx}{3a^3d}$$

$$= -\frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{3\csc(c+dx}{a^3d}$$

$$= \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} -$$

$$= \frac{x}{a^3} + \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+d}{9a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(177) = 354.
time = 1.19, size = 366, normalized size = 2.07

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[2*(c + d*x)]^3*Sec[c/2]*(181440*d*x*Cos[d*x] - 181440*d*x*Cos
[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*x*Cos[3*c + 2*d*x] - 100
80*d*x*Cos[2*c + 3*d*x] + 10080*d*x*Cos[4*c + 3*d*x] - 60480*d*x*Cos[3*c +
4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] - 30240*d*x*Cos[4*c + 5*d*x] + 30240*d*
x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 5040*d*x*Cos[7*c + 6*d*x]
- 169344*Sin[c] - 338112*Sin[d*x] + 675036*Sin[c + d*x] + 506277*Sin[2*(c +
 d*x)] - 37502*Sin[3*(c + d*x)] - 225012*Sin[4*(c + d*x)] - 112506*Sin[5*(c
 + d*x)] - 18751*Sin[6*(c + d*x)] - 431424*Sin[2*c + d*x] - 375552*Sin[c +
2*d*x] - 201600*Sin[3*c + 2*d*x] - 41248*Sin[2*c + 3*d*x] + 84000*Sin[4*c +
 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c + 4*d*x] + 98016*Sin[4*c
 + 5*d*x] + 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c + 6*d*x]))/(80640*a^3*d*
(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 111, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{29\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{64\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d\,a^3}$
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{29\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{64\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d\,a^3}$
risch	$\frac{x}{a^3}-\frac{2i\left(945\,\mathrm{e}^{11i(dx+c)}+3150\,\mathrm{e}^{10i(dx+c)}+2625\,\mathrm{e}^{9i(dx+c)}-6300\,\mathrm{e}^{8i(dx+c)}-13482\,\mathrm{e}^{7i(dx+c)}-5292\,\mathrm{e}^{6i(dx+c)}+10566\,\mathrm{e}^{5i(}\right.}{315d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^9\left(\mathrm{e}^{i(dx+c)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7-29/5*tan(1/2
*d*x+1/2*c)^5+64/3*tan(1/2*d*x+1/2*c)^3-99*tan(1/2*d*x+1/2*c)+128*arctan(ta
n(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+8/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 177, normalized size = 1.00

$$-\frac{\frac{\frac{31185\sin(dx+c)}{\cos(dx+c)+1}-\frac{6720\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1827\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{360\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3}-\frac{40320\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{105\left(\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20160*((31185*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 1827*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 360*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 -
40320*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 105*(24*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Fricas [A]
time = 2.89, size = 216, normalized size = 1.22

$$\frac{668\cos(dx+c)^6+1059\cos(dx+c)^5-573\cos(dx+c)^4-1813\cos(dx+c)^3-393\cos(dx+c)^2+315\left(dx\cos(dx+c)^5+3\,dx\cos(dx+c)^4+2\,dx\cos(dx+c)^3-2\,dx\cos(dx+c)^2-3\,dx\cos(dx+c)-dx\right)\sin(dx+c)+789\cos(dx+c)+368}{315\left(a^3d\cos(dx+c)^5+3\,a^3d\cos(dx+c)^4+2\,a^3d\cos(dx+c)^3-2\,a^3d\cos(dx+c)^2-3\,a^3d\cos(dx+c)-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(668*cos(d*x + c)^6 + 1059*cos(d*x + c)^5 - 573*cos(d*x + c)^4 - 1813
*cos(d*x + c)^3 - 393*cos(d*x + c)^2 + 315*(d*x*cos(d*x + c)^5 + 3*d*x*cos(
d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 - 3*d*x*cos(d*x +
```

```
c) - d*x)*sin(d*x + c) + 789*cos(d*x + c) + 368)/((a^3*d*cos(d*x + c)^5 + 3
*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3
*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.62, size = 131, normalized size = 0.74

$$\frac{\frac{20160\,(dx+c)}{a^3}+\frac{105\left(24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{35\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1827\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6720\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+31185\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{27}}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/20160*(20160*(d*x + c)/a^3 + 105*(24*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*tan
(1/2*d*x + 1/2*c)^3) - (35*a^24*tan(1/2*d*x + 1/2*c)^9 - 360*a^24*tan(1/2*d
*x + 1/2*c)^7 + 1827*a^24*tan(1/2*d*x + 1/2*c)^5 - 6720*a^24*tan(1/2*d*x +
1/2*c)^3 + 31185*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

Mupad [B]
time = 2.08, size = 205, normalized size = 1.16

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9(c+dx)-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}(c+dx)}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\right)}-\frac{\frac{668\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{315}-\frac{983\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{210}+\frac{346\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{105}-\frac{2291\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{2520}+\frac{173\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{840}-\frac{19\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{672}+\frac{1}{576}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (cos(c/2 + (d*x)/2)^9*(c + d*x) - cos(c/2 + (d*x)/2)^11*(c + d*x))/(a^3*d*(
cos(c/2 + (d*x)/2)^9 - cos(c/2 + (d*x)/2)^11)) - ((173*cos(c/2 + (d*x)/2)^4
)/840 - (19*cos(c/2 + (d*x)/2)^2)/672 - (2291*cos(c/2 + (d*x)/2)^6)/2520 +
(346*cos(c/2 + (d*x)/2)^8)/105 - (983*cos(c/2 + (d*x)/2)^10)/210 + (668*cos
(c/2 + (d*x)/2)^12)/315 + 1/576)/(a^3*d*(cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*
x)/2) - cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)))
```

3.103 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=215

$$-\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+d}{a^3d}$$

```
[Out] -x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+1/7*c
ot(d*x+c)^7/a^3/d-1/9*cot(d*x+c)^9/a^3/d+4/11*cot(d*x+c)^11/a^3/d+3*csc(d*x
+c)/a^3/d-16/3*csc(d*x+c)^3/a^3/d+34/5*csc(d*x+c)^5/a^3/d-36/7*csc(d*x+c)^7
/a^3/d+19/9*csc(d*x+c)^9/a^3/d-4/11*csc(d*x+c)^11/a^3/d
```

Rubi [A]
time = 0.21, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{19\csc^9(c+dx)}{9a^3d}-\frac{36\csc^7(c+dx)}{7a^3d}+\frac{34\csc^5(c+dx)}{5a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + Cot[c + d*x]^7/(7*a^3*d) - Cot[c + d*x]^9/(9*a^3*d) + (4*Cot[c
 + d*x]^11)/(11*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (16*Csc[c + d*x]^3)/(3*
a^3*d) + (34*Csc[c + d*x]^5)/(5*a^3*d) - (36*Csc[c + d*x]^7)/(7*a^3*d) + (1
9*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \cot^{12}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)-3a^3\cot^{10}(c+dx)\csc^2(\right.}{a^6}$$
$$= -\frac{\int \cot^{12}(c+dx)\,dx}{a^3}+\frac{\int \cot^9(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^{11}(c+dx)\csc(}{a^3}$$
$$= \frac{\cot^{11}(c+dx)}{11a^3d}+\frac{\int \cot^{10}(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2(-1+x^2)^4\,dx,x,\csc(c+dx\right.}{a^3d}$$
$$= -\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int (x^2-4x^4+\right.}{}$$
$$= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+}{3a^3d}$$
$$= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+}{a^3d}$$
$$= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}$$
$$= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}$$
$$= -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+}{9a^3d}$$

Mathematica [A]
time = 3.80, size = 394, normalized size = 1.83

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(231(-25+28\cos(c+dx))\cot^2\left(\frac{c}{2}\right)\csc^4\left(\frac{1}{2}(c+dx)\right)+561145\sec^2\left(\frac{1}{2}(c+dx)\right)-184650\sec^4\left(\frac{1}{2}(c+dx)\right)+41320\sec^6\left(\frac{1}{2}(c+dx)\right)-5425\sec^8\left(\frac{1}{2}(c+dx)\right)+315\sec^{10}\left(\frac{1}{2}(c+dx)\right)-1736335\csc\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+561145\csc\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-184650\csc\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+41320\csc\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-5425\csc\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+315\csc\left(\frac{c}{2}\right)\sec^{11}\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+6468\csc^3\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\sin(c)\sin\left(\frac{dx}{2}\right)+231\cot\left(\frac{c}{2}\right)\left(3840dx-\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\left(743+3\csc^4\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}\right)\right)}{110880a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/110880*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(231*(-25 + 28*Cos[c + d*x])*C
ot[c/2]^2*Csc[(c + d*x)/2]^4 + 561145*Sec[(c + d*x)/2]^2 - 184650*Sec[(c +
d*x)/2]^4 + 41320*Sec[(c + d*x)/2]^6 - 5425*Sec[(c + d*x)/2]^8 + 315*Sec[(c
 + d*x)/2]^10 - 1736335*Csc[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 561145*Csc
[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 184650*Csc[c/2]*Sec[(c + d*x)/2]^5*
Sin[(d*x)/2] + 41320*Csc[c/2]*Sec[(c + d*x)/2]^7*Sin[(d*x)/2] - 5425*Csc[c/
2]*Sec[(c + d*x)/2]^9*Sin[(d*x)/2] + 315*Csc[c/2]*Sec[(c + d*x)/2]^11*Sin[(
d*x)/2] + 6468*Csc[c/2]^3*Csc[(c + d*x)/2]^3*Sin[c]*Sin[(d*x)/2] + 231*Cot[
c/2]*(3840*d*x - Csc[c/2]*Csc[(c + d*x)/2]*(743 + 3*Csc[(c + d*x)/2]^4)*Sin
[(d*x)/2]))*Tan[c/2]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 137, normalized size = 0.64

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{10\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{46\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+26\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{256\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+382\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-51}{256d\,a^{3}}$
default	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{10\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{46\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+26\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{256\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+382\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-51}{256d\,a^{3}}$
risch	$-\frac{x}{a^{3}}+\frac{2i\left(10395\,\mathrm{e}^{15i(dx+c)}+31185\,\mathrm{e}^{14i(dx+c)}+1155\,\mathrm{e}^{13i(dx+c)}-148995\,\mathrm{e}^{12i(dx+c)}-190113\,\mathrm{e}^{11i(dx+c)}+117117\,\mathrm{e}^{10i(dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/256/d/a^3*(-1/11*tan(1/2*d*x+1/2*c)^11+10/9*tan(1/2*d*x+1/2*c)^9-46/7*tan(1/2*d*x+1/2*c)^7+26*tan(1/2*d*x+1/2*c)^5-256/3*tan(1/2*d*x+1/2*c)^3+382*tan(1/2*d*x+1/2*c)-512*arctan(tan(1/2*d*x+1/2*c))-1/5/tan(1/2*d*x+1/2*c)^5+10/3/tan(1/2*d*x+1/2*c)^3-46/tan(1/2*d*x+1/2*c))

Maxima [A]
time = 0.49, size = 218, normalized size = 1.01

$$\frac{\frac{5\left(\frac{264726\sin(dx+c)}{\cos(dx+c)+1}-\frac{59136\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{18018\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{4554\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{770\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^{3}}-\frac{1774080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}+\frac{231\left(\frac{50\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{690\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-3\right)(\cos(dx+c)+1)^{5}}{a^{3}\sin(dx+c)^{5}}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/887040*(5*(264726*sin(d*x + c)/(cos(d*x + c) + 1) - 59136*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18018*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4554*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3 - 1774080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 231*(50*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 690*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^3*sin(d*x + c)^5))/d

Fricas [A]
time = 3.07, size = 282, normalized size = 1.31

$$-\frac{7453\cos(dx+c)^{8}+11964\cos(dx+c)^{7}-11866\cos(dx+c)^{6}-30542\cos(dx+c)^{5}+90\cos(dx+c)^{4}+26438\cos(dx+c)^{3}+8539\cos(dx+c)^{2}+3465\left(dx\cos(dx+c)^{7}+3\,dx\cos(dx+c)^{6}+dx\cos(dx+c)^{5}-5\,dx\cos(dx+c)^{4}-5\,dx\cos(dx+c)^{3}+dx\cos(dx+c)^{2}+3\,dx\cos(dx+c)+dx\right)\sin(dx+c)-7671\cos(dx+c)-3712}{3465\left(a^{3}d\cos(dx+c)^{7}+3\,a^{3}d\cos(dx+c)^{6}+a^{3}d\cos(dx+c)^{5}-5\,a^{3}d\cos(dx+c)^{4}-5\,a^{3}d\cos(dx+c)^{3}+a^{3}d\cos(dx+c)^{2}+3\,a^{3}d\cos(dx+c)+a^{3}d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/3465*(7453*cos(d*x + c)^8 + 11964*cos(d*x + c)^7 - 11866*cos(d*x + c)^6 - 30542*cos(d*x + c)^5 + 90*cos(d*x + c)^4 + 26438*cos(d*x + c)^3 + 8539*co

```
s(d*x + c)^2 + 3465*(d*x*cos(d*x + c)^7 + 3*d*x*cos(d*x + c)^6 + d*x*cos(d*
x + c)^5 - 5*d*x*cos(d*x + c)^4 - 5*d*x*cos(d*x + c)^3 + d*x*cos(d*x + c)^2
 + 3*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 7671*cos(d*x + c) - 3712)/((a^3
*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d
*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [**A**]
time = 0.63, size = 160, normalized size = 0.74

$$-\frac{\frac{887040\,(dx+c)}{a^3}+\frac{231\left(690\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-50\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{5\left(63\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-770\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4554\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-18018\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+59136\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-264726\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{33}}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(887040*(d*x + c)/a^3 + 231*(690*tan(1/2*d*x + 1/2*c)^4 - 50*tan(
1/2*d*x + 1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) + 5*(63*a^30*tan(1/2*d
*x + 1/2*c)^11 - 770*a^30*tan(1/2*d*x + 1/2*c)^9 + 4554*a^30*tan(1/2*d*x +
1/2*c)^7 - 18018*a^30*tan(1/2*d*x + 1/2*c)^5 + 59136*a^30*tan(1/2*d*x + 1/2
*c)^3 - 264726*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

Mupad [**B**]
time = 3.24, size = 254, normalized size = 1.18

$$-\frac{693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+315\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-3850\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+22770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-90090\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+295680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-1323630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+159390\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-11550\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+887040\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5(c+dx)}{887040\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 - 3850*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 22770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)
/2)^12 - 90090*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680*cos(c/2
+ (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 1323630*cos(c/2 + (d*x)/2)^10*sin(c/2 +
 (d*x)/2)^6 + 159390*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 11550*cos
(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 887040*cos(c/2 + (d*x)/2)^11*sin(
c/2 + (d*x)/2)^5*(c + d*x))/(887040*a^3*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (
d*x)/2)^5)
```

3.104 $\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=310

$$\frac{ae^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{ae^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] 1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2*
a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*a*e^
(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)
+1/4*a*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)-6/5*a*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*c
)^(1/2)-6/5*a*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/15*e*(5*a+3*a*sec(d*x+c
))*(e*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3966, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{ae^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{ae^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{6ae^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}-\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}+\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (6*a*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*d*x]]) - (6
*a*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) + (2*e*(5*a + 3*a*Sec[c + d
*x])*(e*Tan[c + d*x])^(3/2))/(15*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{1}{5}\left(2e^2\right)\int \left(\frac{5a}{2}+\right.$$

$$= \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{1}{5}\left(3ae^2\right)\int \sec(c$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))}{15d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))}{15d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))}{15d}$$

$$= \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos}{}$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{ae^{5/2}\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.21, size = 186, normalized size = 0.60

$$\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12+20\cos(c+dx)-36\cos^2(c+dx)+\frac{24\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)}{\sqrt{\sec^2(c+dx)}}+15\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\cot^2(c+dx)\sqrt{\sin(2(c+dx))}+15\cot^2(c+dx)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)(e\tan(c+dx))^{5/2}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*(12 + 20*Cos[c + d*x]
 - 36*Cos[c + d*x]^2 + (24*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2
])/Sqrt[Sec[c + d*x]^2] + 15*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*
x]^2*Sqrt[Sin[2*(c + d*x)]] + 15*Cot[c + d*x]^2*Log[Cos[c + d*x] + Sin[c +
d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])*(e*Tan[c + d*x])^(5/
2))/(60*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.36, size = 1519, normalized size = 4.90

method	result	size
default	Expression too large to display	1519

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*a/d*(-1+cos(d*x+c))^2*(15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^3*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))+36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*cos(d*x+c)^3*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))+36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*El
lipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-15*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d
*x+c)^3*2^(1/2)+24*cos(d*x+c)^2*2^(1/2)-10*2^(1/2)*cos(d*x+c)-6*2^(1/2))*(e
*sin(d*x+c)/cos(d*x+c))^(5/2)*(1+cos(d*x+c))^2/sin(d*x+c)^7*2^(1/2)
```

Maxima [A]
time = 0.23, size = 122, normalized size = 0.39

$$-\frac{\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+3\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-8\tan\left(dx+c\right)^{\frac{3}{2}}\right)ae^{\frac{5}{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 3*sqrt(2)*lo
g(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 3*sqrt(2)*log(-sqrt(2)*s
qrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*tan(d*x + c)^(3/2))*a*e^(5/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}\,dx+\int\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(5/2), x) + Integral((e*tan(c + d*x))**(5/2)*
sec(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan(c+d\,x))^{5/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.105 $\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=282

$$\frac{ae^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{ae^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{\right.}$$

```
[Out] 1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2*
a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*a*e^
(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)
-1/4*a*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)+1/3*a*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF
(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c)
)^(1/2)+2/3*e*(3*a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{ae^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{ae^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3d\sqrt{e\tan(c+dx)}}+\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) + (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a*e^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d*x]]) + (2*e
*(3*a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx &= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}\left(2e^2\right)\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec\cdots}{\sqrt{e\tan(c\cdots}}\\
&= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}\left(ae^2\right)\int \frac{\sec(c+\cdots}{\sqrt{e\tan(c\cdots}}\\
&= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1\cdots}{\sqrt{x}\,(e\cdots}\right.}{\cdots}\\
&= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4\cdots}\right.}{\cdots}\\
&= -\frac{ae^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}}+\frac{2e(3a+\cdots}{\cdots}\\
&= -\frac{ae^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}}+\frac{2e(3a+\cdots}{\cdots}\\
&= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d}-\frac{a\cdots}{\cdots}\\
&= \frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{ae^{3/2}\tan^{-1}\left(1+\frac{\sqrt{\cdots}}{\cdots}\right.}{\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.18, size = 214, normalized size = 0.76

$$
-\frac{ae\cos(2(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\sqrt{e\tan(c+dx)}\left(4\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\middle|-1\right)\sqrt{\tan(c+dx)}+\sqrt{\sec^2(c+dx)}\left(12\sin(c+dx)+3\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}-3\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}+4\tan(c+dx)\right)\right)}{12d\left(-1+\tan^2(c+dx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(a*e*Cos[2*(c + d*x)]*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c +
d*x]^2]*Sqrt[e*Tan[c + d*x]]*(4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]] + Sqrt[Sec[c + d*x]^2]*(12*Sin[c
 + d*x] + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - 3*
Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d
*x)]] + 4*Tan[c + d*x])))/(d*(-1 + Tan[c + d*x]^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 698, normalized size = 2.48

method	result
default	$\frac{a(-1+\cos(dx+c))\left(-3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/6*a/d*(-1+cos(d*x+c))*(-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)-4*2^(1/2)*cos(d*x+c)-2*2^(1/2))*(e*sin(d*x+c)/cos(d*x+c))^(3/2)*(1+cos(d*x+c))^2/sin(d*x+c)^5*2^(1/2)

Maxima [A]
time = 0.24, size = 121, normalized size = 0.43

$$-\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-8\sqrt{\tan\left(dx+c\right)}\right)ae^{\frac{3}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sqrt(tan(d*x + c)))*a*e^(3/2)/d

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}\,dx+\int\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(3/2), x) + Integral((e*tan(c + d*x))**(3/2)*
sec(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.106 $\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{a\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a\sqrt{e}\,\log\left(\sqrt{e}+\right.}{}$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*2^(1/2
)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)
/d*2^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*c)^(1/
2)+2*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e
```

Rubi [A]
time = 0.17, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{a\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}+\frac{a\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{a\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
)) + (a*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*d) + (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*d) - (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a*Cos[c + d*x]*EllipticE[
c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*
Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx = a\int \sqrt{e\tan(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - (2a)\int \cos(c+dx)\sqrt{e\tan(c\cdot}$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{(2ae)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{\cdot}\right.}{d}$$

$$= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{(ae)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e}\right.}{d}$$

$$= -\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+}{}$$

$$= \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a\cdot}{}$$

$$= -\frac{a\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a\sqrt{e}\tan^{-1}\left(1+\cdot\right.}{\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.48, size = 182, normalized size = 0.67

$$-\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(3\sqrt{\sec^2(c+dx)}\left(-4\sin^2(c+dx)+\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}+\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)+8\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)\tan^2(c+dx)\right)}{12d\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(-4*Sin[c + d*x]^2 + ArcSin[Cos[c + d*x] - Si
n[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt
[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hypergeometric2F1[3/4, 3/2,
 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*Sqrt[Sec[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 1429, normalized size = 5.25

method	result	size

default	Expression too large to display	1429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*(I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I
*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+cos(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+cos(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*cos(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*cos(d*x+c)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))+(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+(-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))-4*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*
2^(1/2)*cos(d*x+c)-2*2^(1/2))/sin(d*x+c)^5*2^(1/2)
```

Maxima [A]

time = 0.23, size = 111, normalized size = 0.41

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)\right)ae^{\frac{1}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sq
rt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(ta
n(d*x + c)) + tan(d*x + c) + 1))*a*e^(1/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\tan\left(c+dx\right)}\,dx+\int\sqrt{e\tan\left(c+dx\right)}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(sqrt(e*tan(c + d*x))*sec(c
+ d*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{e\tan\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.107 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+\right.}{2\sqrt{2}}$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/e^(1/2
)+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/
2)/e^(1/2)-a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+
1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt
[e]) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*Sqrt[e]) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a*EllipticF[c - Pi/4 + d*x,
 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{(ae)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{d} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+d}}}{\sqrt{\cos(c+dx)}\,\sqrt{e\tan}}$$

$$= \frac{(2ae)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(a\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int}{\sqrt{e\tan(c+dx}}$$

$$= \frac{aF\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(}\right.}{d}$$

$$= \frac{aF\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx, x\right.}{2d}$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d\sqrt{e}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

$$= -\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} - \frac{}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 1.77, size = 220, normalized size = 0.90

$$\frac{20aF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\sin(c+dx)}{d\left(2\left(2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+5F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)\sqrt{e\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (20*a*AppellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x])/(d*(2*(2*AppellF1[5/4, 1
/2, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/4, 3/2, 1
, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*Ap
pellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[
c + d*x]))*Sqrt[e*Tan[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 289, normalized size = 1.18

method	result
default	$-\frac{a\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)^2/cos(d*x+c)*(1+cos(d*x+c))^2/(
e*sin(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)
```

Maxima [A]
time = 0.23, size = 111, normalized size = 0.45

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)ae^{\left(-\frac{1}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*log(sq
rt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)*sqrt(ta
n(d*x + c)) + tan(d*x + c) + 1))*a*e^(-1/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + \int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
[Out] a*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*tan(c
 + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*tan(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2), x)
```

3.108 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)\right.}{2\sqrt{2}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/2*
a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*a*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2^(1/2)
+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2
)*2^(1/2)-2*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c
+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2)
)*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d*x+2*c)^(1/2)+2*a*cos(d*x+c)*(e*tan(d*x
+c))^(3/2)/d/e^3
```

Rubi [A]
time = 0.21, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}}-\frac{a\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}+\frac{a\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}-\frac{2a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(3/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (2*(a + a*Sec[c + d*x]))/(d*e*S
qrt[e*Tan[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[
e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*Cos[c + d*x]*(e*Tan[
c + d*x])^(3/2))/(d*e^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2\int\left(-\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{a\int\sqrt{e\tan(c+dx)}\,dx}{e^2} + \frac{a\int\sec(c+dx)\sqrt{e\tan(c+dx}}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} - \frac{(2a)\int\cos(c+dx)\sqrt{}}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} - \frac{(2a)\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,d\right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} + \frac{a\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x\right.}{de}$$

$$= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} + \frac{2a\,c}{}$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{3/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}} - \frac{a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.59, size = 196, normalized size = 0.64

$$-\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(3\sqrt{\sec^2(c+dx)}\left(2+4\cos(c+dx)+2\cos(2(c+dx))-\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}-\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)+8\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)\tan^2(c+dx)\right)}{12de^2\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(2 + 4*Cos[c + d*x] + 2*Cos[2*(c + d*x)] - Ar
cSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - Log[Cos[c + d*x]
 + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hyper
geometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*e^2*Sqrt[S
ec[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1414, normalized size = 4.64

method	result	size
default	Expression too large to display	1414

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-c
os(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*cos(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-(-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))+2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(
1/2))+4*2^(1/2)*cos(d*x+c))*sin(d*x+c)/cos(d*x+c)^2/(e*sin(d*x+c)/cos(d*x+c
))^(3/2)*2^(1/2)
```

Maxima [A]
time = 0.24, size = 121, normalized size = 0.40

$$-\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\frac{8}{\sqrt{\tan\left(dx+c\right)}}\right)ae^{\left(-\frac{3}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(t
an(d*x + c)) + tan(d*x + c) + 1) + 8/sqrt(tan(d*x + c)))*a*e^(-3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\,\tan\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2), x)
```

3.109 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=282

$$\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}\,a}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/2*
a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*a*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2^(1/2)
-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2
)*2^(1/2)+1/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos
(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*tan(d*x+c)
)^(1/2)-2/3*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3de^2\sqrt{e\tan(c+dx)}}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(5/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (2*(a + a*Sec[c + d*x]))/(3*d*e
*(e*Tan[c + d*x])^(3/2)) - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqr
t[Sin[2*c + 2*d*x]])/(3*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2} - \frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{\left(a\sqrt{\text{si}\right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{(2a)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{\left(a\sec(\right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} - \frac{aS}{}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{aF\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} + \frac{aS}{}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{5/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\,\tan\right.}{}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} +$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 11.62, size = 200, normalized size = 0.71

$$-\frac{a\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}\left(2\cot\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)\csc\left(\frac{1}{2}(c+dx)\right)-3\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}+3\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)-4\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\middle|-1\right)\sqrt{\tan(c+dx)}\right)\sqrt{e\tan(c+dx)}}{6de^3\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/6*(a*Csc[c + d*x]*(Sqrt[Sec[c + d*x]^2]*(2*Cot[(c + d*x)/2] + 2*Cos[(3*(
c + d*x))/2]*Csc[(c + d*x)/2] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[
Sin[2*(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x
)]]]*Sqrt[Sin[2*(c + d*x)]]) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]])*Sqrt[e*Tan[c + d*x]])/(d*e^3*S
qrt[Sec[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 658, normalized size = 2.33

method	result
default	$\frac{a(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(3i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\ \sqrt{\ }\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*EllipticPi((-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*sin(d*x+c)*EllipticPi((-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-4*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)+2*2^(1/2)*cos(d*x+c))/cos(d*x+c)^3/(e*sin(d*x+c)/cos(d*
x+c))^(5/2)/sin(d*x+c)*2^(1/2)
```

Maxima [A]
time = 0.23, size = 122, normalized size = 0.43

$$-\frac{\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-3\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{8}{\tan(dx+c)^{\frac{3}{2}}}\right)ae^{\left(-\frac{5}{2}\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*lo
g(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*log(-sqrt(2)*s
qrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8/tan(d*x + c)^(3/2))*a*e^(-5/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{(e\tan{(c+dx)})^{\frac{5}{2}}}\,dx+\int \frac{\sec{(c+dx)}}{(e\tan{(c+dx)})^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{(e\,\tan\left(c+d\,x\right))}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2), x)
```

3.110 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+\ldots\right.}{2\sqrt{2}\ldots}$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*2^(1/2
)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/
2)*2^(1/2)+2/5*(5*a+3*a*sec(d*x+c))/d/e^3/(e*tan(d*x+c))^(1/2)-6/5*a*cos(d*
x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d
*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^4/sin(2*d*x+2*c)^(1/2)-2/5*(a+a*sec(d
*x+c))/d/e/(e*tan(d*x+c))^(5/2)-6/5*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e^5
```

Rubi [A]
time = 0.27, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{7/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-\frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}+\frac{6a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}+\frac{2(3a\sec(c+dx)+5a)}{5de^3\sqrt{e\tan(c+dx)}}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7
/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*e^(7/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (2*(a + a*Sec[c + d*x]))/(5*
d*e*(e*Tan[c + d*x])^(5/2)) + (2*(5*a + 3*a*Sec[c + d*x]))/(5*d*e^3*Sqrt[e*
Tan[c + d*x]]) + (6*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[
c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]]) - (6*a*Cos[c + d*x]*(e*Tan[c +
d*x])^(3/2))/(5*d*e^5)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2\int \frac{-\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{4\int \left(\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{}}{5e^4}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{(3a)\int \sec(c+dx)\sqrt{e\tan(c}}{5e^4}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx}{5de^5}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx}{5de^5}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx}{5de^5}$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+d\right.}{5de^4\sqrt{\sin(2c}}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{7/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

$$= -\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.57, size = 254, normalized size = 0.73

$$-\frac{a\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\left(2\cot\left(\frac{1}{2}(c+dx)\right)-19\sin(c+dx)+12\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)-8\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(c+dx)\right)\sqrt{\sec^2(c+dx)}\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)+5\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)+5\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)+5\sin(c+dx)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{20de^3\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/20*(a*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1 + Sec[c + d*x])*(2*Cot[(c + d
*x)/2] - 19*Sin[c + d*x] + 12*Sin[c + d*x]^2*Tan[(c + d*x)/2] - 8*Hypergeom
etric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Sqrt[Sec[c + d*x]^2]*Sin[c + d*x]^
2*Tan[(c + d*x)/2] + 5*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c +
```

```
d*x)]]*Tan[(c + d*x)/2] + 5*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c
 + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Tan[(c + d*x)/2] + 5*Sin[c + d*x]*Tan[(c
+ d*x)/2]^2))/(d*e^3*Sqrt[e*Tan[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 1451, normalized size = 4.19

method	result	size
default	Expression too large to display	1451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a/d*(-5*I*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))+5*I*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))+6*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-1
2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellipti
cE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*I*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*(-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-6*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*((-1+cos(d*
```

```
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+18*cos(d*x+c)^2*2^(1/2)-16*2^(1/2)*cos
(d*x+c))*sin(d*x+c)^3/(-1+cos(d*x+c))/cos(d*x+c)^4/(e*sin(d*x+c)/cos(d*x+c)
)^(7/2)*2^(1/2)
```

Maxima [**A**]
time = 0.24, size = 134, normalized size = 0.39

$$\frac{\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+5\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\frac{8\left(5\tan\left(dx+c\right)^{2}-1\right)}{\tan\left(dx+c\right)^{\frac{5}{2}}}\right)ae^{\left(-\frac{7}{2}\right)}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 10*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 5*sqrt(2)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 5*sqrt(2)*log(-sqrt(2)*
sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(5*tan(d*x + c)^2 - 1)/tan(d*x +
 c)^(5/2))*a*e^(-7/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\frac{\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-7/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(7/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\,\tan\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2), x)
```

3.111 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=366

$$\frac{a^2e^{5/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{a^2e^{5/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}+\right.}{}$$

```
[Out] 1/2*a^2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
2*a^2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*
a^2*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)+1/4*a^2*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/d*2^(1/2)-12/5*a^2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/si
n(2*d*x+2*c)^(1/2)+2/3*a^2*e*(e*tan(d*x+c))^(3/2)/d-12/5*a^2*e*cos(d*x+c)*(
e*tan(d*x+c))^(3/2)/d+4/5*a^2*e*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/7*a^2*(
e*tan(d*x+c))^(7/2)/d/e
```

Rubi [A]
time = 0.29, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3971, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2691, 2693, 2695, 2652, 2719, 2687, 32}

$$\frac{a^2e^{5/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{a^2e^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{12a^2e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}+\frac{2a^2(e\tan(c+dx))^{7/2}}{7de}+\frac{2a^2e(e\tan(c+dx))^{3/2}}{3d}-\frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}+\frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (12*a^2*e^2*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) - (12*a^2*e*Cos[c + d*x]*(e
*Tan[c + d*x])^(3/2))/(5*d) + (4*a^2*e*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))
/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \int \left(a^2(e\tan(c+dx))^{5/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{5/2}+\right.$$

$$= a^2\int (e\tan(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{5/}$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} +$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$

$$= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$

$$= \frac{12a^2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} + \frac{2a^2e(}{}$$

$$= -\frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d}$$

$$= \frac{a^2e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{a^2e^{5/2}\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 18.22, size = 117, normalized size = 0.32

$$\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\mathrm{ArcTan}(\tan(c+dx))\right)(e\tan(c+dx))^{3/2}\left(35-42\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-35\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)+42\sqrt{\sec^2(c+dx)}+15\tan^2(c+dx)\right)}{105d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*e*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^
(3/2)*(35 - 42*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 35*Hyper
geometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2] + 42*Sqrt[Sec[c + d*x]^2] + 15*T
an[c + d*x]^2))/(105*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 1542, normalized size = 4.21

method	result	size
default	Expression too large to display	1542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210*a^2/d*(-1+cos(d*x+c))^2*(105*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-105*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-504*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))+252*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^4*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2*2^(1/2))-105*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2-1/2*I,1/2*2^(1/2))+105*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
```

```
),1/2+1/2*I,1/2*2^(1/2))+105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))+105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-504*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*cos(d*x+c)^3*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))+252*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*c
os(d*x+c)^3*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))+212*cos(d*x+c)^4*2^(1/2)-336*cos(d*x+c)^3*2^(1/2)+10*cos(d*x+c)^2*2^
(1/2)+84*2^(1/2)*cos(d*x+c)+30*2^(1/2))*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(
d*x+c))^(5/2)/cos(d*x+c)/sin(d*x+c)^7*2^(1/2)
```

Maxima [A]
time = 0.23, size = 140, normalized size = 0.38

$$\frac{\left(24\,a^2\tan\left(dx+c\right)^{\frac{7}{2}}-7\left(6\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+2\,\sqrt{\tan\left(dx+c\right)}\right)\right)+6\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-2\,\sqrt{\tan\left(dx+c\right)}\right)\right)-3\,\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+3\,\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-8\,\tan\left(dx+c\right)^{\frac{3}{2}}\right)a^2\right)e^{\frac{5}{2}}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*a^2*tan(d*x + c)^(7/2) - 7*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2*sqrt(tan(d*x + c)))) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(
tan(d*x + c)))) - 3*sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
 1) + 3*sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*tan
(d*x + c)^(3/2))*a^2)*e^(5/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\tan(c+d\,x)\right)}^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.112 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=335

$$\frac{a^2e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{a^2e^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a^2e^{3/2}\log\left(\sqrt{e}+\right.}{}$$

```
[Out] 1/2*a^2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
2*a^2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*
a^2*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)-1/4*a^2*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/d*2^(1/2)+2/3*a^2*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*
tan(d*x+c))^(1/2)+2*a^2*e*(e*tan(d*x+c))^(1/2)/d+4/3*a^2*e*sec(d*x+c)*(e*ta
n(d*x+c))^(1/2)/d+2/5*a^2*(e*tan(d*x+c))^(5/2)/d/e
```

Rubi [A]
time = 0.27, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 32}

$$\frac{a^2e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{a^2e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}+\frac{a^2e^{3/2}\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{a^2e^{3/2}\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{2a^2e^2\sqrt{\sin(2c+2dx)}\,\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3d\sqrt{e\tan(c+dx)}}+\frac{2a^2(e\tan(c+dx))^{5/2}}{5de}+\frac{2a^2e\sqrt{e\tan(c+dx)}}{d}+\frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a^2*e^2*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d
*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d + (4*a^2*e*Sec[c + d*x]*Sqrt[e*Tan
[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx = \int \left(a^2(e\tan(c+dx))^{3/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}+\right.$$

$$= a^2\int (e\tan(c+dx))^{3/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} +$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} +$$

$$= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} +$$

$$= -\frac{2a^2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e}{}$$

$$= -\frac{2a^2e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e}{}$$

$$= \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d}$$

$$= \frac{a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{a^2e^{3/2}\tan^{-1}\left(1\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 53.80, size = 257, normalized size = 0.77

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\mathrm{ArcTan}(\tan(c+dx))\right)(e\tan(c+dx))^{3/2}\left(30\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-30\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+15\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-15\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+120\sqrt{\tan(c+dx)}-80\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)\sqrt{\tan(c+dx)}+80\sqrt{\sec^2(c+dx)}\sqrt{\tan(c+dx)}+24\tan^{\frac{5}{2}}(c+dx)\right)}{60d\tan^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^(3/2
)*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 30*Sqrt[2]*ArcTan[1
```

```
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] + 120*Sqrt[Tan[c + d*x]] - 80*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan
[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 80*Sqrt[Sec[c + d*x]^2]*Sqrt[Tan[c + d*x]
] + 24*Tan[c + d*x]^(5/2)))/(60*d*Tan[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 731, normalized size = 2.18

method	result
default	$\frac{a^2(-1+\cos(dx+c))\left(15i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}(\cos^2(dx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*a^2/d*(-1+cos(d*x+c))*(15*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*I*sin(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*sin(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
-10*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
+24*cos(d*x+c)^3*2^(1/2)-4*cos(d*x+c)^2*2^(1/2)-14*2^(1/2)*cos(d*x+c)-6*2^(
1/2))*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/cos(d*x+c)/sin(d*x+c
)^5*2^(1/2)
```

Maxima [A]
time = 0.23, size = 139, normalized size = 0.41

$$\frac{\left(8\,a^2\tan\left(dx+c\right)^{\frac{5}{2}}-5\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-8\sqrt{\tan\left(dx+c\right)}\right)a^2\right)e^{\frac{3}{2}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(8*a^2*tan(d*x + c)^(5/2) - 5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(t
an(d*x + c)))) + sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
 - sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sqrt(tan
(d*x + c)))*a^2)*e^(3/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (e\tan(c+dx))^{\frac{3}{2}}\,dx+\int 2(e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx+\int (e\tan(c+dx))^{\frac{3}{2}}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(3/2), x) + Integral(2*(e*tan(c + d*x))**(
3/2)*sec(c + d*x), x) + Integral((e*tan(c + d*x))**(3/2)*sec(c + d*x)**2, x
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.113 $\int (a+a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\, dx$

Optimal. Leaf size=309

$$-\frac{a^2\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{e}\,\log\left(\sqrt{e}\right.}{}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*
2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
*e^(1/2)/d*2^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*
x+2*c)^(1/2)+2/3*a^2*(e*tan(d*x+c))^(3/2)/d/e+4*a^2*cos(d*x+c)*(e*tan(d*x+c
))^(3/2)/d/e
```

Rubi [A]
time = 0.23, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 32}

$$-\frac{a^2\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{a^2\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c +
 d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]
]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) + (4*a^2*Cos[c + d*x]*(e*Tan[c
+ d*x])^(3/2))/(d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \int \left(a^2\sqrt{e\tan(c+dx)} + 2a^2\sec(c+dx)\sqrt{e\tan(c+dx)} + a^2\right.$$

$$= a^2\int \sqrt{e\tan(c+dx)}\,dx + a^2\int \sec^2(c+dx)\sqrt{e\tan(c+dx)}$$

$$= \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - (4a^2)\int \cos(c+dx)\sqrt{e\,\mathrm{ta}}$$

$$= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{(2}{}$$

$$= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{(e}{}$$

$$= -\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a^2(e\,\mathrm{t}}{}$$

$$= \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} -$$

$$= -\frac{a^2\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\tan^{-1}\left(1\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.37, size = 106, normalized size = 0.34

$$\frac{4a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\left(1+2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\mathrm{ArcTan}(\tan(c+dx))\right)\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (4*a^2*Cos[(c + d*x)/2]^5*(1 + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c +
d*x]^2] + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*Sec
[ArcTan[Tan[c + d*x]]/2]^4*Sin[(c + d*x)/2]*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 1504, normalized size = 4.87

method	result	size
default	Expression too large to display	1504

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2/d*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^
2*(3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c
)^2-3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)^2+3*I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))-3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+24*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*cos(d*x+c)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*cos(d*x+c)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*cos(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+24*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))-14*cos(d*x+c)^2*2^(1/2)+12*2^(1/2)*cos(d*x+c)+2*2^(1/
2))/sin(d*x+c)^5/cos(d*x+c)*2^(1/2)
```

Maxima [A]
time = 0.25, size = 129, normalized size = 0.42

$$\frac{\left(8\,a^2\tan\left(dx+c\right)^{\frac{3}{2}}+3\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)\right)a^2\right)e^{\frac{1}{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*a^2*tan(d*x + c)^(3/2) + 3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(t
an(d*x + c)))) - sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
 + sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^2)*e^(1/2
)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{e\tan\left(c+dx\right)}\,dx+\int 2\sqrt{e\tan\left(c+dx\right)}\sec\left(c+dx\right)dx+\int\sqrt{e\tan\left(c+dx\right)}\sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(2*sqrt(e*tan(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*tan(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\tan\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.114 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=278

$$-\frac{a^2\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{a^2\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(\right.}{2}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/
e^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/d*2^(1/2)/e^(1/2)-2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d
*x+c))^(1/2)+2*a^2*(e*tan(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.21, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 32}

$$-\frac{a^2\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{a^2\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}+\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[
e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (2*a^2*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]
]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}} + \frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right) dx$$

$$= a^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx + (2a^2)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+d}}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{ex}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(}{d}$$

$$= \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{(2a^2e)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(2a^2\,\text{se}\right.}{}$$

$$= \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{a^2\text{S}}{}$$

$$= \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} + \frac{a^2\text{S}}{}$$

$$= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= -\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 32.56, size = 220, normalized size = 0.79

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\text{ArcTan}(\tan(c+dx))\right)\left(-2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}+16\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}}{4d\sqrt{e\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(-2*Sqrt[2]*ArcTan[1
- Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d*x]]
 + 16*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]])
*Sqrt[Tan[c + d*x]])/(4*d*Sqrt[e*Tan[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.30, size = 663, normalized size = 2.38

method	result
default	$-\frac{a^2(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(I*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)-I*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+sin(d*x+c)*EllipticPi((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)+2*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)-2*2^(1/2)*cos(d*x+c)+2*2^(1/2))/sin(d*x+c)^3/cos(d*x+c)/(e*s
in(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)
```

Maxima [A]
time = 0.24, size = 128, normalized size = 0.46

$$\frac{\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)a^2+8a^2\sqrt{\tan(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)*sqrt(t
an(d*x + c)) + tan(d*x + c) + 1))*a^2 + 8*a^2*sqrt(tan(d*x + c)))*e^(-1/2)/
d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx+\int \frac{2\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx+\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
tan(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*tan(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\tan(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2), x)
```

3.115 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\,\tan(c\right.}{2\sqrt{}}$$

```
[Out] 1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/
2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*
a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2
^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d/e^(3/2)*2^(1/2)-4*a^2/d/e/(e*tan(d*x+c))^(1/2)-4*a^2*cos(d*x+c)/d/e/(e*ta
n(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d
*x+2*c)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 32}

$$\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}+\frac{a^2\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}-\frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}-\frac{4a^2}{de\sqrt{e\tan(c+dx)}}-\frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Tan
[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) - (4*a^2*Cos[
c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin
[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+}{(e\tan(c+}$$

$$= -\frac{2a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} + \frac{a^2\text{Subst}\left(\int \frac{1}{(ex)^{3/2}}\,dx, x, \tan(c+\right.}{d}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+\right.}{de}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\,\text{ta}}\right.}{de}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{de^2\sqrt{\sin(2c+2dx}}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{de^2\sqrt{\sin(2c+2dx}}$$

$$= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{3/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}} - \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.36, size = 238, normalized size = 0.77

$$\frac{a^2\left(-24\left(1+e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}\right)-3\sqrt{-1+e^{4i(c+dx)}}\,\text{ArcTan}\left(\sqrt{-1+e^{4i(c+dx)}}\right)+6\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)+8e^{3i(c+dx)}\sqrt{1-e^{4i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{4i(c+dx)}\right)\right)}{6de\left(1+e^{2i(c+dx)}\right)\sqrt{e\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-24*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))
 - 3*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
+ 6*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sq
rt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]] + 8*E^((3*I)*(c +
 d*x))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))]))/(6*d*e*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Tan[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 1416, normalized size = 4.57

method	result	size
default	Expression too large to display	1416

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*co
s(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-8*cos(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*((-1+cos(d*x+c)
```

```
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))-8*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))+8*2^(1/2)*cos(d*x+c))*sin(d*x+c)/cos(d*x+c)^2/(e*sin(d*x+c)/cos(d*x
+c))^(3/2)*2^(1/2)
```

Maxima [A]
time = 0.23, size = 138, normalized size = 0.45

$$-\frac{\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\frac{8}{\sqrt{\tan\left(dx+c\right)}}\right)a^2+\frac{8a^2}{\sqrt{\tan\left(dx+c\right)}}\right)e^{\left(-\frac{3}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1) + 8/sqrt(tan(d*x + c)))*a^2 + 8*a^2/sqrt(
tan(d*x + c)))*e^(-3/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{2\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec^{2}\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*ta
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(3/2),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2), x)
```

3.116 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=316

$$\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+\right.}{2\sqrt{2}}$$

```
[Out] 1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/
2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*
a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2
^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d/e^(5/2)*2^(1/2)+2/3*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*
tan(d*x+c))^(1/2)-4/3*a^2/d/e/(e*tan(d*x+c))^(3/2)-4/3*a^2*sec(d*x+c)/d/e/(
e*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.27, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 32}

$$\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}+\frac{a^2\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-\frac{a^2\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}-\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3de^2\sqrt{e\tan(c+dx)}}-\frac{4a^2}{3de(e\tan(c+dx))^{3/2}}-\frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Tan[c
 + d*x])^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^(3/2)) - (2*
a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*
e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+}{(e\tan(c+}$$

$$= -\frac{2a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} + \frac{a^2\text{Subst}\left(\int \frac{1}{(ex)^{5/2}}\,dx, x, \tan\right.}{d}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx,\right.}{de}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x,\right.}{de}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c}{3de^2\sqrt{e\tan(}}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c}{3de^2\sqrt{e\tan(}}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} - \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 35.18, size = 224, normalized size = 0.71

$$-\frac{a^2\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\text{ArcTan}(\tan(c+dx))\right)\left(16\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\tan^2(c+dx)\right)+16\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)+3\sqrt{2}\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{24de^2\sqrt{e\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/24*(a^2*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Cot[(c + d*x)/2]*Sec[ArcTan[Tan[
c + d*x]]/2]^4*(16*Hypergeometric2F1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] + 16*
Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*(2*ArcTan[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])*Tan[c + d*x]^(3/2)))/(d*e^2*Sqrt[e*Tan[c + d*x]
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 660, normalized size = 2.09

method	result
default	$\frac{a^2(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(3i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*sin(d*x+c)*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-2*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)+4*2^(1/2)*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^3/(e*sin(
d*x+c)/cos(d*x+c))^(5/2)*2^(1/2)
```

Maxima [A]
time = 0.24, size = 139, normalized size = 0.44

$$-\frac{\left(\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-3\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\frac{8}{\tan\left(dx+c\right)^{\frac{3}{2}}}\right)a^2+\frac{8a^2}{\tan\left(dx+c\right)^{\frac{3}{2}}}\right)e^{\left(-\frac{5}{2}\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

[Out] -1/12*((6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8/tan(d*x + c)^(3/2))*a^2 + 8*a^2/tan(d*x + c)^(3/2))*e^(-5/2)/d

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int \frac{2\sec\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int \frac{\sec^2\left(c+dx\right)}{\left(e\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(5/2),x)

[Out] a**2*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(2*sec(c + d*x)/(e*tan(c + d*x))**(5/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(5/2), x))

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(5/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}{{\left(e\,\tan\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2),x)

[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2), x)

3.117 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c\right.}{2\sqrt{}}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*
2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/d/e^(7/2)*2^(1/2)+2*a^2/d/e^3/(e*tan(d*x+c))^(1/2)+12/5*a^2*cos(d*x+c)/d/e
^3/(e*tan(d*x+c))^(1/2)-12/5*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin
(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/
e^4/sin(2*d*x+2*c)^(1/2)-4/5*a^2/d/e/(e*tan(d*x+c))^(5/2)-4/5*a^2*sec(d*x+c
)/d/e/(e*tan(d*x+c))^(5/2)
```

Rubi [A]

time = 0.33, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2689, 2688, 2695, 2652, 2719, 2687, 32}

$$-\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{7/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}+\frac{12a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}+\frac{2a^2}{de^3\sqrt{e\tan(c+dx)}}+\frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}-\frac{4a^2}{5de(e\tan(c+dx))^{5/2}}-\frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/
2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(5*d*e*(e*Ta
n[c + d*x])^(5/2)) - (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x])^(5/2)) +
(2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x])/(5*d*e^3*Sqrt[
e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e
*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{7/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+}{(e\tan(c+}$$

$$= -\frac{2a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{a^2\text{Subst}\left(\int \frac{1}{(ex)^{7/2}}\,dx, x, \tan\right.}{d}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{1}{5de}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{1}{5de}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{1}{5de}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{1}{5de}$$

$$= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{1}{5de}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,de^{7/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= -\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,de^{7/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 14.32, size = 2820, normalized size = 7.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((7*Cot[c/2])/(10*d) - (Cot[c/
2]*Csc[c/2 + (d*x)/2]^2)/(20*d) - (3*(4*Cos[c/2] - Cos[(3*c)/2] + Cos[(5*c)
/2])*Cos[d*x]*Sec[2*c]*Sin[c/2])/(10*d*(-1 + 2*Cos[c])) - (7*Csc[c/2]*Csc[c
/2 + (d*x)/2]*Sin[(d*x)/2])/(10*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sin[(d*
x)/2])/(20*d) - (3*(2 - 5*Cos[c] + 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*Sin[d*x]
)/(20*d*(-1 + 2*Cos[c])))*Sin[c + d*x]^2*Tan[c + d*x]^2)/(e*Tan[c + d*x])^(
7/2) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*
I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*
Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c +
 d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((
2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x]
)^(7/2)) + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x
)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*T
an[c + d*x])^(7/2)) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
 + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*
(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*C
os[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))
]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1
+ E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 +
 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTa
n[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d
*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)
```

```
))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x)
)]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I
)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c
+ d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(
c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3
*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4
*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[
2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(20*d*
E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2))
- (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E
^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(20*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/
(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan
[c + d*x])^(7/2)) + (E^(I*(c - d*x))*Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*
x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]
^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(8*d*Sqrt[((-I)*(-1 + E^((2*I
)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2
*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x
)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(40*d*E^(I*(3*c + d*x))*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*
(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*d*x)*(1 + E^((6*I)*c)
)*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(
c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + ...
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 1453, normalized size = 3.93

method	result	size
default	Expression too large to display	1453

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^2/d*(-5*I*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))+5*I*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
```

```
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*I*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*(-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))+5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+26*cos(d*x+c)^2*2^(1/2)-22*2^(1/2)
*cos(d*x+c))*sin(d*x+c)^3/(-1+cos(d*x+c))/cos(d*x+c)^4/(e*sin(d*x+c)/cos(d*
x+c))^(7/2)*2^(1/2)
```

Maxima [A]
time = 0.24, size = 151, normalized size = 0.41

$$\frac{\left(\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+5\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)+\frac{8\left(5\tan(dx+c)^{2}-1\right)}{\tan(dx+c)^{\frac{5}{2}}}\right)a^{2}-\frac{8a^{2}}{\tan(dx+c)^{\frac{5}{2}}}\right)e^{\left(-\frac{7}{2}\right)}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*((10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 10
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 5*sqrt(2)*
log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 5*sqrt(2)*log(-sqrt(2)
```

```
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(5*tan(d*x + c)^2 - 1)/tan(d*x
+ c)^(5/2))*a^2 - 8*a^2/tan(d*x + c)^(5/2))*e^(-7/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\tan\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2), x)
```

3.118 $\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=330

$$\frac{e^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{11/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] 1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2
*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^
(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1
/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c)
)/a/d*2^(1/2)-5/21*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipt
icF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d
*x+c))^(1/2)+2/21*e^5*(21-5*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d-2/35*e^3*(
7-5*sec(d*x+c))*(e*tan(d*x+c))^(5/2)/a/d
```

Rubi [A]

time = 0.30, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3973, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{e^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{11/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}+\frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}+\frac{5e^6\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{21ad\sqrt{e\tan(c+dx)}}+\frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad}-\frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
 - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + S
qrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (5*e^6*EllipticF[c - Pi/4 +
 d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(21*a*d*Sqrt[e*Tan[c + d*x]])
 + (2*e^5*(21 - 5*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(21*a*d) - (2*e^3*(7
- 5*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))/(35*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{7/2}\,dx}{a^2}$$

$$= -\frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{(2e^4)\int\left(-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)\right)(e\cdots}{7a^2}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx\cdots}{35ad}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx\cdots}{35ad}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx\cdots}{35ad}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx\cdots}{35ad}$$

$$= \frac{5e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{\cdots}}{21ad}$$

$$= \frac{5e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{\cdots}}{21ad}$$

$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{11/2}\log\left(\sqrt{e}+\cdots\right.}{\cdots}$$

$$= \frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{11/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx\cdots}}{\sqrt{e}}\right.}{\sqrt{2}ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 48.74, size = 332, normalized size = 1.01

$$\frac{e^5\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\sqrt{e\tan(c+dx)}\left(70\sqrt{2}\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-70\sqrt{2}\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+35\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-35\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+280\sqrt{\tan(c+dx)}-320\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\tan^2(c+dx)\right)\sqrt{\tan(c+dx)}+280\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)\sqrt{\tan(c+dx)}+40\sqrt{\sec^2(c+dx)}\sqrt{\tan(c+dx)}-56\tan^{\frac{5}{2}}(c+dx)+40\sqrt{\sec^2(c+dx)}\tan^{\frac{5}{2}}(c+dx)\right)}{70ad(1+\sec(c+dx))^2\sqrt{\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
[Out] (e^5*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[
c + d*x]]*(70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 70*Sqrt[2]*A
```

```
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 35*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - 35*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]] + 280*Sqrt[Tan[c + d*x]] - 320*Hypergeometric2F1[-1/2, 1/4,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 280*Hypergeometric2F1[1/4, 1/2,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 40*Sqrt[Sec[c + d*x]^2]*Sqrt[Ta
n[c + d*x]] - 56*Tan[c + d*x]^(5/2) + 40*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x]^
(5/2)))/(70*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 744, normalized size = 2.25

method	result
default	$\frac{(-1+\cos(dx+c))\left(105i\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210/a/d*(-1+cos(d*x+c))*(105*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)^3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-105*I*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-260*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+105*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+105*(-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))+252*cos(d*x+c)^4*2^(1/2)-332*cos(d*x+c)^3*2^(1/2)+38*cos(d*x+c)^2*2^(1
/2)+72*2^(1/2)*cos(d*x+c)-30*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(
d*x+c)/cos(d*x+c))^(11/2)/sin(d*x+c)^9*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(11/2)*integrate(tan(d*x + c)^(11/2)/(a*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\tan(c+d\,x)\right)}^{11/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(11/2))/(a*(cos(c + d*x) + 1)), x)
```

3.119 $\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=326

$$-\frac{e^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{9/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\,\text{t}\right.}{}$$

```
[Out] -1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/2
*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^(
9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2
)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
/d*2^(1/2)-6/5*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*
c)^(1/2)-6/5*e^3*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d-2/15*e^3*(5-3*sec(d*x+
c))*(e*tan(d*x+c))^(3/2)/a/d
```

Rubi [A]
time = 0.28, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3973, 3966, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{e^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}+\frac{6e^4\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}}-\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad}-\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (6*e^4*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d*Sqrt[Sin[2*c + 2*d*x]]) -
 (6*e^3*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a*d) - (2*e^3*(5 - 3*Sec[c
+ d*x])*(e*Tan[c + d*x])^(3/2))/(15*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{(2e^4)\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{e}}{5a^2}$$

$$= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} - \frac{(3e^4)\int\sec(c+dx)\sqrt{e\tan(c+dx)}}{5a}$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} +$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} +$$

$$= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} -$$

$$= \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} - \frac{6e^3\cos(c+dx)(e\tan(c+d}{5ad}$$

$$= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} - \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= -\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{e^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 22.16, size = 129, normalized size = 0.40

$$\frac{4e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1+{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)(e\tan(c+dx))^{3/2}}{3ad(1+\sec(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*e^3*Cos[(c + d*x)/2]^2*(-1 + Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c +
d*x]^2] - Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + Hypergeometri
c2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])
*(e*Tan[c + d*x])^(3/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1529, normalized size = 4.69

method	result	size
default	Expression too large to display	1529

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/a/d*(-1+cos(d*x+c))^2*(-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))+36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^3*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))+15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))-18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))+36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*E
```

```
llipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-28*cos
(d*x+c)^3*2^(1/2)+24*cos(d*x+c)^2*2^(1/2)+10*2^(1/2)*cos(d*x+c)-6*2^(1/2))*
cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(9/2)/sin(d*x+c)^9*
2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(9/2)*integrate(tan(d*x + c)^(9/2)/(a*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)\ \left(e\tan (c+d\,x)\right)^{9/2}}{a\ \left(\cos (c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(9/2))/(a*(cos(c + d*x) + 1)), x)
```

3.120 $\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=295

$$-\frac{e^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{7/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] -1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/2
*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^(
7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2
)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
/d*2^(1/2)+1/3*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(
cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c
))^(1/2)-2/3*e^3*(3-sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.25, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3973, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$-\frac{e^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}+\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{e^4\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3ad\sqrt{e\tan(c+dx)}}-\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (e^4*EllipticF[c - Pi/4 + d*
x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]]) - (
2*e^3*(3 - Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{(2e^4)\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{e^4\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{e^4\int \frac{}{\sqrt{e\tan}}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^5\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+\right.}{ad}$$

$$= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{(2e^5)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+}\right.}{ad}$$

$$= -\frac{e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan}}{3ad}$$

$$= -\frac{e^4F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan}}{3ad}$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= -\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{e^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 79.69, size = 271, normalized size = 0.92

$$\frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\left(-2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-8\sqrt{\tan(c+dx)}+8\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\tan^2(c+dx)\right)\sqrt{\tan(c+dx)}-8\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)\sqrt{\tan(c+dx)}\right)\sqrt{e\tan(c+dx)}}{2ad(1+\sec(c+dx))^2\sqrt{\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*(-2*Sqrt[2]
*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
```

```
] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 8*Sqrt[Tan
[c + d*x]] + 8*Hypergeometric2F1[-1/2, 1/4, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[
c + d*x]] - 8*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c
+ d*x]])*Sqrt[e*Tan[c + d*x]])/(2*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x
]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 708, normalized size = 2.40

method	result
default	$-\frac{(-1+\cos(dx+c))\left(3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/d*(-1+cos(d*x+c))*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*sin(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*cos(d*x+c)^2*2
^(1/2)-8*2^(1/2)*cos(d*x+c)+2*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2*(e*sin
(d*x+c)/cos(d*x+c))^(7/2)/sin(d*x+c)^7*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(tan(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)\ {(e\tan(c+d\,x))}^{7/2}}{a\ (\cos (c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

3.121 $\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\,\text{ta}\right.}{}$$

```
[Out] 1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2*
e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^(5
/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)
+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/
d*2^(1/2)+2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*El
lipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^
(1/2)+2*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3973, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}+\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{2e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}+\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (2*e^2*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*e
*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int(-a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{a^2}$$

$$= -\frac{e^2\int\sqrt{e\tan(c+dx)}\,dx}{a} + \frac{e^2\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{(2e^2)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} - \frac{e^3\mathrm{Su}}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{(2e^3)\,\mathrm{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} -$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} + \frac{e^3\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{e^3}{}$$

$$= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+d}{ad}$$

$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} - \frac{e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 15.05, size = 105, normalized size = 0.37

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left({}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\left(1+\sqrt{\sec^2(c+dx)}\right)(e\tan(c+dx))^{5/2}}{3ad(1+\sec(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Csc[c + d*x]*(Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c
 + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(1 + Sqrt[Sec
[c + d*x]^2])*(e*Tan[c + d*x])^(5/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1443, normalized size = 5.06

method	result	size
default	Expression too large to display	1443

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))+I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(d*x+c)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*cos(d*x+c)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cos(d*x+c)-2*2^(1/2))*cos(d*x+c)^2*(1+cos
(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(5/2)/sin(d*x+c)^7*2^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(5/2)*integrate(tan(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x) + 1), x)/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{5/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.122 $\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=257

$$\frac{e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\,\text{ta}\right.}{}$$

```
[Out] 1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2*
e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^(3
/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)
-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/
d*2^(1/2)-e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c
+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3973, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}+\frac{e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{ad\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (e^2*EllipticF[c - Pi/4 + d*x,
2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a*d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

$$= -\frac{e^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} + \frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} + \frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+}}}{a\sqrt{\cos(c+dx)}\,\sqrt{e\,\text{t}}}$$

$$= -\frac{(2e^3)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\left(e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right.}{a\sqrt{e\tan(c+}}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} - \frac{e^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan}\right.}{ad}$$

$$= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} + \frac{e^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}\,x-x^2}\right.}{2\sqrt{2}\,a}$$

$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} - \frac{e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 22.83, size = 1211, normalized size = 4.71

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Csc[c + d*x]*((8*Cos[c]*Cos[d*x]*Sec[2*c]*Sin[c/2]^2)
/d - (16*Cos[c/2]*Sec[2*c]*Sin[c/2]^3*Sin[d*x])/d)*(e*Tan[c + d*x])^(3/2))/
(a + a*Sec[c + d*x]) - (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Se
c[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*
```

```
Tan[c + d*x]^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)
*(c + d*x)))]*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)
))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2
*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]
^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))
]*(Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/
2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2*d*E^((2*I)*
c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2)) + (S
qrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 +
(d*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*E^(I*(2
*c + d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/
2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Co
s[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 +
 E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4
, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*
E^(I*d*x)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2
)) - (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt
[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*(e*Tan[c + d*x])^(3/2))/(d*(a + a*Sec[c
 + d*x])*Tan[c + d*x]^(3/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 325, normalized size = 1.26

method	result
default	$\frac{(1+\cos(dx+c))^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/d*(1+cos(d*x+c))^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF((-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(e*sin(d*x+c)/cos(d*x+c))^(3/
2)*cos(d*x+c)/sin(d*x+c)^4*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(3/2)*integrate(tan(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.123 $\int \frac{\sqrt{e \tan(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(1/2
)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a
/d*2^(1/2)+2*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))
*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^(1/2)+2*cos(d*x+c)*(e*tan(d*x+c))^
(3/2)/a/d/e
```

Rubi [A]
time = 0.26, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}+\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade}+\frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*e*(1 - Sec[c + d*x]))/(a*
d*Sqrt[e*Tan[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt
[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c +
 d*x])^(3/2))/(a*d*e)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}$$
$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{a^2}$$
$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} + \frac{\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a}$$
$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} - \frac{2\int \cos(c+dx)\sqrt{e\tan(c}{a}$$
$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} + \frac{(2e)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x,\right.}{ad}$$
$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} - \frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{\right.}{ad}$$
$$= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + 2\cos(c$$
$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$
$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.75, size = 2715, normalized size = 8.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*((-2*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Sin[c
/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(d*(1 + 2*Cos[c])) - (4*Sec[c/2]*Sec[c/
```

2 + (d*x)/2]*Sin[(d*x)/2])/d - ((-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec
[2*c]*Sin[d*x])/(d*(1 + 2*Cos[c])) - (4*Tan[c/2])/d)*Sqrt[e*Tan[c + d*x]])/
(a + a*Sec[c + d*x]) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[
Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)
*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d
*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c
+ d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqr
t[Tan[c + d*x]]) - ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*
x]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*
Sqrt[Tan[c + d*x]]) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*
I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x
])*Sqrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^(
(4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*T
an[c + d*x]])/(3*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))
/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a
*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*
(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c +

```
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec
[c + d*x]*Sqrt[e*Tan[c + d*x]])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[
c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (5*E^(I*(c - d*x))*Cos[c/2 +
 (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sq
rt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c +
d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*Sqrt[((-I)*(-1 + E
^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(
1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) - (Cos[c/2 + (d*x)/2
]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*E^(I*(3*c + d*x))*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (2
*Cos[c/2 + (d*x)/2]^2*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)
*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]
])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt
[Tan[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 359, normalized size = 1.14

method	result
default	$-\frac{\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{e\sin(dx+c)}{\cos(dx+c)}}\,\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(I*EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*Elli
pticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*Ellipti
cF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-Ellipt
icPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))
/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\tan{(c+dx)}}}{\sec{(c+dx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(c+d\,x)}\,\sqrt{e\tan{(c+d\,x)}}}{a\,(\cos{(c+d\,x)}+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}-\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)\right.\cdots}{2\sqrt{2}\,ad\cdots}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(1/2
)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/
2)/e^(1/2)+1/3*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(
c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c))^(
1/2)+2/3*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.25, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$,
Rules used = {3973, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad\sqrt{e}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad\sqrt{e}}+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad\sqrt{e}}+\frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}}-\frac{\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3ad\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) + (2*e*(1 - Sec[c + d*x]))/(3*a*d*(e*T
an[c + d*x])^(3/2)) - (EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2
*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{\int \frac{1}{\sqrt{e\tan(c}}}{a} \quad \text{[cut off]}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+\right.}{ad} \quad \text{[cut off]}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+}\right.}{ad} \quad \text{[cut off]}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c}}{3ad\sqrt{e\tan(c+dx)}} \quad \text{[cut off]}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c}}{3ad\sqrt{e\tan(c+dx)}} \quad \text{[cut off]}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad\sqrt{e}} + \frac{\log}{} \quad \text{[cut off]}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan}}{\sqrt{}}\right.}{\sqrt{2}\,ad\sqrt{e}} \quad \text{[cut off]}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 19.14, size = 1253, normalized size = 4.32

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c +
```

```
 d*x]])/(3*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^((4*I)
*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqr
t[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2
]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4*I)*(c
+ d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt[-1 + E
^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*
I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))
*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) - (Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1 + E^((4*I
)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c
 + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2*I)*(c +
d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3 - 3*E^(
(4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*
I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2
*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1 + E^((2*I)*(c + d*x
)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(-4/(3*d) + (2*(3 - 2*Cos[c] + 3*Cos[2*c])*Cos[d*x]*Sec[2*c])/(3*d
) + (2*Sec[c/2 + (d*x)/2]^2)/(3*d) - (2*Sec[2*c]*(-2*Sin[c] + 3*Sin[2*c])*S
in[d*x])/(3*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) +
 (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan
[c + d*x]]], -1]*Sec[c + d*x]^4*Sqrt[Tan[c + d*x]])/(3*d*(a + a*Sec[c + d*x
])*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 1289, normalized size = 4.44

method	result	size
default	Expression too large to display	1289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*
x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+
```

```
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(d*x
+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*sin(d*x+
c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*cos(d*x+c)*
sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos
(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))-8*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+
3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*s
in(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-8*sin(
d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)^2*2^(1/
2)-2*2^(1/2)*cos(d*x+c))/sin(d*x+c)^5/cos(d*x+c)/(e*sin(d*x+c)/cos(d*x+c))^
(1/2)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)*sqrt(tan(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\tan\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\tan\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{a\,\sqrt{e\,\tan\left(c+d\,x\right)}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=359

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{3/2}}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{3/2}}-\frac{\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\right.}{2\sqrt{2}\,ade}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/2*
arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/4*ln(e
^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2)*2^(1/2)
+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2
)*2^(1/2)-2/5*(5-3*sec(d*x+c))/a/d/e/(e*tan(d*x+c))^(1/2)-6/5*cos(d*x+c)*(s
in(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(
1/2))*(e*tan(d*x+c))^(1/2)/a/d/e^2/sin(2*d*x+2*c)^(1/2)+2/5*e*(1-sec(d*x+c)
)/a/d/(e*tan(d*x+c))^(5/2)-6/5*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d/e^3
```

Rubi [A]
time = 0.32, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{3/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ade^{3/2}}-\frac{\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ade^{3/2}}+\frac{\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ade^{3/2}}-\frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}+\frac{6\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}}-\frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}}+\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(3/2)) + (2*e*(1 - Sec[c + d*x]))/(5*a*d*(e*Tan[
c + d*x])^(5/2)) - (2*(5 - 3*Sec[c + d*x]))/(5*a*d*e*Sqrt[e*Tan[c + d*x]])
+ (6*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d
*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (6*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a
*d*e^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{4\int\left(-\frac{5a}{4}-\frac{3}{4}\right.}{}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{3\int\sec(c+d}{}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)}{5}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)}{5}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} - \frac{6\cos(c+dx)}{5}$$

$$= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{6\cos(c+dx)}{5}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ade^{3/2}} + \frac{\log}{}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{3/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(}}{\sqrt{e}}\right.}{\sqrt{2}\,ade^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 24.58, size = 180, normalized size = 0.50

$$-\frac{4\csc(c+dx)\left(15\cot^2(c+dx)-3\cot^4(c+dx)+3\cot^4(c+dx)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\tan^2(c+dx)\right)-15\cot^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\tan^2(c+dx)\right)-5\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+5\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}}{15ade^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (-4*Csc[c + d*x]*(15*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 3*Cot[c + d*x]^4*H
ypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 15*Cot[c + d*x]^2*Hyp
ergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 5*Hypergeometric2F1[1/2,
 3/4, 7/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x
]^2])*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]])/(
15*a*d*e^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 2149, normalized size = 5.99

method	result	size
default	Expression too large to display	2149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/a/d*(-1+cos(d*x+c))*(-5*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+5*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
-6*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellipt
icF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE((-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*I*cos(d*x+c)^2*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*cos(d*x+c)^2*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*cos(d*x+c)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-si
```

```
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+24*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))-10*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+10*I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^2*2^(1/2)+4*2^(1/2)*cos(d*x+c))/sin(d*x
+c)/cos(d*x+c)^2/(e*sin(d*x+c)/cos(d*x+c))^(3/2)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate(1/((a*sec(d*x + c) + a)*tan(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*sec(c + d*x) + (e*tan(c + d*x))**(3/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\tan(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=328

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{5/2}}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{5/2}}+\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\right.}{2\sqrt{2}\,ade^{5/}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)-1/2*
arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)+1/4*ln(e
^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2)*2^(1/2)
-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2
)*2^(1/2)-5/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(
c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/e^2/(e*tan(d*x+c
))^(1/2)+2/7*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(7/2)-2/21*(7-5*sec(d*x+c)
)/a/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.30, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3973, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ade^{5/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ade^{5/2}}+\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ade^{5/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ade^{5/2}}+\frac{5\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{21ade^2\sqrt{e\tan(c+dx)}}-\frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}}+\frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) +
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(5/2)) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(5/2)) + (2*e*(1 - Sec[c + d*x]))/(7*a*d*(e*Tan[
c + d*x])^(7/2)) - (2*(7 - 5*Sec[c + d*x]))/(21*a*d*e*(e*Tan[c + d*x])^(3/2
)) + (5*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
21*a*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{e^2\int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} + \frac{2\int \frac{\frac{7a}{2}-\frac{5}{2}a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{7a^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{4\int \frac{-\frac{21a}{4} \cdots}{\sqrt{e\,\mathrm{ta}\cdots}}}{2\cdots}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\int \frac{\mathrm{se}\cdots}{\sqrt{e\,\mathrm{ta}\cdots}}}{\cdots}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{2\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5F\left(c-\frac{\pi}{4}\cdots\right.}{\cdots}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5F\left(c-\frac{\pi}{4}\cdots\right.}{\cdots}$$

$$= \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} - \frac{\log\left(\cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan\cdots}}{\sqrt{\cdots}}\right.}{\sqrt{2}ade^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 19.22, size = 1299, normalized size = 3.96

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (-10*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
 E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*
x]^(5/2))/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)
) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^(
(4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x)
)]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTan
h[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]])*Cos[c/2 + (d
*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4
*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt
[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 +
 E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]])*Cos[c/2 + (d*x)/2]^2*Sec
[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c +
 d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Sqrt[((-I)*(-1 + E^
((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1
+ E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^
((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*S
ec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2
*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((-I)*
(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2
*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
x))])*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*d*x)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Cos[c/2 + (d
*x)/2]^2*Sec[c + d*x]*(40/(21*d) - Csc[c/2 + (d*x)/2]^2/(6*d) - (2*(21 - 10
*Cos[c] + 21*Cos[2*c])*Cos[d*x]*Sec[2*c])/(21*d) - (13*Sec[c/2 + (d*x)/2]^2
)/(14*d) + Sec[c/2 + (d*x)/2]^4/(14*d) + (2*Sec[2*c]*(-10*Sin[c] + 21*Sin[2
*c])*Sin[d*x])/(21*d))*Tan[c + d*x]^3)/((a + a*Sec[c + d*x])*(e*Tan[c + d*x
])^(5/2)) - (20*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1
/4)*Sqrt[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*Tan[c + d*x]^(5/2))/(21*d*(a +
a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1926, normalized size = 5.87

method	result	size
default	Expression too large to display	1926

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(-42*I*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-(-1+cos(d*
```

x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*I*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*sin(d*x+c)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*sin
(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))+52*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))+42*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*c
os(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-21*I*sin(d*x+c)*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-42*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-42*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+104*cos(d*x+c)*sin(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+21*I*sin(d*x+c)*cos(d*x+c)^2*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)-21*sin(d*x+c)*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-21*sin(d*x+c)*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+52*sin(d*x+c)*Ellipt
icF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-20*cos(d*x+c)^3*2^(1/2)-4*cos(d*x
+c)^2*2^(1/2)+10*2^(1/2)*cos(d*x+c))/sin(d*x+c)^5/cos(d*x+c)^3/(e*sin(d*x+c

```
)/cos(d*x+c))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate(1/((a*sec(d*x + c) + a)*tan(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
[Out] Integral(1/((e*tan(c + d*x))**(5/2)*sec(c + d*x) + (e*tan(c + d*x))**(5/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\tan(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.127 $\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=372

$$\frac{e^{13/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{13/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] 1/2*e^(13/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(13/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
4*e^(13/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)+1/4*e^(13/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)+12/5*e^6*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/
d/sin(2*d*x+2*c)^(1/2)+2/3*e^5*(e*tan(d*x+c))^(3/2)/a^2/d+12/5*e^5*cos(d*x+
c)*(e*tan(d*x+c))^(3/2)/a^2/d-4/5*e^5*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/a^2/d
+2/7*e^3*(e*tan(d*x+c))^(7/2)/a^2/d
```

Rubi [A]
time = 0.36, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3973, 3971, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2691, 2693, 2695, 2652, 2719, 2687, 32}

$$\frac{e^{13/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{13/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{e^{13/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}+\frac{e^{13/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{12e^6\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(13/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(13/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) - (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (12*e^6*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c
+ 2*d*x]]) + (2*e^5*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) + (12*e^5*Cos[c + d*x
]*(e*Tan[c + d*x])^(3/2))/(5*a^2*d) - (4*e^5*Sec[c + d*x]*(e*Tan[c + d*x])^
(3/2))/(5*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{5/2}-2a^2\sec(c+dx)(e\tan(c+dx))^{5/2}+a^2\sec^2(c+dx)\right.}{a^4}$$

$$= \frac{e^4\int(e\tan(c+dx))^{5/2}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx}{a^2}-\frac{(2e^4)\int\sec}{}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{e^4\text{Subst}\left(\int(ex)^{5/2}\,d\right.}{a^2}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e}{5a^2}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e}{5a^2}$$

$$= \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e}{5a^2}$$

$$= -\frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+$$

$$= -\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d}+\frac{e^{13/2}\log\left(\sqrt{e}+\right.}{}$$

$$= \frac{e^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{13/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx}}{\sqrt{e}}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [F]
time = 12.81, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 1542, normalized size = 4.15

method	result	size
default	Expression too large to display	1542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/210/a^2/d*(-1+cos(d*x+c))^2*(-105*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*I*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4+105*I*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-105*I*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^3-105*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-105*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticPi((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-504*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticE((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+252*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*cos(d*x+c)^4*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))-105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))-105*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-504*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^3*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2*2^(1/2))+252*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
*cos(d*x+c)^3*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*
2^(1/2))+292*cos(d*x+c)^4*2^(1/2)-336*cos(d*x+c)^3*2^(1/2)-10*cos(d*x+c)^2*
```

```
2^(1/2)+84*2^(1/2)*cos(d*x+c)-30*2^(1/2))*(e*sin(d*x+c)/cos(d*x+c))^(13/2)*
cos(d*x+c)^3*(1+cos(d*x+c))^2/sin(d*x+c)^11*2^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(13/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(13/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan(c+d\,x)\right)}^{13/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(13/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(13/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.128 $\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=339

$$\frac{e^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{11/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] 1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/
4*e^(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)-2/3*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d
/(e*tan(d*x+c))^(1/2)+2*e^5*(e*tan(d*x+c))^(1/2)/a^2/d-4/3*e^5*sec(d*x+c)*(
e*tan(d*x+c))^(1/2)/a^2/d+2/5*e^3*(e*tan(d*x+c))^(5/2)/a^2/d
```

Rubi [A]

time = 0.33, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3973, 3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 32}

$$\frac{e^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{11/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{e^{11/2}\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{11/2}\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}+\frac{2e^6\sqrt{\sin(2c+2dx)}\,\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (2*e^6*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*Sqrt[e*Tan[c
 + d*x]]) + (2*e^5*Sqrt[e*Tan[c + d*x]])/(a^2*d) - (4*e^5*Sec[c + d*x]*Sqrt
[e*Tan[c + d*x]])/(3*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(5/2))/(5*a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{3/2}-2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}+a^2\sec^2(c+dx)\right.}{a^4}$$

$$= \frac{e^4\int(e\tan(c+dx))^{3/2}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx}{a^2}-\frac{(2e^4)\int\sec}{}$$

$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{e^4\text{Subst}\left(\int(ex)^{3/2}\,dx,\right.}{a^2d}$$

$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}$$

$$= \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d}+\frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}$$

$$= \frac{2e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5}{}$$

$$= \frac{2e^6F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d}-\frac{4e^5}{}$$

$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}-\frac{e^{11/2}\log\left(\sqrt{e}+\right.}{}$$

$$= \frac{e^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{11/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx}}{\sqrt{e}}\right.}{\sqrt{2}a^2d}$$

Mathematica [F]

time = 104.52, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 731, normalized size = 2.16

method	result
default	$\frac{(-1+\cos(dx+c))\left(15i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,(\cos^2(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/30/a^2/d*(-1+cos(d*x+c))*(15*I*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*I*sin(d*x+c)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*sin(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
-50*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
+24*cos(d*x+c)^3*2^(1/2)-44*cos(d*x+c)^2*2^(1/2)+26*2^(1/2)*cos(d*x+c)-6*2^
(1/2))*cos(d*x+c)^3*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(11/2)/sin(d
*x+c)^9*2^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan(c+d\,x)\right)}^{11/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(11/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.129 $\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=312

$$-\frac{e^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] -1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/2*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4
*e^(9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*
2^(1/2)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/a^2/d*2^(1/2)-4*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin(
2*d*x+2*c)^(1/2)+2/3*e^3*(e*tan(d*x+c))^(3/2)/a^2/d-4*e^3*cos(d*x+c)*(e*tan
(d*x+c))^(3/2)/a^2/d
```

Rubi [A]
time = 0.30, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3973, 3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 32}

$$-\frac{e^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}+\frac{4e^4\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (4*e^4*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*e^3*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) - (4*e^3*Cos[c + d*x]*(e*
Tan[c + d*x])^(3/2))/(a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2\sqrt{e\tan(c+dx)}-2a^2\sec(c+dx)\sqrt{e\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{\right.}{a^4}$$

$$= \frac{e^4\int\sqrt{e\tan(c+dx)}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}-\frac{(2e^4)\int\sec(c}{}$$

$$= -\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{(4e^4)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}+$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{(2e^5)\,\mathrm{Subst}\left(\int\frac{x^2}{e^2+x}\right.}{}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}-\frac{e^5\mathrm{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx\right.}{}$$

$$= \frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e}{}$$

$$= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{\right.}{}$$

$$= -\frac{e^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx}}{\sqrt{e}}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [F]
time = 4.37, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 1504, normalized size = 4.82

method	result	size
default	Expression too large to display	1504

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2)*cos(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2*2^(1/2))-24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*c
os(d*x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))+3*I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))-3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))-3*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-3*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+1
2*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-24*cos(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
```

```
(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+10*cos(d*x+c)^2*2^(1/2)-12*
2^(1/2)*cos(d*x+c)+2*2^(1/2))*cos(d*x+c)^3*(1+cos(d*x+c))^2*(e*sin(d*x+c)/c
os(d*x+c))^(9/2)/sin(d*x+c)^9*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(9/2)*integrate(tan(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{9/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(9/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.130 $\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=281

$$-\frac{e^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{7/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\,\text{t}\right.}{}$$

```
[Out] -1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/2*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4
*e^(7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*
2^(1/2)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/a^2/d*2^(1/2)+2*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*t
an(d*x+c))^(1/2)+2*e^3*(e*tan(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.27, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3973, 3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 32}

$$-\frac{e^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}+\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{2e^4\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (2*e^4*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*Sqrt[e*Tan[c +
d*x]]) + (2*e^3*Sqrt[e*Tan[c + d*x]])/(a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}} - \frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx}}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{1}{\sqrt{ex}}\,dx, x, \tan(c+dx)\right)}{a^2d} + \frac{e^5\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c-$$

$$= \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} + \frac{(2e^5)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} - \frac{(2e^4\,\text{se}}{}$$

$$= -\frac{2e^4F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} +$$

$$= -\frac{2e^4F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d} -$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} + \frac{e^{7/2}\log\left(\sqrt{e}\,+\right.}{}$$

$$= -\frac{e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} + \frac{e^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+d}}{\sqrt{e}}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [F]
time = 44.03, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 663, normalized size = 2.36

method	result
default	$-\frac{\left(i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/d*(I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
Pi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*si
n(d*x+c)-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d
*x+c)+sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
+sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-6*si
n(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-2*2^(1/2)*cos(d*x+
c)+2*2^(1/2))*(-1+cos(d*x+c))*cos(d*x+c)^3*(1+cos(d*x+c))^2*(e*sin(d*x+c)/c
os(d*x+c))^(7/2)/sin(d*x+c)^7*2^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan(c+d\,x)\right)}^{7/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.131 $\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=310

$$\frac{e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan\right.}{}$$

```
[Out] 1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4*
e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2
^(1/2)+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/a^2/d*2^(1/2)-4*e^3/a^2/d/(e*tan(d*x+c))^(1/2)+4*e^3*cos(d*x+c)/a^2/d/(
e*tan(d*x+c))^(1/2)-4*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*
Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin
(2*d*x+2*c)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 32}

$$\frac{e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}+\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(a^2*d*Sqrt[e*T
an[c + d*x]]) + (4*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*Tan[c + d*x]]) + (4*e^2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt
[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{e^2\int \sqrt{e\tan(c+dx)}\,dx}{a^2} + \frac{(4$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{e^3\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(\right.}{a^2d}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{\right.}{a^2d}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\right.}{a^2d\sqrt{\sin(2c+}}$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\right.}{a^2d\sqrt{\sin(2c+}}$$

$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\right.}{}$$

$$= \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx}}{\sqrt{e}}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.75, size = 812, normalized size = 2.62

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^2(c+dx)\left(\frac{32\cos\left(\frac{c}{2}\right)\cos(dx)\sec(2c)\sin\left(\frac{c}{2}\right)}{d}+\frac{16\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}-\frac{16\cos(c)\sec(2c)\sin(dx)}{d}+\frac{16\tan\left(\frac{c}{2}\right)}{d}\right)(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}+\frac{e^{-2ic}\left(-e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\,\mathrm{ArcTan}\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec^2(c+dx)(e\tan(c+dx))^{5/2}}{d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(c+dx)}-\frac{e^{-2ic}\left(\sqrt{-1+e^{4i(c+dx)}}\,\mathrm{ArcTan}\left(\sqrt{-1+e^{4i(c+dx)}}\right)-2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec^2(c+dx)(e\tan(c+dx))^{5/2}}{d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(c+dx)}-\frac{8e^{i(c-dx)}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{4idx}\left(1+e^{4ic}\right)\sqrt{1-e^{4i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{4i(c+dx)}\right)\right)\sec(2c)\sec^2(c+dx)(e\tan(c+dx))^{5/2}}{3d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*((32*Cos[c/2]*Cos[d*x]*Sec[2*c]*Sin[c/
2])/d + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (16*Cos[c]*Sec[2*
c]*Sin[d*x])/d + (16*Tan[c/2])/d)*(e*Tan[c + d*x])^(5/2))/(a + a*Sec[c + d*
x])^2 + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^(
(4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))
]])*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d
*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)
))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) -
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d*E^((2*I)*c
)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^
((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) - (8*E^(I*(c
- d*x))*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1
+ E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(3
*d*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 367, normalized size = 1.18

method	result
default	$(1+\cos(dx+c))^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2/d*(1+cos(d*x+c))^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+8*EllipticE((-(-1+cos(d*x+c)-sin(d
```

```
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(e*sin(d*x+c)/cos(d*x+c))^(5/2
)*cos(d*x+c)^2/sin(d*x+c)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(5/2)*integrate(tan(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan(c+d\,x)\right)}^{5/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.132 $\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$\frac{e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\ \text{ta}\right.}{}$$

```
[Out] 1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4*
e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2
^(1/2)-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/a^2/d*2^(1/2)-2/3*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*
tan(d*x+c))^(1/2)-4/3*e^3/a^2/d/(e*tan(d*x+c))^(3/2)+4/3*e^3*sec(d*x+c)/a^2
/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.33, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3973, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 32}

$$\frac{e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}+\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}}+\frac{2e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3a^2d\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(3*a^2*d*(e*Tan
[c + d*x])^(3/2)) + (4*e^3*Sec[c + d*x])/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) +
 (2*e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
3*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{(2e^2)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,d}{3a^2}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{e^3\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx,\right.}{a^2d}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{(2e^3)\,\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x,\right.}{a^2d}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c}{3a^2d\sqrt{e\tan}}$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c}{3a^2d\sqrt{e\tan}}$$

$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{}\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} - \frac{e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [F]
time = 76.83, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1287, normalized size = 4.07

method	result	size
default	Expression too large to display	1287

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(d*x+c)*EllipticPi
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*cos(d*x+c)*sin(d*x+c)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*sin(d*x+c)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*cos(d*x+c)*sin(d*
x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(d*x+
c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
F((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*sin(d*x+c)*
EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*Ell
```

```
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-10*sin(d*x+c)*Ellip
ticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+4*cos(d*x+c)^2*2^(1/2)-4*2^(1/2)
*cos(d*x+c))*cos(d*x+c)*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)/si
n(d*x+c)^7*2^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{3/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.133 $\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{\sqrt{e}\,\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1
/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/4
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a^2/d*
2^(1/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(
1/2)/a^2/d*2^(1/2)+2*e/a^2/d/(e*tan(d*x+c))^(1/2)-12/5*e*cos(d*x+c)/a^2/d/(
e*tan(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin(
2*d*x+2*c)^(1/2)-4/5*e^3/a^2/d/(e*tan(d*x+c))^(5/2)+4/5*e^3*sec(d*x+c)/a^2/
d/(e*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.39, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2689, 2688, 2695, 2652, 2719, 2687, 32}

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d}-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{2e}{a^2d\sqrt{e\tan(c+dx)}}+\frac{\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d}-\frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}}-\frac{12\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(5*a^2*d*(e*
Tan[c + d*x])^(5/2)) + (4*e^3*Sec[c + d*x])/(5*a^2*d*(e*Tan[c + d*x])^(5/2)
) + (2*e)/(a^2*d*Sqrt[e*Tan[c + d*x]]) - (12*e*Cos[c + d*x])/(5*a^2*d*Sqrt[
e*Tan[c + d*x]]) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan
[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$= -\frac{2e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} - \frac{e^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} + \frac{(6 \cdots}{\cdots}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\cdots}{5a\cdots}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\cdots}{5a\cdots}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\cdots}{5a\cdots}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\cdots}{5a\cdots}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{\cdots}{5a\cdots}$$

$$= \frac{\sqrt{e}\,\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d} - \frac{\sqrt{e}\,\log\left(\sqrt{e}+\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d} + \frac{\sqrt{e}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+d\cdots}}{\sqrt{e}}\right.}{\sqrt{2}\,a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.37, size = 2792, normalized size = 7.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]

[Out] (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*((-24*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Si
n[c/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(5*d*(1 + 2*Cos[c])) - (56*Sec[c/2]*
Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*S
in[(d*x)/2])/(5*d) - (12*(-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*S
in[d*x])/(5*d*(1 + 2*Cos[c])) - (56*Tan[c/2])/(5*d) + (4*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(5*d))*Sqrt[e*Tan[c + d*x]])/(a + a*Sec[c + d*x])^2 + ((E^((2*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
 - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[S
qrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)
/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt[((-I)*(
-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*
x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E^((4*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x
)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c
 + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E
^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*
x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + ((
Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E
^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
+ (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*
```

```
(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4,
 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(
5*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^
2*Sqrt[Tan[c + d*x]]) + (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x))
+ E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2
*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a +
a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (2*E^(I*(c - d*x))*Cos[c/2 + (d*x)/
2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 -
E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])
*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[((-I)*(-1 + E^((2*I)
*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*C
os[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - (2*Cos[c/2 + (d*x)/2]^4
*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1
- E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))
])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*(3*c + d*x))*Sqr
t[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I
)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (
8*Cos[c/2 + (d*x)/2]^4*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I
)*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/
2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d
*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*...
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 2153, normalized size = 5.93

method	result	size
default	Expression too large to display	2153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a^2/d*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c)
)^3*(-5*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*I*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*I*cos(d*x+c)^2*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*cos(d*x+c)^2*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
```

```
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-10*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))+48*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2*2^(1/2))-24*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))-5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))+2*cos(d*x+c)^2*2^(1/2)-2*2^(1/2)*cos(d*x+c))/sin(d*x+c)^7*2^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e \tan{\left(c + dx \right)}}}{\sec^{2}{\left(c + dx \right)} + 2 \sec{\left(c + dx \right)} + 1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\sqrt{e\,\mathrm{tan}\left(c+d\,x\right)}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.134 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=365

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d\sqrt{e}}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d\sqrt{e}}-\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx\right.}{2\sqrt{2}\,a^2}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1
/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1/4
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2^(1/2)/
e^(1/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2
/d*2^(1/2)/e^(1/2)+10/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*t
an(d*x+c))^(1/2)-4/7*e^3/a^2/d/(e*tan(d*x+c))^(7/2)+4/7*e^3*sec(d*x+c)/a^2/
d/(e*tan(d*x+c))^(7/2)+2/3*e/a^2/d/(e*tan(d*x+c))^(3/2)-20/21*e*sec(d*x+c)/
a^2/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.40, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3973, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 32}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d\sqrt{e}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,a^2d\sqrt{e}}-\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{2e}{3a^2d(e\tan(c+dx))^{3/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d\sqrt{e}}+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,a^2d\sqrt{e}}-\frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}}-\frac{10\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{21a^2d\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt[e]
)) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt
[e]) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(
2*Sqrt[2]*a^2*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - (4*e^3)/(7*a^2*d*(e*Tan[c +
d*x])^(7/2)) + (4*e^3*Sec[c + d*x])/(7*a^2*d*(e*Tan[c + d*x])^(7/2)) + (2*e
)/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) - (20*e*Sec[c + d*x])/(21*a^2*d*(e*Tan[c
 + d*x])^(3/2)) - (10*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*
c + 2*d*x]])/(21*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{9/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{9/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{9/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c}{(e\tan(c}}{a^2}$$

$$= -\frac{2e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} - \frac{e^2\int}{(e\tan}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{}{3a^2d(e\,t}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{}{3a^2d(e\,t}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{}{3a^2d(e\,t}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{}{3a^2d(e\,t}$$

$$= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{}{3a^2d(e\,t}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a^2d\sqrt{e}} + \frac{\log}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a^2d\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\,t}}{}\right)}{\sqrt{2}\,a^2d\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 18.91, size = 1281, normalized size = 3.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (40*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[
c + d*x]])/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]
]) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^
((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x
))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (
d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((
4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqr
t[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1
+ E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^4*Se
c[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 + E^((2*I)*(c
+ d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) - (2*Sqrt[((-I)*(-1 +
 E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^4*(3*(
-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 -
 E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))]
)*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*
x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c
))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*
(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1
 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-104/(21*d) + (4*(21 - 20*Cos[c] + 21*Cos
[2*c])*Cos[d*x]*Sec[2*c])/(21*d) + (64*Sec[c/2 + (d*x)/2]^2)/(21*d) - (2*Se
c[c/2 + (d*x)/2]^4)/(7*d) - (4*Sec[2*c]*(-20*Sin[c] + 21*Sin[2*c])*Sin[d*x]
)/(21*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (80
*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^4*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c
+ d*x]]], -1]*Sec[c + d*x]^5*Sqrt[Tan[c + d*x]])/(21*d*(a + a*Sec[c + d*x])
^2*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 1926, normalized size = 5.28

method	result	size
default	Expression too large to display	1926

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(-42*I*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)
```

-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*I*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+42*I*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*
sin(d*x+c)*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))-21*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*
cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-21*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))+62*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2*2^(1/2))+21*I*sin(d*x+c)*cos(d*x+c)^2*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-21*I*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)-42*cos(d*x+c)*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-42*cos(d*x+c)*sin(d*x+c
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+124*cos(d*x+c)
*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-21*sin(d*x+c)*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-21*sin(d*x+c)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+62*sin(d*x+c)*Ellip
ticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-26*cos(d*x+c)^3*2^(1/2)+6*cos(d*

```
x+c)^2*2^(1/2)+20*2^(1/2)*cos(d*x+c))/sin(d*x+c)^7/cos(d*x+c)/(e*sin(d*x+c)
/cos(d*x+c))^(1/2)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)^2*sqrt(tan(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\tan\left(c+dx\right)}\sec^{2}\left(c+dx\right)+2\sqrt{e\tan\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\tan\left(c+dx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*tan(c + d*x))*s
ec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d x)^2}{a^2 \sqrt{e \tan (c+d x)}\, (\cos (c+d x)+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.135 $\int \sqrt{a+a\sec(c+dx)}\,\tan^5(c+dx)\,dx$

Optimal. Leaf size=147

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+d}{5a^2d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d-6/7*(a+a*se
c(d*x+c))^(7/2)/a^3/d+2/9*(a+a*sec(d*x+c))^(9/2)/a^4/d-2*arctanh((a+a*sec(d
*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 90, 52, 65, 213}

$$\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^4d}-\frac{6(a\sec(c+dx)+a)^{7/2}}{7a^3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}+\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a^2*d) - (6*(a + a*Sec[c + d*x])^(7/2))/(7*a^3*d) + (2*(a
+ a*Sec[c + d*x])^(9/2))/(9*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{5/2}+\frac{a^2(a+ax)^{5/2}}{x}+a(a+ax)^{7/2}\right)dx, x, \text{s}\right.}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}+\frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d}+\frac{2(a+a\sec(}{9a^4}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{6(a+a\sec(}{7a^3}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c}{5a^2d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c}{5a^2d}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.58, size = 102, normalized size = 0.69

$$\frac{2\sqrt{a(1+\sec(c+dx))}\left(-315\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(383-34\sec(c+dx)-132\sec^2(c+dx)+5\sec^3(c+dx)+35\sec^4(c+dx))\right)}{315d\sqrt{1+\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(-315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(383 - 34*Sec[c + d*x] - 132*Sec[c + d*x]^2 + 5*Sec[c + d
*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*Sqrt[1 + Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(123) = 246$.
time = 0.40, size = 359, normalized size = 2.44

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315(\cos^4(dx+c))\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}\sqrt{2}+1260(\cos^3(dx+c))\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*cos(d*x+c)^4*arctan(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(9/2)*2^(1/2)+1260*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)+1890*cos(d*x+c)
^2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(9/2)*2^(1/2)+1260*cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)+31
5*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)+12256*cos(d*x+c)^4-1088*cos(d*x+c)^3-4224*cos(
d*x+c)^2+160*cos(d*x+c)+1120)/cos(d*x+c)^4
```

Maxima [A]
time = 0.51, size = 145, normalized size = 0.99

$$\frac{315\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+630\sqrt{a+\frac{a}{\cos(dx+c)}}+\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^4}-\frac{270\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^3}+\frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^2}+\frac{210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(315*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos
(d*x + c)) + sqrt(a))) + 630*sqrt(a + a/cos(d*x + c)) + 70*(a + a/cos(d*x +
 c))^(9/2)/a^4 - 270*(a + a/cos(d*x + c))^(7/2)/a^3 + 126*(a + a/cos(d*x +
c))^(5/2)/a^2 + 210*(a + a/cos(d*x + c))^(3/2)/a)/d
```

Fricas [A]
time = 4.52, size = 299, normalized size = 2.03

$$\left[\frac{315\sqrt{a}\cos\left(dx+c\right)^4\log\left(-8\,a\cos\left(dx+c\right)^2+4\left(2\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\left(383\cos\left(dx+c\right)^4-34\cos\left(dx+c\right)^3-132\cos\left(dx+c\right)^2+5\cos\left(dx+c\right)+35\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{630\,d\cos\left(dx+c\right)^4}, \frac{315\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+a}\right)\cos\left(dx+c\right)^4+2\left(383\cos\left(dx+c\right)^4-34\cos\left(dx+c\right)^3-132\cos\left(dx+c\right)^2+5\cos\left(dx+c\right)+35\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{315\,d\cos\left(dx+c\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*
x + c)^2 + 5*cos(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d
*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^4 +
2*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 + 5*cos(d*x
+ c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**5, x)
```

Giac [A]
time = 2.06, size = 193, normalized size = 1.31

$$\frac{\sqrt{2}\left(\frac{315\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(315\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4a-210\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3a^2+252\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^3+1080\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^4+560\,a^5\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(315*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a - 2
10*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^2 + 252*(a*tan(1/2*d*x + 1/2*c)^2 - a
)^2*a^3 + 1080*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^4 + 560*a^5)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

3.136 $\int \sqrt{a+a\sec(c+dx)}\,\tan^3(c+dx)\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+a\sec(c+dx)}}{d}-\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}$$

```
[Out] -2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d+2*arctanh(
(a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 81, 52, 65, 213}

$$\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}-\frac{2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + a*Sec
[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c\cdots}{5a^2\cdots}$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c\cdots}{5a^2\cdots}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+a\sec(c+dx)}}{d} - \cdots$$

Mathematica [A]
time = 0.17, size = 80, normalized size = 0.81

$$\frac{2\sqrt{a(1+\sec(c+dx))}\left(15\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(-17+\sec(c+dx)+3\sec^2(c+dx))\right)}{15d\sqrt{1+\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[1
+ Sec[c + d*x]]*(-17 + Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*Sqrt[1 + Se
c[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(83) = 166$.
time = 0.17, size = 221, normalized size = 2.23

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\left(\cos^2(dx+c)\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+30\cos(dx+c)\sqrt{2}\,\operatorname{arct}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*cos(d*x+c)^2*2^(1/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)+30*cos(d*x+c)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*2^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+136*cos(d*x+c)^2-8*cos(d*x+c)-24)/cos(d*x+c)^2
```

Maxima [A]
time = 0.51, size = 107, normalized size = 1.08

$$-\frac{15\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}-\frac{6\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^2}+\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(
d*x + c)) + sqrt(a))) + 30*sqrt(a + a/cos(d*x + c)) - 6*(a + a/cos(d*x + c)
)^(5/2)/a^2 + 10*(a + a/cos(d*x + c))^(3/2)/a)/d
```

Fricas [A]
time = 3.20, size = 259, normalized size = 2.62

$$\left[\frac{15\sqrt{a}\,\cos\left(dx+c\right)^2\log\left(-8\,a\cos\left(dx+c\right)^2-4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)-4\left(17\,\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{30\,d\cos\left(dx+c\right)^2},\ -\frac{15\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos\left(dx+c\right)^2+2\left(17\,\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{15\,d\cos\left(dx+c\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 4*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), -1/15*(15*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a))*cos(d*x + c)^2 + 2*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**3, x)
```

Giac [A]
time = 1.08, size = 152, normalized size = 1.54

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^2-10\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^3-12\,a^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)\mathrm{sgn}(\cos\left(dx+c\right))}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(15*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^2 -
 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^3 - 12*a^4)/((a*tan(1/2*d*x + 1/2*c)^2
 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(cos(d*x + c))/(a*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{tan}\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.137 $\int \sqrt{a+a\sec(c+dx)}\,\tan(c+dx)\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 52, 65, 213}

$$\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$
$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.18

$$\frac{\sqrt{a(1+\sec(c+dx))}\left(-2\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+2\sqrt{1+\sec(c+dx)}\right)}{d\sqrt{1+\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[1
+ Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A]
time = 0.05, size = 42, normalized size = 0.82

method	result	size

derivativedivides	$\frac{2\sqrt{a+a\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+a\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a+a*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1
/2)))
```

Maxima [A]
time = 0.50, size = 67, normalized size = 1.31

$$\frac{\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{a}{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
 + sqrt(a))) + 2*sqrt(a + a/cos(d*x + c)))/d
```

Fricas [A]
time = 3.98, size = 184, normalized size = 3.61

$$\left[\frac{\sqrt{a}\,\log\left(-8\,a\cos(dx+c)^2+4\,\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)+4\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{2\,d},\ \frac{\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)+2\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))
 + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x), x)
```

Giac [A]
time = 0.82, size = 75, normalized size = 1.47

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\,a}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)\mathrm{sgn}(\cos\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/s
qrt(-a))/sqrt(-a) + 2*a/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))*sgn(cos(d*x +
c))/d
```

Mupad [B]
time = 1.48, size = 47, normalized size = 0.92

$$\frac{2\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{d}-\frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + a/cos(c + d*x))^(1
/2)/a^(1/2)))/d
```

3.138 $\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh(1/2*(a+a*sec(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3965, 88, 65, 213}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[2]*Sqrt[a]*
ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\text{Subst}\left(\int \frac{}{(-a+ax)\sqrt{}}\right.}{}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(2a)\text{Subst}\left(\int \right.}{}$$
$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{d}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 0.99

$$\frac{\left(2\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\right)\sqrt{a(1+\sec(c+dx))}}{d\sqrt{1+\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]
]/Sqrt[2]])*Sqrt[a*(1 + Sec[c + d*x])])/(d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A]
time = 0.14, size = 98, normalized size = 1.34

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\right)}{d}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+arctan
(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c), x)
```

Fricas [A]
time = 3.76, size = 242, normalized size = 3.32

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+2\sqrt{a}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-a\right)}{2d},\ \frac{\sqrt{2}\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 2*sqr
t(a)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c) - a))/d, (sqrt(2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqr
t(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(a*cos(d*x + c) + a)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x), x)
```

Giac [A]
time = 0.79, size = 88, normalized size = 1.21

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)\operatorname{sgn}(\cos\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/sqrt(-a))*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

3.139 $\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{7\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}+\frac{a}{4d\sqrt{a+a\sec(c+dx)}}+\frac{}{2}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+7/8*arctanh(1/2*(a+a*s
ec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d+1/4*a/d/(a+a*sec(d*x+c)
)^(1/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3965, 105, 157, 162, 65, 213}

$$\frac{a}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a}{2d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{7\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (7*Sqrt[a]*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*d) + a/(4*d*Sqrt
[a + a*Sec[c + d*x]]) + a/(2*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{3a^2x}{2}}{x(-a+ax)(a+ax)}\right.}{2}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a}}{\sqrt{2}}\right.}{4\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.30, size = 87, normalized size = 0.66

$$\frac{\cot^2(c+dx)\left(-2-7\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))\right)\sqrt{a(1+\sec(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^2*(-2 - 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/
2]*(-1 + Sec[c + d*x]) + 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]
]*(-1 + Sec[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(106) = 212$.
time = 0.20, size = 267, normalized size = 2.04

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(8\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)(\cos^2(dx+c))\sqrt{2}+7\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \text{ar}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-8*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+6*cos(d*x+c)^2-2*cos(d*x+c))/sin(d*x+c)^4*(cos(d*x+c)^2-1)

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^3, x)

<u>Fricas</u> [A]
time = 3.27, size = 426, normalized size = 3.25

$$\left[\frac{8\left(\cos(dx+c)^2-1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)+7\left(\sqrt{2}\cos(dx+c)^2-\sqrt{2}\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3a\cos(dx+c)+a}{\cos(dx+c)-1}\right)+4\left(3\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{16\left(d\cos(dx+c)^2-d\right)},\ -\frac{7\left(\sqrt{2}\cos(dx+c)^2-\sqrt{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-8\left(\cos(dx+c)^2-1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)-2\left(3\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{8\left(d\cos(dx+c)^2-d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*(8*(cos(d*x + c)^2 - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d), -1/8*(7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 8*(cos(d*x + c)^2 - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(

```
d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(3*cos(d*x + c)^2 - cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**3, x)
```

Giac [A]
time = 0.84, size = 140, normalized size = 1.07

$$\frac{\sqrt{2}\left(\frac{8\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{7\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(8*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) - 7*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sq
rt(-a))/sqrt(-a) + 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)/tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.140 $\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{107\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 43/96*a^2/d/(a+a*sec(d*x+c))^(3/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+
c))^(3/2)-15/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)+2*arctanh((a+a*
sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-107/128*arctanh(1/2*(a+a*sec(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-21/64*a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\frac{43a^2}{96d(a\sec(c+dx)+a)^{3/2}}-\frac{15a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/2}}-\frac{21a}{64d\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{107\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (107*Sqrt[a]*ArcT
anh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]*d) + (43*a^2)/
(96*d*(a + a*Sec[c + d*x])^(3/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a + a*Se
c[c + d*x])^(3/2)) - (15*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) - (21*a)/(64*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a}{x(-a+a}\right.}{}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+d}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{107\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{64\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.32, size = 102, normalized size = 0.53

$$\frac{\cot^4(c+dx)\left(-2\left(57+32\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))^2-45\sec(c+dx)\right)+107\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2\right)\sqrt{a(1+\sec(c+dx))}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(57 + 32*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d
*x]]*(-1 + Sec[c + d*x])^2 - 45*Sec[c + d*x]) + 107*Hypergeometric2F1[-3/2,
 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2)*Sqrt[a*(1 + Sec[c +
d*x])])/(96*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(160) = 320$.
time = 0.28, size = 407, normalized size = 2.11

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(384\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^4(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c
))^2*(384*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^4*2^(1/2)+321*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^4
-768*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)-642*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+410*
cos(d*x+c)^4+384*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-142*cos(d*x+c)^3+321*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2
98*cos(d*x+c)^2+126*cos(d*x+c))/sin(d*x+c)^8
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.75, size = 529, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(384*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*log(-8*a*cos(d*
x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
```

```
 + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 321*(sqrt(2)*cos(d*x + c)^4 -
 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos
(d*x + c) - 1)) - 4*(205*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x +
 c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d), 1/384*(321*(sqrt(2)*cos(d*x + c)^4 - 2*
sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 384*(c
os(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(205
*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 + 63*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^2 + d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**5, x)
```

Giac [**A**]
time = 0.85, size = 201, normalized size = 1.04

$$-\frac{\sqrt{2}\left(\frac{384\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{321\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{8\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^2+15\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^3\right)}{a^3}+\frac{3\left(21\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a-19\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^2\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(384*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) - 321*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) + 8*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^2 + 15
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3)/a^3 + 3*(21*(-a*tan(1/2*d*x + 1/2
*c)^2 + a)^(3/2)*a - 19*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2)/(a^2*tan(1
/2*d*x + 1/2*c)^4))*sgn(cos(d*x + c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^5\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

3.141 $\int \sqrt{a+a\sec(c+dx)}\,\tan^6(c+dx)\,dx$

Optimal. Leaf size=222

$$-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^3\tan^5(c+\ldots}{5d(a+a\sec(c+\ldots}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2
/5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2*a^4*tan(d*x+c)^7/d/(a+a*sec(
d*x+c))^(7/2)+10/9*a^5*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^6*tan(d
*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

Rubi [A]
time = 0.08, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$\frac{2a^6\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{10a^5\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{2a^4\tan^7(c+dx)}{d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(d*(a + a*Sec[c + d*x])^(7/2)) + (10*a
^5*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^6*Tan[c + d*x]^1
1)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^6(c+dx)\,dx = -\frac{(2a^4)\,\mathrm{Subst}\left(\int \frac{x^6(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\mathrm{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+7x^6+5ax^8+a^2x^{10}-\frac{1}{a^3(1+ax^2)}\right)\right.}{d}$$

$$= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^3\tan^5(c}{5d(a+a\sec(c}$$

$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} -$$

Mathematica [A]
time = 7.24, size = 134, normalized size = 0.60

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(1+\sec(c+dx))}\left(3960\sqrt{2}\,\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{11}{2}}(c+dx)+792\sin\left(\frac{1}{2}(c+dx)\right)-1386\sin\left(\frac{3}{2}(c+dx)\right)+495\sin\left(\frac{5}{2}(c+dx)\right)-616\sin\left(\frac{7}{2}(c+dx)\right)-247\sin\left(\frac{11}{2}(c+dx)\right)\right)}{3960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] -1/3960*(Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(3960*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) + 792*Sin[(c +
d*x)/2] - 1386*Sin[(3*(c + d*x))/2] + 495*Sin[(5*(c + d*x))/2] - 616*Sin[(7
*(c + d*x))/2] - 247*Sin[(11*(c + d*x))/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(196) = 392$.
time = 0.26, size = 566, normalized size = 2.55

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(495\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sin(dx+c)\left(\cos^5(dx+c)\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{11}{2}}+2$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] -1/15840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(495*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+2475*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+4950*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+4950*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+2475*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+495*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)+31616*cos(d*x+c)^6-15808*cos(d*x+c)^5-27712*cos(d*x+c)^4+1984*cos(d*x+c)^3+13120*cos(d*x+c)^2-320*cos(d*x+c)-2880)/sin(d*x+c)/cos(d*x+c)^5

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="maxima")

[Out] 1/990*(495*((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), co

```
s(2*d*x + 2*c) + 1)) - 1) - 2*(d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4
+ 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2
 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)
*integrate((((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*co
s(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*
c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x +
4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x +
 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2
*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*
cos(13/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*s
in(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x +
2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d
*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*co
s(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) -
15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))
*sin(13/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(14*d*x + 14*
c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x
 + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6
*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(
2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c
)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x +
6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(13/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2
*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d
*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(
2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + s
in(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c)
+ 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x +
2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(13/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(6*
cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6
*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + c
os(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*
cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c
) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d
*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2
*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + c
os(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x +
 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*
sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6
```

```
*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + s
in(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*
sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c
) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c)
 + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d
*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + s
in(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x +
 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + ...
```

<u>Fricas</u> [**A**]
time = 3.70, size = 371, normalized size = 1.67

$$\left[\frac{495\,\left(\cos\left(dx+c\right)^{6}+\cos\left(dx+c\right)^{5}\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}+2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\,\left(494\,\cos\left(dx+c\right)^{5}+247\,\cos\left(dx+c\right)^{4}-186\,\cos\left(dx+c\right)^{3}-155\,\cos\left(dx+c\right)^{2}+50\,\cos\left(dx+c\right)+45\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{495\,\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)},\ \frac{2\,\left(495\,\left(\cos\left(dx+c\right)^{6}+\cos\left(dx+c\right)^{5}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+\left(494\,\cos\left(dx+c\right)^{5}+247\,\cos\left(dx+c\right)^{4}-186\,\cos\left(dx+c\right)^{3}-155\,\cos\left(dx+c\right)^{2}+50\,\cos\left(dx+c\right)+45\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{495\,\left(d\cos\left(dx+c\right)^{6}+d\cos\left(dx+c\right)^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(494*cos(d*x + c)^5 +
 247*cos(d*x + c)^4 - 186*cos(d*x + c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x
+ c) + 45)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^6 + d*cos(d*x + c)^5), 2/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) + (494*cos(d*x + c)^5 + 247*cos(d*x + c)^4 - 186*cos(d*x +
c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x + c) + 45)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

<u>Sympy</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**6,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**6, x)
```

<u>Giac</u> [**A**]

time = 2.67, size = 284, normalized size = 1.28

$$\frac{\sqrt{2}\left(\frac{495\,\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\,|a|-6\,a\right|}\right)}{|a|}-\frac{4\left(495\,a^{6}-\left(2805\,a^{6}-\left(6666\,a^{6}-\left(4158\,a^{6}+\left(221\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1463\,a^{6}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{5}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{990\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/990*sqrt(2)*(495*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 4*(495*a^6 - (2805*a^6 - (6666*a^6 - (41
58*a^6 + (221*a^6*tan(1/2*d*x + 1/2*c)^2 - 1463*a^6)*tan(1/2*d*x + 1/2*c)^2
)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*t
an(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^6\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

3.142 $\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^3\tan^5(c+}{5d(a+a\sec(c+}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-2*a*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+6/
5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^4*tan(d*x+c)^7/d/(a+a*sec
(d*x+c))^(7/2)
```

Rubi [A]

time = 0.06, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$\frac{2a^4\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d*
(a + a*Sec[c + d*x])^(3/2)) + (6*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*
x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx = -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{}{\sqrt{a}}\right.}{d}$$

$$= -\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^3\tan^5}{5d(a+a\sec}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{}{3}$$

Mathematica [A]
time = 5.80, size = 110, normalized size = 0.69

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(1+\sec(c+dx))}\left(105\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{7}{2}}(c+dx)+35\sin\left(\frac{1}{2}(c+dx)\right)-28\sin\left(\frac{3}{2}(c+dx)\right)-23\sin\left(\frac{7}{2}(c+dx)\right)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(105*Sqrt[2]*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 35*Sin[(c + d*x)/2] - 2
8*Sin[(3*(c + d*x))/2] - 23*Sin[(7*(c + d*x))/2]))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(140) = 280$.
time = 0.18, size = 317, normalized size = 1.98

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\sin(dx+c)(\cos^3(dx+c))\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+210\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/420/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*sin(d*x+c)*cos(d*x+c)^3*2
^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+
c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+210*cos(d*x+c)^2*sin(d*x+c
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+105*cos(d*x+c)*sin(d*x+
c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)-736*cos(d*x+c)^4+368*c
os(d*x+c)^3+512*cos(d*x+c)^2-24*cos(d*x+c)-120)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1) - 2*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*
d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) +
4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) +
 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10
*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x +
6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2
*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x +
2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x
 + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x
 + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(
2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos
```

```
(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*co
s(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*c
os(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos
(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*co
s(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))/(((2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos
(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2
 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(
4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c)
+ 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x +
10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d
*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16
*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c
)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos
(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))
*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(
4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 +
 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x
 + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*
(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*
c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + 8*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x
+ 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*
d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2
*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + si
n(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6
*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
(cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c)
+ 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
 - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c
) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos...
```

Fricas [A]
time = 3.79, size = 331, normalized size = 2.07

$$\left[\frac{105\,{\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)}\sqrt{-a}\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)-2\,{\left(92\,\cos\left(dx+c\right)^{3}+46\,\cos\left(dx+c\right)^{2}-18\,\cos\left(dx+c\right)-15\right)}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{105\,{\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}},-\frac{2\,{\left(105\,{\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)}\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+{\left(92\,\cos\left(dx+c\right)^{3}+46\,\cos\left(dx+c\right)^{2}-18\,\cos\left(dx+c\right)-15\right)}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}}{105\,{\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(92*cos(d*x + c)^3 +
46*cos(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/105*(105*(co
s(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (92*cos(d*x + c)^3 + 46*c
os(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**4, x)
```

Giac [A]
time = 1.58, size = 246, normalized size = 1.54

$$-\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)}^{2}-4\,\sqrt{2}\left|a\right|-6\,a\right|}{\left|2\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)}^{2}+4\,\sqrt{2}\left|a\right|-6\,a\right|}\right)}{\left|a\right|}-\frac{4\,{\left(105\,a^{4}-{\left(385\,a^{4}+{\left(43\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-203\,a^{4}\right)}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)}^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/210*sqrt(2)*(105*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
```

```
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 4*(105*a^4 - (385*a^4 + (43*a^4*tan(1/2
*d*x + 1/2*c)^2 - 203*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx$

Optimal. Leaf size=96

$$-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3972, 470, 327, 209}

$$\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,\tan^2(c+dx)\,dx = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(\ldots}{\sqrt{a+a\,\text{se}\ldots}}\right.}{d}$$

$$= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{(2a)\text{Subst}\left(\int \ldots\right.}{}$$

$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.10, size = 226, normalized size = 2.35

$$\frac{8\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{7/2}\sqrt{a(1+\sec(c+dx))}\left(-\frac{\cos(c+dx)(7+3\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-3\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\cos(c+dx)+(-1+4\cos(c+dx))\sqrt{1-\sec(c+dx)}\right)}{24\sqrt{1-\sec(c+dx)}} - \frac{4}{7}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-2\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{3d\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (8*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(-1/2
4*(Cos[c + d*x]*(7 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2*
(-3*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + (-1 + 4*Cos[c + d*x])*Sq
rt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2, 7/2
```

```
, 9/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^2)
/7)*Tan[c + d*x]^3)/(3*d*(1 - Tan[(c + d*x)/2]^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(84) = 168$.
time = 0.15, size = 210, normalized size = 2.19

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\sin(dx+c)\cos(dx+c)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\right.}{6d\sin(dx+c)\cos(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*sin(d*x+c)*cos(d*x+c)*2^(1/2)
*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(
1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*2^(1/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*sin(d*x+c)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)/sin(d*x+c)/co
s(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*(2*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4
*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*
c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*
c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*
c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)
*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^
2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

```
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*cos(4*d
*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 4*cos
(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
+ 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x +
6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2
*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
 (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x +
6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2
*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)), x) - 4*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*c
os(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*si
n(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin
(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*si
n(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2
*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((
2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*
c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d
*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) +
cos(6*d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c
)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)), x) + 2*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x
+ 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d
*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*
x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d
```

```
*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))/(((2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + co
s(6*d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos...
```

Fricas [**A**]
time = 2.18, size = 283, normalized size = 2.95

$$\left[\frac{3\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(2\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}, \frac{2\left(3\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(2\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\tan^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(84) = 168.

time = 1.15, size = 208, normalized size = 2.17

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{4\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)\operatorname{sgn}(\cos(dx+c))}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/abs(a) + 4*(a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2)*tan(1
/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)))*sgn(cos(d*x + c))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.144 $\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{2\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}\,d}-\frac{\cot(c+dx)\sqrt{a+a\sec}}{d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/2*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-cot
(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3972, 491, 536, 209}

$$-\frac{2\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\,d}-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[2]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{\text{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, \cdots\right.}{d}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\cdots}{\sqrt{a\cdots}}\right.}{d}$$

$$= -\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan\cdots}{\sqrt{2}\sqrt{a+a\cdots}}\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.26, size = 5502, normalized size = 50.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(93) = 186.
time = 0.18, size = 189, normalized size = 1.73

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-2\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)-\ln\left(-\frac{-\sqrt{-}}{}\right.\right.}{2d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+2*cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Fricas [A]
time = 3.60, size = 422, normalized size = 3.87

$$\left[\frac{\sqrt{2}\,\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+2\sqrt{-a}\,\log\left(-\frac{8a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{4d\sin(dx+c)},\ -\frac{\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+2\sqrt{a}\,\arctan\left(\frac{2\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x +
 c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 2*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c))/(d*sin(d*x + c)), -1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(
d*x + c) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin
```

```
(d*x + c) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(
d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(93) = 186.
time = 1.07, size = 236, normalized size = 2.17

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}\,|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}\,|a|-6a\right|}\right)}{|a|}+\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)+\frac{4\sqrt{-a}\,a}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/abs(a) + sqrt(-a)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2) + 4*sqrt(-a)*a/((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))*sgn(cos(d*x +
c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(c+d\,x\right)}^2\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.145 $\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{9\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{2}\,d}+\frac{7\cot(c+dx)\sqrt{a+a\,\text{se}}}{8d}$$

[Out] 1/12*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a/d+2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-9/16*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+7/8*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.14, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 483, 597, 536, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{9\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{2}\,d}+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12ad}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8d}-\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{4ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]

[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (9*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(8*Sqrt[2]*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*d) + (Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12*a*d) - (Cos[c + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(4*a*d)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad}$$

$$= \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+dx)\cot^3(c+dx)\sec}{}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+d}{12ad}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+d}{12ad}$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{9\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,\text{ta}}{\sqrt{2}\sqrt{a+c}}\right.}{8\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.11, size = 5552, normalized size = 28.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(166) = 332.
time = 0.22, size = 381, normalized size = 1.94

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(48\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^2(dx+c))\sin(dx+c)\sqrt{2}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/48/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(48*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+27*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-48*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-27*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-62*cos(d*x+c)^3+4*cos(d*x+c)^2+42*cos(d*x+c))/sin(d*x+c)^5*(cos(d*x+c)^2-1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^4, x)

Fricas [A]
time = 3.91, size = 547, normalized size = 2.79

$$\left[\frac{27\left(\sqrt{2}\cos\left(dx+c\right)^{2}-\sqrt{2}\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^{2}+2a\cos(dx+c)-a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)\sin\left(dx+c\right)+48\left(\cos\left(dx+c\right)^{2}-1\right)\sqrt{-a}\log\left(-\frac{8a\cos(dx+c)^{3}-4\left(2\cos(dx+c)^{2}-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin\left(dx+c\right)+4\left(31\cos\left(dx+c\right)^{3}-2\cos\left(dx+c\right)^{2}-21\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{96\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)},\ \frac{48\left(\cos\left(dx+c\right)^{2}-1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^{2}+a\cos(dx+c)-a}\right)\sin\left(dx+c\right)+27\left(\sqrt{2}\cos\left(dx+c\right)^{2}-\sqrt{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin\left(dx+c\right)+2\left(31\cos\left(dx+c\right)^{3}-2\cos\left(dx+c\right)^{2}-21\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{48\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/96*(27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 48*(cos(d*x + c)^2 - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - d)*sin(d*x + c)), 1/48*(48*(cos(d*x + c)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*a

```
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c)))*sin(d*x + c) + 2*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2
- d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(166) = 332.
time = 1.13, size = 365, normalized size = 1.86

$$-\frac{\sqrt{2}\left(\frac{48\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+27\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)+6\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{8\left(15\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}\,a-24\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^2+13\sqrt{-a}\,a^3\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^3}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(48*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/abs(a) + 27*sqrt(-a)*log((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2) + 6*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 8*(15*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a - 24*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^2 + 13*sqrt(
-a)*a^3)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a)^3)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(c+d\,x\right)}^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.146 $\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=280

$$-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{151\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}\,d}-\frac{105\cot(c+dx)\sqrt{a-}}{128d}$$

```
[Out] -23/192*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d+87/160*cot(d*x+c)^5*(a+a*se
c(d*x+c))^(5/2)/a^2/d-17/32*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a
+a*sec(d*x+c))^(5/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^4*(a+a*sec(d*x+c))^(5/2)/a^2/d-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c
))^(1/2))*a^(1/2)/d+151/256*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-105/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.19, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\frac{87\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{160a^2d}-\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{16a^2d}-\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{32a^2d}-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{151\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}\,d}-\frac{23\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{192ad}-\frac{105\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
51*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(128*Sqrt[2]*d) - (105*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*d) -
(23*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(192*a*d) + (87*Cot[c + d*x]
^5*(a + a*Sec[c + d*x])^(5/2))/(160*a^2*d) - (17*Cos[c + d*x]*Cot[c + d*x]^
5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(32*a^2*d) - (Cos[c + d*x]
^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2))/(16*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}$$

$$= -\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d}$$

$$= -\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/}}{32a^2d}$$

$$= \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx}{}$$

$$= -\frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec}{160a^2d}$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(}{192ad}$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(}{192ad}$$

$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{151\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\,\sqrt{a}}\right.}{128\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.05, size = 5594, normalized size = 19.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(241) = 482.
time = 0.18, size = 573, normalized size = 2.05

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(-3840\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^4(dx+c))\sin(dx+c)\operatorname{arctanh}\left(\sqrt{-\frac{2}{1}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2*(-3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-2265*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+7680*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+5642*cos(d*x+c)^5+4530*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-556*cos(d*x+c)^4-3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-7928*cos(d*x+c)^3-2265*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+460*cos(d*x+c)^2+3150*cos(d*x+c))/sin(d*x+c)^9

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

Fricas [**A**]
time = 2.27, size = 646, normalized size = 2.31

$$\left[\frac{2265\left(\sqrt{2}\cos(dx+c)^4-2\sqrt{2}\cos(dx+c)^2+\sqrt{2}\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+3840\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sqrt{-a}\log\left(-\frac{8a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-4\left(2821\cos(dx+c)^5-278\cos(dx+c)^4-3964\cos(dx+c)^3+230\cos(dx+c)^2+1575\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{7680\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)\sin(dx+c)}, -\frac{3840\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+2265\left(\sqrt{2}\cos(dx+c)^4-2\sqrt{2}\cos(dx+c)^2+\sqrt{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+2\left(2821\cos(dx+c)^5-278\cos(dx+c)^4-3964\cos(dx+c)^3+230\cos(dx+c)^2+1575\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{3840\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] [1/7680*(2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 3840*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2
 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*(2821*cos(
d*x + c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2
+ 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x +
 c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c)), -1/3840*(3840*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x +
 c) - a))*sin(d*x + c) + 2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x +
 c)^2 + sqrt(2))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(2821*cos(d*x +
c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2 + 1575
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cot^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**6, x)
```

Giac [A]
time = 1.22, size = 476, normalized size = 1.70

$$-\frac{\sqrt{2}\left(30\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-25\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{3840\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-2265\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)-\frac{64\left(165\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{8}\sqrt{-a}\,a-555\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{6}\sqrt{-a}\,a^{2}+785\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}\sqrt{-a}\,a^{3}-505\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{4}+134\sqrt{-a}\,a^{5}\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\right)^{5}}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/7680*sqrt(2)*(30*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/
2*c)^2 - 25)*tan(1/2*d*x + 1/2*c) - 3840*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt
(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 2265*sqrt(-a)*log(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2) -
```

```
64*(165*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*sqrt(-a)*a - 555*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^6*sqrt(-a)*a^2 + 785*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^3 - 505*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^4 + 134*sqrt(
-a)*a^5)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a)^5)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^6\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

3.147 $\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx$

Optimal. Leaf size=169

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+d}{5ad}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^(
3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2/3*
(a+a*sec(d*x+c))^(9/2)/a^3/d+2/11*(a+a*sec(d*x+c))^(11/2)/a^4/d+2*a*(a+a*se
c(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 90, 52, 65, 213}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{11/2}}{11a^4d}-\frac{2(a\sec(c+dx)+a)^{9/2}}{3a^3d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5ad}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{2a\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d) - (2*(a +
a*Sec[c + d*x])^(9/2))/(3*a^3*d) + (2*(a + a*Sec[c + d*x])^(11/2))/(11*a^4*
d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{7/2} + \frac{a^2(a+ax)^{7/2}}{x} + a(a+ax)^{9/2}\right) dx, x, \text{se}\right.}{a^4 d}$$

$$= -\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c}{11a}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c}{3a^3}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c}{7a^2}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c}{5ad}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c}{5ad}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.52, size = 112, normalized size = 0.66

$$\frac{2(a(1+\sec(c+dx)))^{3/2}\left(-1155\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(1656+327\sec(c+dx)-534\sec^2(c+dx)-325\sec^3(c+dx)+140\sec^4(c+dx)+105\sec^5(c+dx))\right)}{1155d(1+\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-1155*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sq
rt[1 + Sec[c + d*x]]*(1656 + 327*Sec[c + d*x] - 534*Sec[c + d*x]^2 - 325*Se
c[c + d*x]^3 + 140*Sec[c + d*x]^4 + 105*Sec[c + d*x]^5)))/(1155*d*(1 + Sec[
c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(141) = 282$.
time = 0.18, size = 429, normalized size = 2.54

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(1155(\cos^5(dx+c))\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{11}{2}}\sqrt{2}+5775(\cos^4(dx+c))\mathrm{ar}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/36960/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1155*cos(d*x+c)^5*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(11/2)*2^(1/2)+5775*cos(d*x+c)^4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*2^(1/2)+11550*cos(
d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(11/2)*2^(1/2)+11550*cos(d*x+c)^2*arctan(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)
*2^(1/2)+5775*cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*2^(1/2)+1155*2^(1/2)*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(11/2)-105984*cos(d*x+c)^5-20928*cos(d*x+c)^4+34176*cos(d*x+c)^3+20800*
cos(d*x+c)^2-8960*cos(d*x+c)-6720)/cos(d*x+c)^5*a
```

Maxima [A]
time = 0.49, size = 162, normalized size = 0.96

$$\frac{1155\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}+\frac{210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{11}{2}}}{a^4}-\frac{770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^3}+\frac{330\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^2}+\frac{462\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a}+2310\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/1155*(1155*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/c
os(d*x + c)) + sqrt(a))) + 770*(a + a/cos(d*x + c))^(3/2) + 210*(a + a/cos(
d*x + c))^(11/2)/a^4 - 770*(a + a/cos(d*x + c))^(9/2)/a^3 + 330*(a + a/cos(
d*x + c))^(7/2)/a^2 + 462*(a + a/cos(d*x + c))^(5/2)/a + 2310*sqrt(a + a/co
s(d*x + c))*a)/d
```

Fricas [A]
time = 3.58, size = 334, normalized size = 1.98

$$\left[\frac{1155\,a^{\frac{3}{2}}\cos(dx+c)^5\log\left(-8\,a\cos(dx+c)^2+4\,(2\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)+4\,(1656\,a\cos(dx+c)^5+327\,a\cos(dx+c)^4-534\,a\cos(dx+c)^3-325\,a\cos(dx+c)^2+140\,a\cos(dx+c)+105\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{2310\,d\cos(dx+c)^5},\ \frac{1155\sqrt{-a}\,a\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^5+2\,(1656\,a\cos(dx+c)^5+327\,a\cos(dx+c)^4-534\,a\cos(dx+c)^3-325\,a\cos(dx+c)^2+140\,a\cos(dx+c)+105\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{1155\,d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/2310*(1155*a^(3/2)*cos(d*x + c)^5*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x
 + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + 4*(1656*a*cos(d*x + c)^5 + 327*a*cos(d*x + c)^4 - 53
4*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(d*x + c) + 105*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^5), 1/1155*(1155*sqrt
(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^5 + 2*(1656*a*cos(d*x + c)^5 + 327*
a*cos(d*x + c)^4 - 534*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(
d*x + c) + 105*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^
5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**5, x)
```

Giac [A]
time = 2.69, size = 218, normalized size = 1.29

$$\frac{\sqrt{2}\left(\frac{1155\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(1155\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^5a-770\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4a^2+924\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3a^3-1320\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^4-6160\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^5-3360\,a^6\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^5\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)a\,\mathrm{sgn}(\cos(dx+c))}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/1155*sqrt(2)*(1155*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(1155*(a*tan(1/2*d*x + 1/2*c)^2 - a)^5*a
- 770*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a^2 + 924*(a*tan(1/2*d*x + 1/2*c)^2
- a)^3*a^3 - 1320*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^4 - 6160*(a*tan(1/2*d*
x + 1/2*c)^2 - a)*a^5 - 3360*a^6)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a)))*a*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

3.148 $\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a+a\sec(c+dx)}}{d}-\frac{2(a+a\sec(c+dx))^{3/2}}{3d}-\frac{2(a+a\sec(c+dx}{5ad}$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*(a+a*sec(d*x+c))^(3
/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2*a*(
a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 81, 52, 65, 213}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5ad}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}-\frac{2a\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a*Sqrt[a + a*S
ec[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\,\text{se}}{7}$$

$$= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\,\text{se}}{}$$

$$= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\,\text{se}}{}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A]

time = 0.26, size = 92, normalized size = 0.76

$$\frac{2(a(1+\sec(c+dx)))^{3/2}\left(105\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(-146-32\sec(c+dx)+24\sec^2(c+dx)+15\sec^3(c+dx))\right)}{105d(1+\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(105*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-146 - 32*Sec[c + d*x] + 24*Sec[c + d*x]^2 + 15*Sec[c +
 d*x]^3)))/(105*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(101) = 202$.

time = 0.16, size = 291, normalized size = 2.40

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105(\cos^3(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+315(\cos^2(dx+c))\sqrt{2}\,\text{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*cos(d*x+c)^3*2^(1/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(7/2)+315*cos(d*x+c)^2*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+315*cos(d*x+c)*2^
(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(7/2)+105*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-2336*cos(d*x+c)^3-5
12*cos(d*x+c)^2+384*cos(d*x+c)+240)/cos(d*x+c)^3*a
```

Maxima [A]

time = 0.50, size = 124, normalized size = 1.02

$$-\frac{105\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}-\frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^2}+\frac{42\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a}+210\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/105*(105*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 70*(a + a/cos(d*x + c))^(3/2) - 30*(a + a/cos(d*x
```

```
+ c))^(7/2)/a^2 + 42*(a + a/cos(d*x + c))^(5/2)/a + 210*sqrt(a + a/cos(d*x
+ c))*a)/d
```

Fricas [A]
time = 2.55, size = 290, normalized size = 2.40

$$\left[\frac{105\,a^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-8\,a\cos\left(dx+c\right)^{2}-4\,\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)-4\,\left(146\,a\cos\left(dx+c\right)^{3}+32\,a\cos\left(dx+c\right)^{2}-24\,a\cos\left(dx+c\right)-15\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{210\,d\cos\left(dx+c\right)^{3}},\ -\frac{105\,\sqrt{-a}\,a\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+a}\right)\cos\left(dx+c\right)^{3}+2\,\left(146\,a\cos\left(dx+c\right)^{3}+32\,a\cos\left(dx+c\right)^{2}-24\,a\cos\left(dx+c\right)-15\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{105\,d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^(3/2)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 4*(146*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*c
os(d*x + c) - 15*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c
)^3), -1/105*(105*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(146*a*
cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*cos(d*x + c) - 15*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\tan^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**3, x)
```

Giac [A]
time = 1.56, size = 173, normalized size = 1.43

$$-\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\,\left(105\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}a^{2}-70\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}a^{3}+84\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a^{4}+120\,a^{5}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(105*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^
2 - 70*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a^3 + 84*(a*tan(1/2*d*x + 1/2*c)^2
- a)*a^4 + 120*a^5)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.149 $\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx$

Optimal. Leaf size=73

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3d}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^(
3/2)/d+2*a*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 52, 65, 213}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a\sqrt{a\sec(c+dx)+a}}{d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a^2\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{(2a)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.14, size = 70, normalized size = 0.96

$$\frac{2(a(1+\sec(c+dx)))^{3/2}\left(-3\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(4+\sec(c+dx))\right)}{3d(1+\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-3*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(4 + Sec[c + d*x])))/(3*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A]
time = 0.04, size = 57, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+a\sec\left(dx+c\right)}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{d}$	57
default	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+a\sec\left(dx+c\right)}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*(a+a*sec(d*x+c))^(3/2)+2*a*(a+a*sec(d*x+c))^(1/2)-2*a^(3/2)*arctan
h((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 1.18

$$\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)+2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}\,a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x
 + c)) + sqrt(a))) + 2*(a + a/cos(d*x + c))^(3/2) + 6*sqrt(a + a/cos(d*x +
c))*a)/d
```

Fricas [A]
time = 2.44, size = 238, normalized size = 3.26

$$\left[\frac{3\,a^{\frac{3}{2}}\cos\left(dx+c\right)\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\left(4\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{6\,d\cos\left(dx+c\right)},\ \frac{3\sqrt{-a}\,a\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos\left(dx+c\right)+2\left(4\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) + 4*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c)))/(d*cos(d*x + c)), 1/3*(3*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
```

```
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
+ 2*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(
d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^{\frac{3}{2}} \tan (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x), x)
```

Giac [A]
time = 1.04, size = 122, normalized size = 1.67

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a-2\,a^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)a\mathrm{sgn}(\cos (dx+c))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) + 2*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a - 2*a^2)/((
a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*a*sgn(c
os(d*x + c))/d
```

Mupad [B]
time = 1.46, size = 67, normalized size = 0.92

$$\frac{2\left(a+\frac{a}{\cos (c+d\,x)}\right)^{3/2}}{3\,d}-\frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos (c+d\,x)}}}{\sqrt{a}}\right)}{d}+\frac{2\,a\,\sqrt{a+\frac{a}{\cos (c+d\,x)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(3/2))/(3*d) - (2*a^(3/2)*atanh((a + a/cos(c + d*x)
)^(1/2)/a^(1/2)))/d + (2*a*(a + a/cos(c + d*x))^(1/2))/d
```

3.150 $\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2*a^(3/2)*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3965, 85, 65, 213}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]

[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{}{(-a}\right.}{}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(4a^2)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{d}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 0.99

$$\frac{\left(2\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right) - 2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\right)(a(1+\sec(c+dx)))^{3/2}}{d(1+\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*
x]]/Sqrt[2]])*(a*(1 + Sec[c + d*x]))^(3/2))/(d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A]
time = 0.19, size = 101, normalized size = 1.38

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+2\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\right)a}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2*arct
an(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.
time = 2.33, size = 243, normalized size = 3.33

$$\left[\frac{\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{3}{2}}\log\left(-2a\cos(dx+c)-2\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-a\right)}{d},\ \frac{2\left(\sqrt{2}\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-\sqrt{-a}\,a\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a^(3/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(3/2)*l
og(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) - a))/d, 2*(sqrt(2)*sqrt(-a)*a*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a
)*a*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a
*cos(d*x + c) + a)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x), x)
```

Giac [A]
time = 0.91, size = 89, normalized size = 1.22

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)a\mathrm{sgn}(\cos\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - 2*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/sqrt(-a))*a*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

3.151 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}+\frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+5/4*a^(3/2)*arctanh(1/
2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+1/2*a*(a+a*sec(d*x+c))^
(1/2)/d/(1-sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 105, 162, 65, 213}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}+\frac{a\sqrt{a\sec(c+dx)+a}}{2d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (5*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d) + (a*Sqrt[a +
 a*Sec[c + d*x]])/(2*d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{a^2x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c\right.}{2d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right.}{d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c\right.}{d}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a}}{\sqrt{}}\right.}{2\sqrt{2}\,d}$$

Mathematica [A]
time = 0.34, size = 99, normalized size = 0.91

$$\frac{(a(1+\sec(c+dx)))^{3/2}\left(-8\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)-\frac{2\sqrt{1+\sec(c+dx)}}{-1+\sec(c+dx)}\right)}{4d(1+\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(3/2)*(-8*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 5*Sqrt[
2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] - (2*Sqrt[1 + Sec[c + d*x]])/(-1
 + Sec[c + d*x])))/(4*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(88) = 176$.
time = 0.18, size = 258, normalized size = 2.37

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(4\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)(\cos^2(dx+c))\sqrt{2}+5\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)
^2*2^(1/2)+5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+2*cos(d*x+c
)^2-5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))+2*cos(d*x+c))/sin(d*x+c)^2*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(86) = 172$.
time = 2.82, size = 378, normalized size = 3.47

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+8\,(a\cos(dx+c)-a)\sqrt{a}\log\left(-2\,a\cos(dx+c)+2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-a\right)+5\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3\,a\cos(dx+c)+a}{\cos(dx+c)-1}\right)}{8\,(d\cos(dx+c)-d)},\ -\frac{5\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-8\,(a\cos(dx+c)-a)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{4\,(d\cos(dx+c)-d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 8*(a*cos(d
*x + c) - a)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c) - a) + 5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)
*a)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(d*x + c) -
 d), -1/4*(5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c
) + a)) - 8*(a*cos(d*x + c) - a)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*a*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*cos(d*x + c) - d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.93, size = 138, normalized size = 1.27

$$-\frac{\frac{5\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{8\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\;a\,\mathrm{sgn}(\cos(dx+c))}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(5*sqrt(2)*a^2*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*sg
n(cos(d*x + c))/sqrt(-a) - 8*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) + sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.152 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=171

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{71a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}+\frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}}-$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-71/64*a^(3/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+7/32*a^2/d/(a+a*sec(d*
x+c))^(1/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(1/2)-13/16*a^2/d/(
1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{71a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}+\frac{7a^2}{32d\sqrt{a\sec(c+dx)+a}}-\frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (71*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*d) + (7*a^2)/(3
2*d*Sqrt[a + a*Sec[c + d*x]]) - a^2/(4*d*(1 - Sec[c + d*x])^2*Sqrt[a + a*Se
c[c + d*x]]) - (13*a^2)/(16*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{a^3\text{Subst}\left(\int \frac{4a}{x(-a+a}\right.}{}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{}{16d(1-\sec(c+d}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+}}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+}}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+}}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a}}{\sqrt{}}\right.}{32\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.31, size = 104, normalized size = 0.61

$$\frac{a^2\left(-34+71\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2-64\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))^2+26\sec(c+dx)\right)}{32d(-1+\sec(c+dx))^2\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-34 + 71*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2]*(-1 +
Sec[c + d*x])^2 - 64*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]]*(-1
+ Sec[c + d*x])^2 + 26*Sec[c + d*x]))/(32*d*(-1 + Sec[c + d*x])^2*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(142) = 284$.
time = 0.22, size = 502, normalized size = 2.94

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))(1+\cos(dx+c))^2\left(64(\cos^3(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^2
*(64*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+71*cos(d*x+c)^3*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-64*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)-71*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-64*cos(d*x+c
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*2^(1/2)+54*cos(d*x+c)^3-71*cos(d*x+c)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+64*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2))*2^(1/2)-24*cos(d*x+c)^2+71*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-14*cos(d*x+c))/sin(d*x+c
)^6*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(138) = 276.
time = 3.63, size = 589, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(
a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 71*(sqrt(2)
*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a*cos(d*x + c) + sqr
t(2)*a)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(27*a*cos
(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
 d), 1/64*(71*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2
)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 64*(a*cos(
d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
+ a)) - 2*(27*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2
 - d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.99, size = 211, normalized size = 1.23

$$\frac{\frac{71\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{128\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-8\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\;a\mathrm{sgn}(\cos(dx+c))-\frac{17\sqrt{2}\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}a^2\mathrm{sgn}(\cos(dx+c))-15\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\;a^3\mathrm{sgn}(\cos(dx+c))}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(71*sqrt(2)*a^2*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*s
gn(cos(d*x + c))/sqrt(-a) - 128*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) - 8*sqrt(2)*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c)) - (17*sqrt(2)*(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)^(3/2)*a^2*sgn(cos(d*x + c)) - 15*sqrt(2)*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*a^3*sgn(cos(d*x + c)))/(a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^5\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

3.153 $\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx$

Optimal. Leaf size=258

$$-\frac{2a^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^4\tan^5(c-}{5d(a+a\sec(c-}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+30/7*a^5*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+34/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+14/11*a^7
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+2/13*a^8*tan(d*x+c)^13/d/(a+a*sec(
d*x+c))^(13/2)
```

Rubi [A]

time = 0.09, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$-\frac{2a^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^8\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{14a^7\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{30a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^3*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (30*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(34*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (14*a^7*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (2*a^8*Tan[c + d*x]^13)/(13*d
*(a + a*Sec[c + d*x])^(13/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = -\frac{(2a^5)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^5)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+15x^6+17ax^8+7a^2x^{10}+a^3x^{12}\right.\right.}{d}$$

$$= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(}{5d(a+a\sec(}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} -$$

Mathematica [A]
time = 8.41, size = 147, normalized size = 0.57

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-1441440\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{13}{2}}(c+dx)+164736\sin\left(\frac{1}{2}(c+dx)\right)+81081\sin\left(\frac{3}{2}(c+dx)\right)+134849\sin\left(\frac{5}{2}(c+dx)\right)+98176\sin\left(\frac{9}{2}(c+dx)\right)+45045\sin\left(\frac{11}{2}(c+dx)\right)+32429\sin\left(\frac{13}{2}(c+dx)\right)\right)}{1441440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*(-1441440*Sqr
t[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(13/2) + 164736*Sin[(c +
 d*x)/2] + 81081*Sin[(3*(c + d*x))/2] + 134849*Sin[(5*(c + d*x))/2] + 98176
*Sin[(9*(c + d*x))/2] + 45045*Sin[(11*(c + d*x))/2] + 32429*Sin[(13*(c + d*
x))/2]))/(1441440*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(226) = 452.
time = 0.25, size = 656, normalized size = 2.54

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(45045\sin(dx+c)\left(\cos^6(dx+c)\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{13}{2}}+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2882880/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(45045*sin(d*x+c)*cos(d*x+c
)^6*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+270270*sin(d*x+c)*co
s(d*x+c)^5*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+675675*sin(d*
x+c)*cos(d*x+c)^4*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+900900
*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)
+675675*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(13/2)+270270*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(13/2)+45045*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*sin(d
*x+c)-4150912*cos(d*x+c)^7-807424*cos(d*x+c)^6+5563904*cos(d*x+c)^5+2781952
*cos(d*x+c)^4-2585600*cos(d*x+c)^3-1809920*cos(d*x+c)^2+564480*cos(d*x+c)+4
43520)/cos(d*x+c)^6/sin(d*x+c)*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.13, size = 415, normalized size = 1.61

$$\left[\frac{45045\left(a\cos(dx+c)^7+a\cos(dx+c)^6\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(32429a\cos(dx+c)^6+38737a\cos(dx+c)^5-4731a\cos(dx+c)^4-26465a\cos(dx+c)^3-6265a\cos(dx+c)^2+7875a\cos(dx+c)+3465a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{45045\left(d\cos(dx+c)^7+d\cos(dx+c)^6\right)}, \frac{2\left(45045\left(a\cos(dx+c)^7+a\cos(dx+c)^6\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(32429a\cos(dx+c)^6+38737a\cos(dx+c)^5-4731a\cos(dx+c)^4-26465a\cos(dx+c)^3-6265a\cos(dx+c)^2+7875a\cos(dx+c)+3465a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)\right)}{45045\left(d\cos(dx+c)^7+d\cos(dx+c)^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(32429*a*cos(
d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c)^4 - 26465*a*cos(d
*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c) + 3465*a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7 + d*cos(d*x
+ c)^6), 2/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(a)*arcta
n(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) + (32429*a*cos(d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c
)^4 - 26465*a*cos(d*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c)
+ 3465*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**6,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**6, x)
```

Giac [A]
time = 3.52, size = 369, normalized size = 1.43

$$\frac{\frac{45045\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{|a|}+\frac{2\left(45045\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))-\left(300300\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))-\left(861861\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))-\left(573144\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))-\left(236951\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))+\left(4751\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-53404\sqrt{2}a^8\operatorname{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^6\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/45045*(45045*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a) + 2*(45045*sqrt(2)*a^8*sgn(cos(d*x
 + c)) - (300300*sqrt(2)*a^8*sgn(cos(d*x + c)) - (861861*sqrt(2)*a^8*sgn(co
s(d*x + c)) - (573144*sqrt(2)*a^8*sgn(cos(d*x + c)) - (236951*sqrt(2)*a^8*s
gn(cos(d*x + c)) + (4751*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)
^2 - 53404*sqrt(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d
*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^6\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

3.154 $\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx$

Optimal. Leaf size=194

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{14a^4\tan^5(c+d}{5d(a+a\sec(c+d}$$

[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+14/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+10/7*a^5*tan(d*x+c)^7/d/(a+a*sec(d*x+c))^(7/2)+2/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)

Rubi [A]
time = 0.07, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{10a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{14a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]

[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(3*d*(a + a*Sec[c + d*x])^(3/2)) + (14*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x])^(5/2)) + (10*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (2*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+7x^4+5ax^6+a^2x^8+\frac{1}{a^2(1+ax^2)}\right) d}{d}$$

$$= -\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{14a^4\,\text{ta}}{5d(a+a\,\text{s}}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+$$

Mathematica [A]
time = 6.50, size = 123, normalized size = 0.63

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(1+\sec(c+dx))}\left(2520\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{9}{2}}(c+dx)+126\sin\left(\frac{1}{2}(c+dx)\right)-288\sin\left(\frac{5}{2}(c+dx)\right)-315\sin\left(\frac{7}{2}(c+dx)\right)-169\sin\left(\frac{9}{2}(c+dx)\right)\right)}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*(2520*Sqrt[2]
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(9/2) + 126*Sin[(c + d*x)/2]
 - 288*Sin[(5*(c + d*x))/2] - 315*Sin[(7*(c + d*x))/2] - 169*Sin[(9*(c + d*
x))/2]))/(2520*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(170) = 340$.
time = 0.18, size = 407, normalized size = 2.10

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\sin(dx+c)\left(\cos^4(dx+c)\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+945\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/2520/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+945*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+945*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+2704*cos(d*x+c)^5+1168*cos(d*x+c)^4-3488*cos(d*x+c)^3-1744*cos(d*x+c)^2+800*cos(d*x+c)+560)/cos(d*x+c)^4/sin(d*x+c)*a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="maxima")

[Out] -1/630*(315*((a*cos(2*d*x + 2*c)^4 + a*sin(2*d*x + 2*c)^4 + 4*a*cos(2*d*x + 2*c)^3 + 6*a*cos(2*d*x + 2*c)^2 + 2*(a*cos(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*sin(2*d*x + 2*c)^2 + 4*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a*cos(2*d*x + 2*c)^4 + a*sin(2*d*x + 2*c)^4 + 4*a*cos(2*d*x + 2*c)^3 + 6*a*cos(2*d*x + 2*c)^2 + 2*(a*cos(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*sin(2*d*x + 2*c)^2 + 4*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 2*(a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrate((cos(2*d*x + 2*c)

^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*
c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6
*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*
c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x +
 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d
*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*
d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2
*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c))*sin(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin
(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*
sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)
*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c
)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(11/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c)
 + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c
) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x +
 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x
 + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x
+ 2*c)^2)*sin(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(
2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*cos(10*d*x + 10*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 36*(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 16*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*sin(10*d*x + 10*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 36*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 +
16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4
*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 4*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x
 + 8*c) + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c)
)*cos(10*d*x + 10*c) + 8*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x +
 2*c)^2 + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c)
)*cos(8*d*x + 8*c) + 12*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x +
2*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +

```
1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 8*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 +
2*(sin(2*d*x + 2*c)^3 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(8*d*x + 8*c) + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)...
```

Fricas [A]
time = 3.30, size = 371, normalized size = 1.91

$$\left[\frac{315\,\left(a\cos\left(dx+c\right)^5+a\cos\left(dx+c\right)^4\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\,\left(169\,a\cos\left(dx+c\right)^4+242\,a\cos\left(dx+c\right)^3+24\,a\cos\left(dx+c\right)^2-85\,a\cos\left(dx+c\right)-35\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)},\ -\frac{2\,\left(315\,\left(a\cos\left(dx+c\right)^5+a\cos\left(dx+c\right)^4\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{\sqrt{a}\,\sin(dx+c)}\right)+\left(169\,a\cos\left(dx+c\right)^4+242\,a\cos\left(dx+c\right)^3+24\,a\cos\left(dx+c\right)^2-85\,a\cos\left(dx+c\right)-35\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)\right)}{315\,\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(169*a*cos(d*x +
c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 85*a*cos(d*x + c) - 35*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
+ d*cos(d*x + c)^4), -2/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt
(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*si
n(d*x + c))) + (169*a*cos(d*x + c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x
+ c)^2 - 85*a*cos(d*x + c) - 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\tan^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**4, x)
```

Giac [A]
time = 2.12, size = 310, normalized size = 1.60

$$-\frac{\frac{315\,\sqrt{-a}\,a^2\log\left(\frac{\left|2\,\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\,\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\,\left(315\,\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))-\left(1470\,\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))-\left(756\,\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))+\left(\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-162\,\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/315*(315*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(d*x + c))/abs(a) + 2*(315*sqrt(2)*a^6*sgn(cos(d*x + c)
) - (1470*sqrt(2)*a^6*sgn(cos(d*x + c)) - (756*sqrt(2)*a^6*sgn(cos(d*x + c)
) + (sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 162*sqrt(2)*a^6
*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2
*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

3.155 $\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx$

Optimal. Leaf size=128

$$-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}}+\frac{2a^4\tan^5(c+d}{5d(a+a\sec(c+a}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+2*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2
/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = -\frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{x^2(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a(1+ax^2)}\right)\,dx, x, -\frac{\tan(c}{\sqrt{a+a\,\mathrm{se}}}\right.}{d}$$

$$= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(}{5d(a+a\sec(}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [**A**]
time = 5.56, size = 97, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-10\sqrt{2}\,\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+dx)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(-10*Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 5*Sin[(3*(c + d*x))/2
] + Sin[(5*(c + d*x))/2]))/(10*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(114) = 228$.
time = 0.14, size = 300, normalized size = 2.34

method	result
default	$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5(\cos^2(dx+c))\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}+10\cos(\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/20/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*cos(d*x+c)^2*sin(d*x+c)*arcta
nh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+10*cos(d*x+c)*sin(d*x+c)*arcta
nh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+5*2^(1/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-8*cos(d*x+c)^3-16*cos(d*x+c)^2+16*cos(d*x+
c)+8)/sin(d*x+c)/cos(d*x+c)^2*a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="maxima")

[Out] 1/10*(5*((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c
) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a*cos(2*d*x + 2*
c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1) - 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x +
 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d
*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(
6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2
*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(
2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6
*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(
2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4
*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*

```
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3
 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2
+ cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*
x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin
(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 +
4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x +
2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*
d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*co
s(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) +
(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2), x) - 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2
*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x
 + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*
d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d
*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*
d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)...
```

Fricas [A]
time = 2.38, size = 321, normalized size = 2.51

$$\left[\frac{5\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}+2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\left(a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)},\ \frac{2\left(5\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)}{\sqrt{a}\,\sin\left(dx+c\right)}\right)+\left(a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a*cos(d*x + c)^2 + 3
*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 2/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x
 + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c))) + (a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(
d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\tan^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**2, x)
```

Giac [A]
time = 1.54, size = 224, normalized size = 1.75

$$\frac{\frac{5\,\sqrt{-a}\,a^{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\left|a\right|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\left|a\right|-6\,a\right|}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{\left|a\right|}-\frac{2\left(\sqrt{2}\,a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-5\,\sqrt{2}\,a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*(5*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/
```

```
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a)
- 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(sqrt(2)*a^4*sgn(cos(d*x + c))*tan(1/2
*d*x + 1/2*c)^4 - 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a
*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.156 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=64

$$-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a*cot(d*x+
c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 331, 209}

$$-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2
*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,\right.}{}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a\cot(c+dx)\sqrt{a+a}}{d}$$

Mathematica [A]
time = 0.39, size = 102, normalized size = 1.59

$$-\frac{2\cot(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}\,(a(1+\sec(c+dx)))^{3/2}\left(\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}}+\text{ArcSin}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*(a*(1 + Sec[c + d*x]))^(3/2)
*(Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)] + ArcSin[Tan[(c + d*x)/2
]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Tan[(c + d*x)/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(56) = 112$.
time = 0.16, size = 115, normalized size = 1.80

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+2\cos(dx+c)\right)}{d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(d*x+c))/sin(d*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.
time = 2.63, size = 264, normalized size = 4.12

$$\left[\frac{\sqrt{-a}\, a \log\left(-\frac{8\, a \cos(dx+c)^{3}+4\left(2\, \cos(dx+c)^{2}-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\, a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c) - 4\, a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\, d\sin(dx+c)}, -\frac{a^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\, a\cos(dx+c)^{2}+a\cos(dx+c)-a}\right)\sin(dx+c) + 2\, a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x +
 c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*co
s(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*a*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(3/2)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*a*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cot^{2}\left(c+dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(56) = 112$.

time = 1.20, size = 197, normalized size = 3.08

$$\frac{\frac{\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{2\,\sqrt{2}\,\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}{\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a)
)*sgn(cos(d*x + c))/abs(a) + 2*sqrt(2)*sqrt(-a)*a^2*sgn(cos(d*x + c))/((sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^2\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.157 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,d}+\frac{3a\cot(c+dx)\sqrt{a+a\sec(}{2d}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-1/3*cot(d*x+c
)^3*(a+a*sec(d*x+c))^(3/2)/d-1/4*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1
/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)+3/2*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 491, 597, 536, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,d}-\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{3a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*d) + (3*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(2*d) - (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} - \frac{\text{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\right.}{}$$

$$= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+}{3d}$$

$$= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+}{3d}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan}{\sqrt{2}\,\sqrt{a+a}\right.}{2\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 24.06, size = 5542, normalized size = 38.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(119) = 238.
time = 0.19, size = 372, normalized size = 2.58

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-12\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^2(dx+c))\sin(dx+c)\sqrt{2}-3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-12*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-3*sin(d*x+c)*cos(d*x+c)^2*l
n(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+12*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))*sin(d*x+c)+22*cos(d*x+c)^3+3*ln(-(-(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2-18*cos(d*x+c))/sin(d*x+c)^3*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.14, size = 531, normalized size = 3.69

$$\left[\frac{3\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\sqrt{-a}\,\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+12(a\cos(dx+c)-a)\sqrt{-a}\,\log\left(-\frac{8a\cos(dx+c)^3-4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)+4\left(11a\cos(dx+c)^2-9a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{24(d\cos(dx+c)-d)\sin(dx+c)},\ \frac{12(a\cos(dx+c)-a)\sqrt{a}\,\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+3\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+2\left(11a\cos(dx+c)^2-9a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{12(d\cos(dx+c)-d)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))*sin(d*x + c) + 12*(a*cos(d*x + c) - a)*sqrt(-a)*log(-(8*a*cos(d*x + c)^
3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(
d*x + c) + 4*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/12*(12*(a*cos(d*
x + c) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*
sin(d*x + c) + 2*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

<u>Sympy</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

<u>Giac</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(119) = 238.
time = 1.56, size = 369, normalized size = 2.56

$$-\frac{3\sqrt{2}\sqrt{-a}\,a\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\frac{24\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{8\sqrt{2}\left(6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))-9\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^3\mathrm{sgn}(\cos(dx+c))+5\sqrt{-a}\,a^4\mathrm{sgn}(\cos(dx+c))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(3*sqrt(2)*sqrt(-a)*a*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 24*sqrt(-a)*a^2*log(abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs
(a) + 8*sqrt(2)*(6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 9*(sqrt(-a)*tan(1/2*d*x +
```

```
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(d*x +
c)) + 5*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

3.158 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=226

$$-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{11a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,d}-\frac{21a\cot(c+dx)\sqrt{a+}}{16d}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+5/24*cot(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)/d+3/20*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/a/d
-1/4*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(5/2)/a/
d+11/32*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/d*2^(1/2)-21/16*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 483, 597, 536, 209}

$$-\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{11a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,d}+\frac{3\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20ad}+\frac{5\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{24d}-\frac{21a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{16d}-\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
1*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(16*Sqrt[2]*d) - (21*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(16*d) + (5
*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(24*d) + (3*Cot[c + d*x]^5*(a +
 a*Sec[c + d*x])^(5/2))/(20*a*d) - (Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*
x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}$$

$$= \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} - \frac{\cos(c+dx)\cot^5(c+dx)\,s}{}$$

$$= \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} + \frac{3\cot^5(c+dx)(a+a\sec(c}{20ad}$$

$$= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec}{24d}$$

$$= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec}{24d}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{a}}\right.}{16\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.01, size = 5582, normalized size = 24.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(192) = 384.
time = 0.26, size = 720, normalized size = 3.19

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))(1+\cos(dx+c))^2\left(-480\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx}{1+\cos(d}}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/480/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^
2*(-480*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
h(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2
^(1/2)-165*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(
-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)+480*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(
1/2)+165*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
480*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)
+898*cos(d*x+c)^4+165*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(
-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
cos(d*x+c)-480*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)
-702*cos(d*x+c)^3-165*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-
730*cos(d*x+c)^2+630*cos(d*x+c))/sin(d*x+c)^7*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.21, size = 708, normalized size = 3.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*
a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 -
 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c)
+ 480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*(449*a*cos(d*x + c)^4 - 351*a*cos(d*x +
c)^3 - 365*a*cos(d*x + c)^2 + 315*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
*sin(d*x + c)), -1/480*(480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*
x + c) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(
2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(4
49*a*cos(d*x + c)^4 - 351*a*cos(d*x + c)^3 - 365*a*cos(d*x + c)^2 + 315*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^3 -
d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(192) = 384.
time = 2.37, size = 519, normalized size = 2.30

$$\frac{165\sqrt{2}\sqrt{-a}\,a\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)\mathrm{sgn}(\cos(dx+c))+30\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a\,\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{960\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{32\sqrt{2}\left(60\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^8\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))-195\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6\sqrt{-a}\,a^3\mathrm{sgn}(\cos(dx+c))+275\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}\,a^4\mathrm{sgn}(\cos(dx+c))-175\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^5\mathrm{sgn}(\cos(dx+c))+47\sqrt{-a}\,a^6\mathrm{sgn}(\cos(dx+c))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/960*(165*sqrt(2)*sqrt(-a)*a*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 30*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c) + 960*sqrt
(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn
```

```
(cos(d*x + c))/abs(a) + 32*sqrt(2)*(60*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 195*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(
-a)*a^3*sgn(cos(d*x + c)) + 275*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 175*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^5
*sgn(cos(d*x + c)) + 47*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^6\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

3.159 $\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx$

Optimal. Leaf size=193

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c}{5d}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c))
^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(a
+a*sec(d*x+c))^(9/2)/a^2/d-6/11*(a+a*sec(d*x+c))^(11/2)/a^3/d+2/13*(a+a*sec
(d*x+c))^(13/2)/a^4/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {3965, 90, 52, 65, 213}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{13/2}}{13a^4d}-\frac{6(a\sec(c+dx)+a)^{11/2}}{11a^3d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}+\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a +
a*Sec[c + d*x])^(9/2))/(9*a^2*d) - (6*(a + a*Sec[c + d*x])^(11/2))/(11*a^3*
d) + (2*(a + a*Sec[c + d*x])^(13/2))/(13*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{9/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{9/2} + \frac{a^2(a+ax)^{9/2}}{x} + a(a+ax)^{11/2}\right)\,dx, x, \ldots}{a^4 d}$$

$$= -\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d} + \frac{2(a+a\sec(c+dx))^{13/2}}{13a^4 d} + \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d} - \frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d} + \frac{2(a+a\sec\ldots}{13\ldots}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d} - \frac{6(a+a\sec\ldots}{11\ldots}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec\ldots}{9a\ldots}$$

$$= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{2(a+a\sec\ldots}{7\ldots}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec\ldots}{\ldots}$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec\ldots}{\ldots}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+d\ldots}}{d}$$

Mathematica [A]
time = 0.74, size = 156, normalized size = 0.81

$$\frac{(a(1+\sec(c+dx)))^{5/2}\left(-2\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+2\sqrt{1+\sec(c+dx)}+\frac{2}{3}(1+\sec(c+dx))^{3/2}+\frac{2}{5}(1+\sec(c+dx))^{5/2}+\frac{2}{7}(1+\sec(c+dx))^{7/2}+\frac{2}{9}(1+\sec(c+dx))^{9/2}-\frac{6}{11}(1+\sec(c+dx))^{11/2}+\frac{2}{13}(1+\sec(c+dx))^{13/2}\right)}{d(1+\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[
1 + Sec[c + d*x]] + (2*(1 + Sec[c + d*x])^(3/2))/3 + (2*(1 + Sec[c + d*x])^
(5/2))/5 + (2*(1 + Sec[c + d*x])^(7/2))/7 + (2*(1 + Sec[c + d*x])^(9/2))/9
- (6*(1 + Sec[c + d*x])^(11/2))/11 + (2*(1 + Sec[c + d*x])^(13/2))/13))/(d*
(1 + Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(161) = 322$.

time = 0.19, size = 500, normalized size = 2.59

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(45045\left(\cos^6(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{13}{2}}\sqrt{2}+270270\left(\cos^5(dx+c)\right)\text{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2882880/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(45045*cos(d*x+c)^6*arctan(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(13/2)*2^(1/2)+270270*cos(d*x+c)^5*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+675675
*cos(d*x+c)^4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+900900*cos(d*x+c)^3*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(13/2)*2^(1/2)+675675*cos(d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+270270*cos(d
*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)+45045*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)+9176192
*cos(d*x+c)^6+4060544*cos(d*x+c)^5-1603968*cos(d*x+c)^4-3468160*cos(d*x+c)^
3-568960*cos(d*x+c)^2+1088640*cos(d*x+c)+443520)/cos(d*x+c)^6*a^2
```

Maxima [A]

time = 0.47, size = 181, normalized size = 0.94

$$\frac{45045\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+18018\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}+\frac{6930\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{13}{2}}}{a^4}-\frac{24570\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{11}{2}}}{a^3}+\frac{10010\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^2}+\frac{12870\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a}+30030\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a+90090\sqrt{a+\frac{a}{\cos(dx+c)}}\,a^2}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/45045*(45045*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a
/cos(d*x + c)) + sqrt(a))) + 18018*(a + a/cos(d*x + c))^(5/2) + 6930*(a + a
/cos(d*x + c))^(13/2)/a^4 - 24570*(a + a/cos(d*x + c))^(11/2)/a^3 + 10010*(
a + a/cos(d*x + c))^(9/2)/a^2 + 12870*(a + a/cos(d*x + c))^(7/2)/a + 30030*
(a + a/cos(d*x + c))^(3/2)*a + 90090*sqrt(a + a/cos(d*x + c))*a^2)/d
```

Fricas [A]

time = 3.93, size = 386, normalized size = 2.00

$$\left[\frac{45045\,a^{\frac{5}{2}}\cos(dx+c)^6\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)+4\left(71689\,a^2\cos(dx+c)^6+31723\,a^2\cos(dx+c)^5-12531\,a^2\cos(dx+c)^4-27095\,a^2\cos(dx+c)^3-4445\,a^2\cos(dx+c)^2+8505\,a^2\cos(dx+c)+3465\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{90090\,d\cos(dx+c)^6},\ \frac{45045\sqrt{-a}\,a^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^6+2\left(71689\,a^2\cos(dx+c)^6+31723\,a^2\cos(dx+c)^5-12531\,a^2\cos(dx+c)^4-27095\,a^2\cos(dx+c)^3-4445\,a^2\cos(dx+c)^2+8505\,a^2\cos(dx+c)+3465\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{45045\,d\cos(dx+c)^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/90090*(45045*a^(5/2)*cos(d*x + c)^6*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
 8*a*cos(d*x + c) - a) + 4*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x +
c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d
*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(d*cos(d*x + c)^6), 1/45045*(45045*sqrt(-a)*a^2*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a))*cos(d*x + c)^6 + 2*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x + c)^
5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d*x
+ c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)))/(d*cos(d*x + c)^6)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**A**]
time = 2.68, size = 244, normalized size = 1.26

$$\frac{\sqrt{2}\left(\frac{45045\,\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(45045\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{6}a-30030\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{5}a^{2}+36036\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{4}a^{3}-51480\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}a^{4}+80080\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}a^{5}+393120\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a^{6}+221760\,a^{7}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{6}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(45045*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(45045*(a*tan(1/2*d*x + 1/2*c)^2 - a)^6
*a - 30030*(a*tan(1/2*d*x + 1/2*c)^2 - a)^5*a^2 + 36036*(a*tan(1/2*d*x + 1/
2*c)^2 - a)^4*a^3 - 51480*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^4 + 80080*(a*t
an(1/2*d*x + 1/2*c)^2 - a)^2*a^5 + 393120*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^
6 + 221760*a^7)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)))*a^2*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

3.160 $\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}-\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}-\frac{2(a+a\sec(c+}{5d}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*a*(a+a*sec(d*x+c))^
(3/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/d-2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(a+
a*sec(d*x+c))^(9/2)/a^2/d-2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 81, 52, 65, 213}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}-\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}-\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a^2*Sqrt[a + a
*Sec[c + d*x]])/d - (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[
c + d*x])^(5/2))/(5*d) - (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a + a
*Sec[c + d*x])^(9/2))/(9*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\,\text{se}}{9}$$

$$= -\frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\,\text{s}}{}$$

$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a}{}$$

$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.61, size = 102, normalized size = 0.70

$$\frac{2(a(1+\sec(c+dx)))^{5/2}\left(315\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(-493-226\sec(c+dx)+12\sec^2(c+dx)+95\sec^3(c+dx)+35\sec^4(c+dx))\right)}{315d(1+\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-493 - 226*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 95*Sec[c
+ d*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(121) = 242$.
time = 0.16, size = 362, normalized size = 2.50

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\left(\cos^4(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}\sqrt{2}+1260\left(\cos^3(dx+c)\right)\arctan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*cos(d*x+c)^4*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(9/2)*2^(1/2)+1260*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)+1890*cos(d*x+c
)^2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(9/2)*2^(1/2)+1260*cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)+3
15*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(9/2)+15776*cos(d*x+c)^4+7232*cos(d*x+c)^3-384*cos(
d*x+c)^2-3040*cos(d*x+c)-1120)/cos(d*x+c)^4*a^2
```

Maxima [A]
time = 0.52, size = 143, normalized size = 0.99

$$-\frac{315\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}-\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^2}+\frac{90\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a}+210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a+630\sqrt{a+\frac{a}{\cos(dx+c)}}\,a^2}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/315*(315*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 126*(a + a/cos(d*x + c))^(5/2) - 70*(a + a/cos(d*
x + c))^(9/2)/a^2 + 90*(a + a/cos(d*x + c))^(7/2)/a + 210*(a + a/cos(d*x +
c))^(3/2)*a + 630*sqrt(a + a/cos(d*x + c))*a^2)/d
```

Fricas [A]
time = 4.13, size = 334, normalized size = 2.30

$$\left[\frac{315\,a^{\frac{5}{2}}\cos(dx+c)^4\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)-4\left(493\,a^2\cos(dx+c)^4+226\,a^2\cos(dx+c)^3-12\,a^2\cos(dx+c)^2-95\,a^2\cos(dx+c)-35\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{630\,d\cos(dx+c)^4},\ -\frac{315\sqrt{-a}\,a^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^4+2\left(493\,a^2\cos(dx+c)^4+226\,a^2\cos(dx+c)^3-12\,a^2\cos(dx+c)^2-95\,a^2\cos(dx+c)-35\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{315\,d\cos(dx+c)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^(5/2)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
```

```
*cos(d*x + c) - a) - 4*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^3 - 1
2*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(d*cos(d*x + c)^4), -1/315*(315*sqrt(-a)*a^2*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^4 + 2*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^
3 - 12*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{5}{2}}\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**3,x)
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**3, x)
```

Giac [A]
time = 1.64, size = 198, normalized size = 1.37

$$-\frac{\sqrt{2}\left(\frac{315\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(315\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4a^2-210\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3a^3+252\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a^4-360\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^5-560\,a^6\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)a\,\mathrm{sgn}(\cos(dx+c))}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="giac")
[Out] -1/315*sqrt(2)*(315*sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/sqrt(-a) + 2*(315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*a^
2 - 210*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*a^3 + 252*(a*tan(1/2*d*x + 1/2*c)^
2 - a)^2*a^4 - 360*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^5 - 560*a^6)/((a*tan(1/
2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*a*sgn(cos(d*x
 + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.161 $\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx$

Optimal. Leaf size=97

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c))
^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3965, 52, 65, 213}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}+\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a^2\text{Subst}\left(\int\right.}{}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec}{}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec}{}$$
$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx}}{d}$$

Mathematica [A]
time = 0.22, size = 82, normalized size = 0.85

$$\frac{2(a(1+\sec(c+dx)))^{5/2}\left(-15\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}\,(23+11\sec(c+dx)+3\sec^2(c+dx))\right)}{15d(1+\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(-15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(23 + 11*Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*(1 + S
ec[c + d*x])^(5/2))
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2a^{2}\sqrt{a+a\sec\left(dx+c\right)}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{d}$
default	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2a^{2}\sqrt{a+a\sec\left(dx+c\right)}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*(a+a*sec(d*x+c))^(5/2)+2/3*a*(a+a*sec(d*x+c))^(3/2)+2*a^2*(a+a*sec
(d*x+c))^(1/2)-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 105, normalized size = 1.08

$$\frac{15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)+6\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}+10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}\,a^{2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/15*(15*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d
*x + c)) + sqrt(a))) + 6*(a + a/cos(d*x + c))^(5/2) + 10*(a + a/cos(d*x + c
))^(3/2)*a + 30*sqrt(a + a/cos(d*x + c))*a^2)/d
```

Fricas [A]
time = 2.67, size = 282, normalized size = 2.91

$$\left[\frac{15\,a^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\left(23\,a^{2}\cos\left(dx+c\right)^{2}+11\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{30\,d\cos\left(dx+c\right)^{2}},\ \frac{15\sqrt{-a}\,a^{2}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos\left(dx+c\right)^{2}+2\left(23\,a^{2}\cos\left(dx+c\right)^{2}+11\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{15\,d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^(5/2)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) + 4*(23*a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c) + 3*a^2)*
```

```
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), 1/15*(15*sqrt(
-a)*a^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^2 + 2*(23*a^2*cos(d*x + c)^2 + 11*
a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d
*x + c)^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{5}{2}}\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x), x)
```

Giac [A]

time = 1.17, size = 148, normalized size = 1.53

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2a-10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a^2+12a^3\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)a^2\operatorname{sgn}(\cos(dx+c))}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/sqrt(-a) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 10*(
a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2 + 12*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a)
^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))*a^2*sgn(cos(d*x + c))/d
```

Mupad [B]

time = 1.82, size = 92, normalized size = 0.95

$$\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{5\,d}+\frac{2\,a\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{3\,d}+\frac{2\,a^2\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{d}+\frac{a^{5/2}\operatorname{atan}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,1\mathrm{i}}{\sqrt{a}}\right)2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(5/2))/(5*d) + (a^(5/2)*atan(((a + a/cos(c + d*x))^
(1/2)*1i)/a^(1/2))*2i)/d + (2*a*(a + a/cos(c + d*x))^(3/2))/(3*d) + (2*a^2*
(a + a/cos(c + d*x))^(1/2))/d
```

3.162 $\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-4*a^(5/2)*arctanh(1/2*(
a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3965, 86, 162, 65, 213}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (4*Sqrt[2]*a^(5/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x(-a+ax)}\,dx,x,\sec(c+dx)\right)}{d}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{a^3+3a^3x}{x(-a+ax)\sqrt{a+ax}}\,dx,x,\sec\right.}{d}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx,x,\sec(c+\right.}{d}$$
$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\,\text{se}}\right.}{d}$$
$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{d}$$

Mathematica [A]
time = 0.10, size = 83, normalized size = 0.87

$$\frac{2(a(1+\sec(c+dx)))^{5/2}\left(\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)+\sqrt{1+\sec(c+dx)}\right)}{d(1+\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + Sqrt[1 + Sec[c + d*x]]))/(d*(1 + Sec[c + d*x])^(5/2))

Maple [A]
time = 0.13, size = 124, normalized size = 1.31

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}+4\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2)*a^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c), x)

Fricas [A]
time = 2.38, size = 300, normalized size = 3.16

$$\left[\frac{2\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{2\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{5}{2}}\log\left(-2\,a\cos(dx+c)-2\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-a\right)+2\,a^{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{d},\ \frac{2\left(2\sqrt{2}\,\sqrt{-a}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-\sqrt{-a}\,a^{2}\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)+a^{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [(2*sqrt(2)*a^(5/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(5/2)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, 2*(2*sqrt(2)*sqrt(-a)*a^2*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(

```
d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a)*a^2*arctan(sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a))
 + a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.96, size = 112, normalized size = 1.18

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\,a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,a}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)a^{2}\mathrm{sgn}(\cos\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - 4*a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/sqrt(-a) - 2*a/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2*sgn(cos(d*x + c)
)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

3.163 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+3/2*a^(5/2)*arctanh(1/
2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+a^2*(a+a*sec(d*x+c))^(1
/2)/d/(1-sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 101, 162, 65, 213}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{a\sec(c+dx)+a}}{d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (3*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d) + (a^2*Sqrt[a +
 a*Sec[c + d*x]])/(d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \frac{a^4\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)^2}\,dx,x,\sec(c+dx)\right)}{d} \\
&= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}+\frac{a^3\text{Subst}\left(\int \frac{-a-\frac{ax}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx,x,\text{se}\right.}{d} \\
&= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}+\frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx,x,\sec(c+\right.}{d} \\
&= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\,\text{se}}\right.}{d} \\
&= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{\right.}}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 115, normalized size = 1.08

$$
-\frac{(a(1+\sec(c+dx)))^{5/2}\left(4\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)(-1+\sec(c+dx))-3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)(-1+\sec(c+dx))+2\sqrt{1+\sec(c+dx)}\right)}{2d(-1+\sec(c+dx))(1+\sec(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/2*((a*(1 + Sec[c + d*x]))^(5/2)*(4*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*(-1 +
 Sec[c + d*x]) - 3*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*(-1 + Se
c[c + d*x]) + 2*Sqrt[1 + Sec[c + d*x]]))/(d*(-1 + Sec[c + d*x])*(1 + Sec[c
+ d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(89) = 178$.
time = 0.17, size = 248, normalized size = 2.34

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(2\cos(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sqrt{2}+3\cos(dx+c)\sqrt{-\frac{2\cos(dx+c}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
2^(1/2)+3*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))-2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-3*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*cos
(d*x+c))/(-1+cos(d*x+c))*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(88) = 176$.
time = 2.58, size = 398, normalized size = 3.75

$$\left[\frac{4\,a^{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)+4\left(a^{2}\cos\left(dx+c\right)-a^{2}\right)\sqrt{a}\log\left(-2\,a\cos\left(dx+c\right)+2\,\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)-a\right)+3\left(\sqrt{2}\,a^{2}\cos\left(dx+c\right)-\sqrt{2}\,a^{2}\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)+3\,a\cos(dx+c)+a}{\cos(dx+c)-1}\right)}{4\left(d\cos\left(dx+c\right)-d\right)}, \frac{2\,a^{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)-3\left(\sqrt{2}\,a^{2}\cos\left(dx+c\right)-\sqrt{2}\,a^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)+4\left(a^{2}\cos\left(dx+c\right)-a^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)}{2\left(d\cos\left(dx+c\right)-d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 4*(a^2*c
os(d*x + c) - a^2)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 3*(sqrt(2)*a^2*cos(d*x + c) -
 sqrt(2)*a^2)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(
d*x + c) - d), 1/2*(2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c) - 3*(sqrt(2)*a^2*cos(d*x + c) - sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c)
 + a)) + 4*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)))/(d*cos(d*x +
c) - d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.94, size = 140, normalized size = 1.32

$$-\frac{\frac{3\sqrt{2}\,a^{3}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{4\,a^{3}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\;a^{2}\mathrm{sgn}(\cos(dx+c))}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(2)*a^3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*sg
n(cos(d*x + c))/sqrt(-a) - 4*a^3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) + sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a^2*sgn(cos(d*x + c))/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.164 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=147

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}-\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2}-1$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-43/32*a^(5/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-1/4*a^2*(a+a*sec(d*x+c
))^(1/2)/d/(1-sec(d*x+c))^2-11/16*a^2*(a+a*sec(d*x+c))^(1/2)/d/(1-sec(d*x+c
))
```

Rubi [A]

time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3965, 105, 156, 162, 65, 213}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}-\frac{11a^2\sqrt{a\sec(c+dx)+a}}{16d(1-\sec(c+dx))}-\frac{a^2\sqrt{a\sec(c+dx)+a}}{4d(1-\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (43*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*d) - (a^2*Sqrt[
a + a*Sec[c + d*x]])/(4*d*(1 - Sec[c + d*x])^2) - (11*a^2*Sqrt[a + a*Sec[c
+ d*x]])/(16*d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{3a^2x}{2}}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x,\right.}{4d}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} - \frac{a^3\text{Subst}}{}$$

$$= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))} - \frac{(2a^2)\,\text{Su}}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{16\sqrt{2}\,d}$$

Mathematica [A]
time = 1.31, size = 138, normalized size = 0.94

$$\frac{(a(1+\sec(c+dx)))^{5/2}\left(32\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)(-1+\sec(c+dx))^2+\sqrt{1+\sec(c+dx)}\,(-15+11\sec(c+dx))-86\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\sec^2(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)\right)}{16d(-1+\sec(c+dx))^2(1+\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(32*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*(-1 + Sec
[c + d*x])^2 + Sqrt[1 + Sec[c + d*x]]*(-15 + 11*Sec[c + d*x]) - 86*Sqrt[2]*
ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^4))
/(16*d*(-1 + Sec[c + d*x])^2*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(122) = 244$.
time = 0.20, size = 376, normalized size = 2.56

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))^2\left(32\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^2(dx+c))\sqrt{2}+43\sqrt{-\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(32*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*cos(d*x+c)^2*2^(1/2)+43*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-64*cos(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2))*2^(1/2)-86*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+32*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+4
3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))+30*cos(d*x+c)^2-22*cos(d*x+c))/sin(d*x+c)^4*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(118) = 236$.
time = 2.64, size = 503, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(a)*log(-2*a*
cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c) - a) + 43*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(15*a^2
```

```
*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d), 1/32*(43*(sqrt(2)*a^2*cos(d
*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
 + c) + a)) - 64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(
d*x + c) + a)) - 2*(15*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.07, size = 177, normalized size = 1.20

$$\frac{\frac{43\sqrt{2}\,a^3\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{64\,a^3\arctan\left(\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\,\sqrt{-a}}\right)\mathrm{sgn}(\cos(dx+c))}{\sqrt{-a}}-\frac{\sqrt{2}\left(13\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}a^3\mathrm{sgn}(\cos(dx+c))-11\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,a^4\mathrm{sgn}(\cos(dx+c))\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(43*sqrt(2)*a^3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))*s
gn(cos(d*x + c))/sqrt(-a) - 64*a^3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)/sqrt(-a))*sgn(cos(d*x + c))/sqrt(-a) - sqrt(2)*(13*(-a*tan(1
/2*d*x + 1/2*c)^2 + a)^(3/2)*a^3*sgn(cos(d*x + c)) - 11*sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)*a^4*sgn(cos(d*x + c)))/(a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^5\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

3.165 $\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx$

Optimal. Leaf size=290

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+}{5d(a+a\sec(c+}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+62/7*a^6*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+98/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+62/11*a^8
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+18/13*a^9*tan(d*x+c)^13/d/(a+a*sec
(d*x+c))^(13/2)+2/15*a^10*tan(d*x+c)^15/d/(a+a*sec(d*x+c))^(15/2)
```

Rubi [A]
time = 0.09, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^{10}\tan^{15}(c+dx)}{15d(a\sec(c+dx)+a)^{15/2}}+\frac{18a^9\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}}+\frac{62a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{98a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{62a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^5\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^4*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (62*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(98*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (62*a^8*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (18*a^9*Tan[c + d*x]^13)/(13*
d*(a + a*Sec[c + d*x])^(13/2)) + (2*a^10*Tan[c + d*x]^15)/(15*d*(a + a*Sec[
c + d*x])^(15/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = -\frac{(2a^6)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^5}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^6)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+31x^6+49ax^8+31a^2x^{10}+9a^3\right.\right.}{d}$$

$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5}{5d(a+a\sec}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 9.94, size = 173, normalized size = 0.60

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-2882880\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{15}{2}}(c+dx)+604890\sin\left(\frac{1}{2}(c+dx)\right)-87230\sin\left(\frac{3}{2}(c+dx)\right)+450450\sin\left(\frac{5}{2}(c+dx)\right)-137670\sin\left(\frac{7}{2}(c+dx)\right)+210210\sin\left(\frac{9}{2}(c+dx)\right)+75450\sin\left(\frac{11}{2}(c+dx)\right)+90090\sin\left(\frac{13}{2}(c+dx)\right)+16066\sin\left(\frac{15}{2}(c+dx)\right)\right)}{2882880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^7*Sqrt[a*(1 + Sec[c + d*x])]*(-2882880*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(15/2) + 604890*Sin[(c
 + d*x)/2] - 87230*Sin[(3*(c + d*x))/2] + 450450*Sin[(5*(c + d*x))/2] - 137
670*Sin[(7*(c + d*x))/2] + 210210*Sin[(9*(c + d*x))/2] + 75450*Sin[(11*(c +
 d*x))/2] + 90090*Sin[(13*(c + d*x))/2] + 16066*Sin[(15*(c + d*x))/2]))/(28
82880*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(254) = 508$.
time = 0.27, size = 747, normalized size = 2.58

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(45045\sin(dx+c)\left(\cos^7(dx+c)\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{15}{2}}+ \cdots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] -1/5765760/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(45045*sin(d*x+c)*cos(d*x+c)^7*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+315315*sin(d*x+c)*cos(d*x+c)^6*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+945945*sin(d*x+c)*cos(d*x+c)^5*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+1576575*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+1576575*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+945945*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+315315*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)+45045*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(15/2)*sin(d*x+c)+4112896*cos(d*x+c)^8+9475072*cos(d*x+c)^7-9162752*cos(d*x+c)^6-12269056*cos(d*x+c)^5+980480*cos(d*x+c)^4+7605248*cos(d*x+c)^3+1860096*cos(d*x+c)^2-1833216*cos(d*x+c)-768768)/cos(d*x+c)^7/sin(d*x+c)*a^2

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="maxima")

[Out] Timed out

Fricas [A]
time = 3.29, size = 477, normalized size = 1.64

$$\left[\frac{45045\left(a^2\cos(dx+c)^8+a^2\cos(dx+c)^7\right)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(16066a^2\cos(dx+c)^7+53078a^2\cos(dx+c)^6+17286a^2\cos(dx+c)^5-30640a^2\cos(dx+c)^4-26810a^2\cos(dx+c)^3+2898a^2\cos(dx+c)^2+10164a^2\cos(dx+c)+3003a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{45045\left(d\cos(dx+c)^8+d\cos(dx+c)^7\right)}, \frac{2\left(45045\left(a^2\cos(dx+c)^8+a^2\cos(dx+c)^7\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(16066a^2\cos(dx+c)^7+53078a^2\cos(dx+c)^6+17286a^2\cos(dx+c)^5-30640a^2\cos(dx+c)^4-26810a^2\cos(dx+c)^3+2898a^2\cos(dx+c)^2+10164a^2\cos(dx+c)+3003a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)\right)}{45045\left(d\cos(dx+c)^8+d\cos(dx+c)^7\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a^2*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(16066*a^
2*cos(d*x + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30
640*a^2*cos(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2
 + 10164*a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7), 2/45045*(45045*(a^2
*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (16066*a^2*cos(d*x
 + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30640*a^2*c
os(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2 + 10164*
a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [A]
time = 3.13, size = 397, normalized size = 1.37

$$\frac{\frac{45045\sqrt{-a}\,a^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{|a|}-\frac{2\left(45045\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(345345\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(1162161\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(611325\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(77935\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\left(109005\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\left(11633\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-64725\sqrt{2}a^{10}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{7}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/45045*(45045*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(45045*sqrt(2)*a^10*sgn(cos(d*
x + c)) - (345345*sqrt(2)*a^10*sgn(cos(d*x + c)) - (1162161*sqrt(2)*a^10*sg
n(cos(d*x + c)) - (611325*sqrt(2)*a^10*sgn(cos(d*x + c)) - (77935*sqrt(2)*a
^10*sgn(cos(d*x + c)) + (109005*sqrt(2)*a^10*sgn(cos(d*x + c)) + (11633*sqr
t(2)*a^10*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 64725*sqrt(2)*a^10*sgn
(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
```

```
 + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^7*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^6\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

3.166 $\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx$

Optimal. Leaf size=224

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^5\tan^5(c+d}{d(a+a\sec(c+d}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^3*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+
6*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+34/7*a^6*tan(d*x+c)^7/d/(a+a*se
c(d*x+c))^(7/2)+14/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^8*tan
(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

Rubi [A]

time = 0.08, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}}+\frac{14a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{34a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^4*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (6*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c + d*
x])^(5/2)) + (34*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (14
*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^8*Tan[c + d*x]
^11)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = -\frac{(2a^5)\,\mathrm{Subst}\left(\int \frac{x^4(2+ax^2)^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^5)\,\mathrm{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+15x^4+17ax^6+7a^2x^8+a^3x^{10}+\frac{}{a^2}\right.\right.}{d}$$

$$= -\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^5\tan^5}{d(a+a\sec}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{}{3}$$

Mathematica [A]
time = 7.40, size = 149, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(1+\sec(c+dx))}\left(5544\sqrt{2}\,\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{11}{2}}(c+dx)-1386\sin\left(\frac{1}{2}(c+dx)\right)+1584\sin\left(\frac{3}{2}(c+dx)\right)-1386\sin\left(\frac{5}{2}(c+dx)\right)-143\sin\left(\frac{7}{2}(c+dx)\right)-693\sin\left(\frac{9}{2}(c+dx)\right)-26\sin\left(\frac{11}{2}(c+dx)\right)\right)}{5544d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(5544*Sqrt[
2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) - 1386*Sin[(c + d*x
)/2] + 1584*Sin[(3*(c + d*x))/2] - 1386*Sin[(5*(c + d*x))/2] - 143*Sin[(7*(
c + d*x))/2] - 693*Sin[(9*(c + d*x))/2] - 26*Sin[(11*(c + d*x))/2]))/(5544*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(198) = 396$.
time = 0.18, size = 498, normalized size = 2.22

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(693\sin(dx+c)\left(\cos^5(dx+c)\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11088/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(693*sin(d*x+c)*cos(d*x+c)^5
*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+2772*sin(d*x+c)*cos(d*x+
c)^4*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+4158*sin(d*x+c)*cos(
d*x+c)^3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+2772*sin(d*x+c)*
cos(d*x+c)^2*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+693*sin(d*x+
c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-1664*cos(d*
x+c)^6-21344*cos(d*x+c)^5+11296*cos(d*x+c)^4+16736*cos(d*x+c)^3+2144*cos(d*
x+c)^2-5152*cos(d*x+c)-2016)/cos(d*x+c)^5/sin(d*x+c)*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.88, size = 425, normalized size = 1.90

$$\left[\frac{693\left(a^2\cos(dx+c)^6+a^2\cos(dx+c)^5\right)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(52a^2\cos(dx+c)^5+719a^2\cos(dx+c)^4+366a^2\cos(dx+c)^3-157a^2\cos(dx+c)^2-224a^2\cos(dx+c)-63a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{693\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)},\ -\frac{2\left(693\left(a^2\cos(dx+c)^6+a^2\cos(dx+c)^5\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(52a^2\cos(dx+c)^5+719a^2\cos(dx+c)^4+366a^2\cos(dx+c)^3-157a^2\cos(dx+c)^2-224a^2\cos(dx+c)-63a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)\right)}{693\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/693*(693*(a^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(52*a^2*cos(d
```

```
*x + c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d
*x + c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), -2/693*(693*(a
^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (52*a^2*cos(d*x
+ c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d*x
+ c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]

time = 2.32, size = 339, normalized size = 1.51

$$-\frac{\frac{693\,\sqrt{-a}\,a^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\left|a\right|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\left|a\right|-6\,a\right|}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{\left|a\right|}-\frac{2\left(693\,\sqrt{2}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(3927\,\sqrt{2}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(462\,\sqrt{2}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\left(1782\,\sqrt{2}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\left(305\,\sqrt{2}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1331\,\sqrt{2}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{5}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/693*(693*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(693*sqrt(2)*a^8*sgn(cos(d*x + c)
) - (3927*sqrt(2)*a^8*sgn(cos(d*x + c)) - (462*sqrt(2)*a^8*sgn(cos(d*x + c)
) + (1782*sqrt(2)*a^8*sgn(cos(d*x + c)) + (305*sqrt(2)*a^8*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)^2 - 1331*sqrt(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.167 $\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx$

Optimal. Leaf size=160

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+14/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2
)+2*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^6*tan(d*x+c)^7/d/(a+a*s
ec(d*x+c))^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}}+\frac{14a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (14*a^4*Tan[c + d*x]^3)/(
3*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c +
d*x])^(5/2)) + (2*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = -\frac{(2a^4)\,\mathrm{Subst}\left(\int \frac{x^2(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^4)\,\mathrm{Subst}\left(\int \left(\frac{1}{a}+7x^2+5ax^4+a^2x^6-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{}{\sqrt{a}}\right.}{d}$$

$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c}{d(a+a\sec(c}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A]
time = 5.80, size = 125, normalized size = 0.78

$$-\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(1+\sec(c+dx))}\left(42\sqrt{2}\,\mathrm{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{7}{2}}(c+dx)-35\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] -1/42*(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(42*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) - 35*Sin[(c + d*
x)/2] + 7*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*
x))/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(142) = 284.
time = 0.15, size = 391, normalized size = 2.44

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(21\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^3(dx+c))\sin(dx+c)\sqrt{2}+63(\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/168/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(21*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+63*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+63*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+21*2^(1/2)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-160*cos(d*x+c)^4+416*cos(d*x+c)^3-64
*cos(d*x+c)^2-144*cos(d*x+c)-48)/cos(d*x+c)^3/sin(d*x+c)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/42*(21*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a^2*cos
(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 2*(a^2*d*cos(2*d*x + 2*c)^
2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(
(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*
c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*c
```

```
os(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c
)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)
*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2
*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*
sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2
)*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x +
2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*
x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2
*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2
+ cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x +
2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*
d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^
2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*
x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 +
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*
(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*
d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - 16*(a^2*d*cos(2*d*
x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*i
ntegrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*
x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*
```

```
x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2
*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*
d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*
d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c...
```

Fricas [A]
time = 2.57, size = 374, normalized size = 2.34

$$\left[\frac{21\left(a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(10\,a^2\cos\left(dx+c\right)^3-16\,a^2\cos\left(dx+c\right)^2-12\,a^2\cos\left(dx+c\right)-3\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)},\ \frac{2\left(21\left(a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(10\,a^2\cos\left(dx+c\right)^3-16\,a^2\cos\left(dx+c\right)^2-12\,a^2\cos\left(dx+c\right)-3\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{21\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(10*a^2*cos(d*x
 + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3), 2/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
 (10*a^2*cos(d*x + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a
^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\tan^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**2, x)
```

Giac [A]
time = 1.91, size = 281, normalized size = 1.76

$$\frac{\frac{21\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(21\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))+\left(35\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))+\left(17\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-49\sqrt{2}\,a^6\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/21*(21*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a
) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) +
(35*sqrt(2)*a^6*sgn(cos(d*x + c)) + (17*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1
/2*d*x + 1/2*c)^2 - 49*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^
2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.168 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-4*a^2*cot(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 464, 209}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (4
*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

tegerQ[n - 1/2]

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{2+ax^2}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,\right.}{}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{4a^2\cot(c+dx)\sqrt{a+a\,\text{s}}}{d}$$

Mathematica [A]
time = 0.78, size = 124, normalized size = 1.88

$$-\frac{\sqrt{2}\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c+dx)-\frac{\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)(-1+\cos(c+dx))}{\sqrt{1-\sec(c+dx)}}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3/2}(a(1+\sec(c+dx)))^{5/2}}{d\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*(2*Cos[c + d*x] - (ArcTanh[Sqrt[
1 - Sec[c + d*x]]]*(-1 + Cos[c + d*x]))/Sqrt[1 - Sec[c + d*x]])*((1 + Sec[c
 + d*x])^(-1))^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2))/(d*Sqrt[1 - Tan[(c + d*x
)/2]^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(58) = 116$.
time = 0.13, size = 192, normalized size = 2.91

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left((\cos^2(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}-\sqrt{2}\sqrt{-\frac{2\cos}{1+\cos}}\right.}{d(\cos^2(dx+c)-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*2^(1/2)-2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))+4*cos(d*x+c)*sin(d*x+c))/(cos(d*x+c)^2-1)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(58) = 116$.
time = 3.52, size = 270, normalized size = 4.09

$$\left[\frac{\sqrt{-a}\, a^{2} \log\left(-\frac{8 \, a \cos\left(dx + c\right)^{3} + 4 \, {\left(2 \, \cos\left(dx + c\right)^{2} - \cos\left(dx + c\right)\right)} \sqrt{-a} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sin\left(dx + c\right) - 7 \, a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right) + 1}\right) \sin\left(dx + c\right) - 8 \, a^{2} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \cos\left(dx + c\right)}{2 \, d \sin\left(dx + c\right)}, -\frac{a^{\frac{5}{2}} \arctan\left(\frac{2 \, \sqrt{a} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \cos\left(dx + c\right) \sin\left(dx + c\right)}{2 \, a \cos\left(dx + c\right)^{2} + a \cos\left(dx + c\right) - a}\right) \sin\left(dx + c\right) + 4 \, a^{2} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \cos\left(dx + c\right)}{d \sin\left(dx + c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x
 + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 8*a^2*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(5/2)*arctan(2*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(
2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 4*a^2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(58) = 116$.
time = 1.38, size = 192, normalized size = 2.91

$$\frac{\sqrt{2}\sqrt{-a}\,a^{4}\left(\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\sqrt{2}\,|a|-6\,a\right|}\right)}{a|a|}+\frac{8}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\right)a}\right)\operatorname{sgn}(\cos(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a)*a^4*(sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/(a*abs(a)) + 8/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*a))*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)^2\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.169 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=96

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}-\frac{2a\cot^3(c+dx)(a+a\sec(c+}{3d}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2/3*a*cot(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)/d+2*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 331, 209}

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-\frac{2a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d - (2*a*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\right.}{}$$

$$= \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a\cot^3(c+dx)(a+a\sec(c}{3d}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\cot(c+dx)\sqrt{a+a\sec}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.23, size = 81, normalized size = 0.84

$$-\frac{2\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sec(c+dx)))^{5/2}}{3d\sqrt{\frac{1}{1+\sec(c+dx)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*((1 + Cos[c + d*x])^(-1))^(3/2)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2,
-3/2, -1/2, 2*Sin[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))/(3*d*Sqrt[(
1 + Sec[c + d*x])^(-1)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(84) = 168.
time = 0.18, size = 214, normalized size = 2.23

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-3\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\ \cos(dx+c)+3\sqrt{\ldots}\right.}{3d\sin(dx+c)(-1+\cos\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*sin(d*x+c)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+3*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*sin(d*x+c)+8*cos(d*x+c)^2-6*cos(d*x+c))/sin(d*x+c)/(-1+
cos(d*x+c))*a^2
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^4, x)
```

<u>Fricas</u> [A]
time = 4.17, size = 355, normalized size = 3.70

$$\left[\frac{3\left(a^{2}\cos(dx+c)-a^{2}\right)\sqrt{-a}\,\log\left(-\frac{8a\cos(dx+c)^{3}-4\left(2\cos(dx+c)^{2}-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)+4\left(4a^{2}\cos(dx+c)^{2}-3a^{2}\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{6\left(d\cos(dx+c)-d\right)\sin(dx+c)},\ \frac{3\left(a^{2}\cos(dx+c)-a^{2}\right)\sqrt{a}\,\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^{2}+a\cos(dx+c)-a}\right)\sin(dx+c)+2\left(4a^{2}\cos(dx+c)^{2}-3a^{2}\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{3\left(d\cos(dx+c)-d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*c
os(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) +
 4*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/3*(3*(a^2*cos(d*x + c)
- a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x +
 c) + 2*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(84) = 168.
time = 2.38, size = 311, normalized size = 3.24

$$-\frac{\frac{3\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}+\frac{\sqrt{2}\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}\,a^3\mathrm{sgn}(\cos(dx+c))-12\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^4\mathrm{sgn}(\cos(dx+c))+7\sqrt{-a}\,a^5\mathrm{sgn}(\cos(dx+c))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(d*x + c))/abs(a) + sqrt(2)*(9*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^3*sgn(cos(d*x + c))
- 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 7*sqrt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/
d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.170 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=176

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\,d}-\frac{7a^2\cot(c+dx)\sqrt{a+a}}{4d}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/2*a*cot(d*
x+c)^3*(a+a*sec(d*x+c))^(3/2)/d-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/d+1
/8*a^(5/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*
2^(1/2)-7/4*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 491, 597, 536, 209}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\,d}-\frac{7a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{5d}+\frac{a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
4*Sqrt[2]*d) - (7*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a*Cot
[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(2*d) - (Cot[c + d*x]^5*(a + a*Sec[
c + d*x])^(5/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} - \frac{\text{Subst}\left(\int \frac{-15a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\right.}{}$$

$$= \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c}{5d}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec}{2d}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec}{2d}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a+}}\right.}{4\sqrt{2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.09, size = 5562, normalized size = 31.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(147) = 294$.
time = 0.25, size = 542, normalized size = 3.08

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))^2\left(-40\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^2(dx+c))\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-40*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-5*sin(d*x+
c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*sin(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+10*sin(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-40*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+98*cos(d*x+c)^3-5*ln(-(-(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-160*cos(d*x+c)^2+70*cos(d*x+c))/sin(d*x+
c)^5*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 3.64, size = 650, normalized size = 3.69

$$\left[\frac{5\left(\sqrt{2}\,a^2\cos\left(dx+c\right)^2-2\sqrt{2}\,a^2\cos\left(dx+c\right)+\sqrt{2}\,a^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin\left(dx+c\right)+40\left(a^2\cos\left(dx+c\right)^2-2a^2\cos\left(dx+c\right)+a^2\right)\sqrt{-a}\log\left(-\frac{8a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin\left(dx+c\right)-4\left(49a^2\cos\left(dx+c\right)^3-80a^2\cos\left(dx+c\right)^2+35a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{80\left(d\cos\left(dx+c\right)^2-2d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)},-\frac{40\left(a^2\cos\left(dx+c\right)^2-2a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin\left(dx+c\right)+5\left(\sqrt{2}\,a^2\cos\left(dx+c\right)^2-2\sqrt{2}\,a^2\cos\left(dx+c\right)+\sqrt{2}\,a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin\left(dx+c\right)+2\left(49a^2\cos\left(dx+c\right)^3-80a^2\cos\left(dx+c\right)^2+35a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{40\left(d\cos\left(dx+c\right)^2-2d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)
*a^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 40*(a^2*cos(d*x + c)
^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos
(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4
```

```
*(49*a^2*cos(d*x + c)^3 - 80*a^2*cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c)
+ d)*sin(d*x + c)), -1/40*(40*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^
2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c)
+ 5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)
*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(49*a^2*cos(d*x + c)^3 - 80*a^2*
cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(147) = 294.
time = 3.91, size = 481, normalized size = 2.73

$$\frac{5\sqrt{2}\sqrt{-a}\,a^{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)\operatorname{sgn}(\cos(dx+c))+\frac{80\sqrt{-a}\,a^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{|a|}+\frac{4\sqrt{2}\left(55\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{8}\sqrt{-a}\,a^{3}\operatorname{sgn}(\cos(dx+c))-170\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{6}\sqrt{-a}\,a^{4}\operatorname{sgn}(\cos(dx+c))+240\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}\sqrt{-a}\,a^{5}\operatorname{sgn}(\cos(dx+c))-150\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{6}\operatorname{sgn}(\cos(dx+c))+41\sqrt{-a}\,a^{7}\operatorname{sgn}(\cos(dx+c))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\right)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/80*(5*sqrt(2)*sqrt(-a)*a^2*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2)*sgn(cos(d*x + c)) + 80*sqrt(-a)*a^3*log(abs(
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/ab
s(a) + 4*sqrt(2)*(55*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 170*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^4*sgn(cos(d*
x + c)) + 240*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 150*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^6*sgn(cos(d*x + c))
 + 41*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^6\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

3.171 $\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))}{5a^3d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d-6/5*(a+a*sec(d*x+c))^(5/2)/a^3/d+2/7*(a+a*
sec(d*x+c))^(7/2)/a^4/d-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
+2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 90, 52, 65, 213}

$$\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^4d}-\frac{6(a\sec(c+dx)+a)^{5/2}}{5a^3d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}+\frac{2\sqrt{a\sec(c+dx)+a}}{ad}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d) - (6*(a + a
*Sec[c + d*x])^(5/2))/(5*a^3*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{3/2}+\frac{a^2(a+ax)^{3/2}}{x}+a(a+ax)^{5/2}\right)\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, s\right.}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}+$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2}{}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+}{3a^2d}$$

Mathematica [A]

time = 0.20, size = 88, normalized size = 0.70

$$\frac{2\left(92+46\sec(c+dx)-64\sec^2(c+dx)-3\sec^3(c+dx)+15\sec^4(c+dx)-105\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{105d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(92 + 46*Sec[c + d*x] - 64*Sec[c + d*x]^2 - 3*Sec[c + d*x]^3 + 15*Sec[c
+ d*x]^4 - 105*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(10
5*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(106) = 212.
time = 0.17, size = 293, normalized size = 2.33

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\left(\cos^3(dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+315\left(\cos^2(dx+c)\right)\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*cos(d*x+c)^3*2^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)+315*cos(d*x+c)^2*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+315*cos(d*x+c)*2
^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(7/2)+105*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-1472*cos(d*x+c)^3+
736*cos(d*x+c)^2+288*cos(d*x+c)-240)/cos(d*x+c)^3/a
```

Maxima [**A**]
time = 0.50, size = 129, normalized size = 1.02

$$\frac{\frac{105\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^4}-\frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^3}+\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^2}+\frac{210\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c
)) + sqrt(a)))/sqrt(a) + 30*(a + a/cos(d*x + c))^(7/2)/a^4 - 126*(a + a/cos
(d*x + c))^(5/2)/a^3 + 70*(a + a/cos(d*x + c))^(3/2)/a^2 + 210*sqrt(a + a/c
os(d*x + c))/a)/d
```

Fricas [A]
time = 3.26, size = 285, normalized size = 2.26

$$\left[\frac{105\,\sqrt{a}\cos\left(dx+c\right)^{3}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\,\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\,\left(92\,\cos\left(dx+c\right)^{3}-46\,\cos\left(dx+c\right)^{2}-18\,\cos\left(dx+c\right)+15\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{210\,ad\cos\left(dx+c\right)^{3}},\ \frac{105\,\sqrt{-a}\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+a}\right)\cos\left(dx+c\right)^{3}+2\,\left(92\,\cos\left(dx+c\right)^{3}-46\,\cos\left(dx+c\right)^{2}-18\,\cos\left(dx+c\right)+15\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{105\,ad\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*(92*cos(d*x + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d*x
+ c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3), 1
/105*(105*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(92*cos(d*x + c)^
3 - 46*cos(d*x + c)^2 - 18*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(a*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 2.11, size = 167, normalized size = 1.33

$$\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\,\left(105\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}-70\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}a-252\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a^{2}-120\,a^{3}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)}{105\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/sqrt(-a) + 2*(105*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3 - 70*(a
*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 252*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2 -
 120*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^5}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

3.172 $\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/
d/a^(1/2)-2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 81, 52, 65, 213}

$$\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d}-\frac{2\sqrt{a\sec(c+dx)+a}}{ad}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*Sqrt[a + a*S
ec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx,\right.}{d}$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x,\right.}{ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+}{3a^2 d}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.85

$$\frac{2\left(-2-\sec(c+dx)+\sec^2(c+dx)+3\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{3d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(-2 - Sec[c + d*x] + Sec[c + d*x]^2 + 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*
Sqrt[1 + Sec[c + d*x]]))/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(66) = 132$.
time = 0.16, size = 155, normalized size = 1.99

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx}{1+\cos(dx}}}{2}\right.\right.}{6d\cos(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*2^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*cos(d*x+c)+4)/cos(d*x+c)/a
```

Maxima [A]
time = 0.49, size = 91, normalized size = 1.17

$$-\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^2}+\frac{6\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/sqrt(a) - 2*(a + a/cos(d*x + c))^(3/2)/a^2 + 6*sqrt(a + a/cos(d
*x + c))/a)/d
```

Fricas [A]
time = 2.87, size = 241, normalized size = 3.09

$$\left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)-1)}{6ad\cos(dx+c)}, -\frac{3\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)\cos(dx+c)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)-1)}{3ad\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) -
1))/(a*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c) +
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) - 1))/(a*d*cos(d
*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.19, size = 126, normalized size = 1.62

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+2a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)}{3ad\,\mathrm{sgn}(\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)/sqrt(-a))/sqrt(-a) + 2*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 2*a^2)/(
(a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a*d*s
gn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

3.173 $\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3965, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.05, size = 44, normalized size = 1.42

$$-\frac{2\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}}{d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [**A**]
time = 0.06, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Maxima [A]
time = 0.48, size = 49, normalized size = 1.58

$$\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(a
)))/(sqrt(a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.
time = 3.08, size = 137, normalized size = 4.42

$$\left[\frac{\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)}{2\sqrt{a}\,d},\ \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)/(sqrt(a)*d)
, sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(2*a*cos(d*x + c) + a))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 0.88, size = 48, normalized size = 1.55

$$\frac{2\,\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a
)*d*sgn(cos(d*x + c)))
```

Mupad [B]
time = 1.41, size = 27, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.174 $\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{1}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2*arctanh(1/2*(a+a*se
c(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-1/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A]

time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3965, 87, 162, 65, 213}

$$-\frac{1}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1/(d*Sqrt[a + a*
Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{2\text{Su}}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{}{d\sqrt{a+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 57, normalized size = 0.62

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1+\sec(c+dx))\right) - 2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\sec(c+dx)\right)}{d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2F
1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(75) = 150$.
time = 0.16, size = 259, normalized size = 2.82

method	result
default	$\frac{\left(2\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^2(dx+c))\sqrt{2}+\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}} \right.\right.\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*2^(1/2)+2*cos(d*x+c)^2-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*cos(d*x+c))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(75) = 150$.
time = 2.55, size = 384, normalized size = 4.17

$$\left[\frac{2\sqrt{a}\left(\cos(dx+c)+1\right)\log\left(-8a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)+\frac{\sqrt{2}\left(a\cos(dx+c)+a\right)\log\left(-\frac{\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}}-3\cos(dx+c)-1}{\cos(dx+c)-1}\right)}{\sqrt{a}}-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{4\left(ad\cos(dx+c)+ad\right)}, \frac{\sqrt{2}\left(a\cos(dx+c)+a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)}{\cos(dx+c)+1}\right)-2\sqrt{-a}\left(\cos(dx+c)+1\right)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + sqrt(2)*(a*cos(d*x + c) + a)*log(-(2*sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/sqrt(a) - 3*cos(d*x + c) - 1)/(
cos(d*x + c) - 1))/sqrt(a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c))/(a*d*cos(d*x + c) + a*d), 1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(
-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos
(d*x + c)/(cos(d*x + c) + 1)) - 2*sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
 + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a*d*cos(d
*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 0.87, size = 111, normalized size = 1.21

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a}\right)}{2\,d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)/sqrt(-a))/sqrt(-a) - arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/sqrt(-a) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/a)/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

3.175 $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{8\sqrt{2}\,\sqrt{a}\,d}-\frac{a}{12d(a+a\sec(c+dx))^{3/2}}+\frac{}{2d(1-}$$

```
[Out] -1/12*a/d/(a+a*sec(d*x+c))^(3/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3
/2)-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+9/16*arctanh(1/2*(a
+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+7/8/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3965, 105, 157, 162, 65, 213}

$$-\frac{a}{12d(a\sec(c+dx)+a)^{3/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}+\frac{7}{8d\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{8\sqrt{2}\,\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (9*ArcTanh[Sqr
t[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*d) - a/(12*d*(
a + a*Sec[c + d*x])^(3/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) + 7/(8*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{5a^2x}{2}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \text{s}\right.}{2d}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\text{Sub}}{}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{}{8d\sqrt{}}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{}{8d\sqrt{}}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{}{8d\sqrt{}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d} - \frac{}{12d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.21, size = 90, normalized size = 0.59

$$\frac{a\left(-6-9\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))\right)}{12d(-1+\sec(c+dx))(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(-6 - 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec
[c + d*x]) + 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]]*(-1 + Sec
[c + d*x])))/(12*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(123) = 246$.
time = 0.23, size = 504, normalized size = 3.32

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))(-1+\cos(dx+c))^2\left(48(\cos^3(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))^
2*(48*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+27*cos(d*x+c)^3*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+48*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+27*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-48*cos(d*x+
c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*2^(1/2))*2^(1/2)+62*cos(d*x+c)^3-27*cos(d*x+c)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-48*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*2^(1/2))*2^(1/2)+4*cos(d*x+c)^2-27*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-42*cos(d*x+c))/sin(d*x+c
)^6/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.
time = 3.41, size = 546, normalized size = 3.59

$$\left[\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3a\cos(dx+c)+a}{\cos(dx+c)-1}\right)+48\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)+4\left(31\cos(dx+c)^3+2\cos(dx+c)^2-21\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{96\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)}, -\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-48\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)-2\left(31\cos(dx+c)^3+2\cos(dx+c)^2-21\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{48\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

[Out] [1/96*(27*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 48*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d), -1/48*(27*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 48*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral(cot(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)

Giac [A]
time = 0.96, size = 174, normalized size = 1.14

$$\frac{\sqrt{2}\left(\frac{48\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{27\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{2\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^4+12\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^5\right)}{a^6}\right)}{48\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/48*sqrt(2)*(48*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/sqrt(-a) - 27*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/sqrt(-a) - 3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*tan(1/2*d*x + 1/2*c)^2) + 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^4 + 12*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^5)/a^6)/(d*sgn(cos(d*x + c)))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

3.176 $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{151\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}+\frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}}-\frac{}{4d(1-}$$

```
[Out] 87/160*a^2/d/(a+a*sec(d*x+c))^(5/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x
+c))^(5/2)-17/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+23/192*a/d/(a+
a*sec(d*x+c))^(3/2)+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-151
/256*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-
105/128/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\frac{87a^2}{160d(a\sec(c+dx)+a)^{5/2}}-\frac{17a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}}+\frac{23a}{192d(a\sec(c+dx)+a)^{3/2}}-\frac{105}{128d\sqrt{a\sec(c+dx)+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{151\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (151*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[a]*d) + (87*a^
2)/(160*d*(a + a*Sec[c + d*x])^(5/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
a*Sec[c + d*x])^(5/2)) - (17*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d
*x])^(5/2)) + (23*a)/(192*d*(a + a*Sec[c + d*x])^(3/2)) - 105/(128*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{9a^2x}{2}}{x(-a+ax)^2(a+ax)^{7/2}}\,dx\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{}{16d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{151\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d} + \frac{}{16}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.29, size = 102, normalized size = 0.48

$$\frac{\cot^4(c+dx)\left(-2\left(105+32\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))^2-85\sec(c+dx)\right)+151\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2\right)}{160d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(105 + 32*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c +
d*x]]*(-1 + Sec[c + d*x])^2 - 85*Sec[c + d*x]) + 151*Hypergeometric2F1[-5/2
```

```
, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2))/(160*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(177) = 354$.
time = 0.32, size = 746, normalized size = 3.49

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^3\left(3840\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)(\cos^5(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c
))^3*(3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^5*2^(1/2)+2265*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)
^5+3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^4*2^(1/2)+2265*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^4-
7680*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-4530*cos(d*x+c)^3*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-768
0*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+5642*cos(d*x+c)^5-4530*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*c
os(d*x+c)^2+3840*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+556*cos(d*x+c)^4+226
5*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2))+3840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-7928*cos(d*x+c)^3+226
5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))-460*cos(d*x+c)^2+3150*cos(d*x+c))/sin(d*x+c)^10/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] integrate(cot(d*x + c)^5/sqrt(a*sec(d*x + c) + a), x)

Fricas [A]
time = 2.80, size = 705, normalized size = 3.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 -
 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(co
s(d*x + c) - 1)) + 3840*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3
 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4
*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)) - 8*a*cos(d*x + c) - a) - 4*(2821*cos(d*x + c)^5 + 278*cos(d*x + c)
^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d), 1/3
840*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 3840*(
cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos
(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(2821*cos(d*x + c)^5 + 278*
cos(d*x + c)^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(
d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)
 + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.00, size = 241, normalized size = 1.13

$$-\frac{\sqrt{2}\left(\frac{3840\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2265\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\left(25\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}-23\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{8\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{12}+25\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^{13}+240\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{14}\right)}{a^{15}}\right)}{3840\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3840*sqrt(2)*(3840*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) - 2265*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)/sqrt(-a))/sqrt(-a) + 15*(25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 23*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^2*tan(1/2*d*x + 1/2*c)^4) + 8*(3*
(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^12 +
 25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^13 + 240*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*a^14)/a^15)/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^5}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

3.177 $\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx)}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)-2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a
^2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+6/7*a^3*tan(d*x+c)^7/d/(a+a*sec(d*
x+c))^(7/2)+2/9*a^4*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

Rubi [A]
time = 0.07, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 472, 209}

$$\frac{2a^4\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6a^3\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2)) + (6*a^3*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (2*a^
4*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+}{\sqrt{a+a\sec}}\right.}{d}$$

$$= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c}{3d(a+a\sec(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 18.99, size = 467, normalized size = 2.47

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{1532}{315}\sin\left(\frac{1}{2}(c+dx)\right)+\frac{136}{315}\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)-\frac{176}{105}\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)-\frac{4}{63}\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+\frac{4}{9}\sec^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sec(c+dx))}} + \frac{16\left(-3-2\sqrt{2}\right)\cos^4\left(\frac{1}{4}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{7-5\sqrt{2}+\left(10-7\sqrt{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)}{1+\cos\left(\frac{1}{2}(c+dx)\right)}}\sqrt{\frac{-1+\sqrt{2}-\left(-2+\sqrt{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)}{1+\cos\left(\frac{1}{2}(c+dx)\right)}}\left(1-\sqrt{2}+\left(-2+\sqrt{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(F\left(\text{ArcSin}\left(\frac{\tan\left(\frac{1}{4}(c+dx)\right)}{\sqrt{3-2\sqrt{2}}}\right)|17-12\sqrt{2}\right)-2\Pi\left(-3+2\sqrt{2};\text{ArcSin}\left(\frac{\tan\left(\frac{1}{4}(c+dx)\right)}{\sqrt{3-2\sqrt{2}}}\right)|17-12\sqrt{2}\right)\right)\sqrt{\left(-1-\sqrt{2}+\left(2+\sqrt{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\right)\sec^2\left(\frac{1}{4}(c+dx)\right)}\sec^2(c+dx)\sqrt{3-2\sqrt{2}-\tan^2\left(\frac{1}{4}(c+dx)\right)}}{d\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*((1532*Sin[(c + d*x)/2])/315 + (136*Sec[c +
d*x]*Sin[(c + d*x)/2])/315 - (176*Sec[c + d*x]^2*Sin[(c + d*x)/2])/105 - (4
*Sec[c + d*x]^3*Sin[(c + d*x)/2])/63 + (4*Sec[c + d*x]^4*Sin[(c + d*x)/2])/
9))/(d*Sqrt[a*(1 + Sec[c + d*x])]) + (16*(-3 - 2*Sqrt[2])*Cos[(c + d*x)/4]^
4*Cos[(c + d*x)/2]*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(c + d*x)/2])
/(1 + Cos[(c + d*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(c + d*x)/
2])/(1 + Cos[(c + d*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(c + d*x)/2])
*(EllipticF[ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]
- 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(c + d*x)/2])*Se
```

c[(c + d*x)/4]^2]*Sec[c + d*x]^2*Sqrt[3 - 2*Sqrt[2] - Tan[(c + d*x)/4]^2])/(d*Sqrt[a*(1 + Sec[c + d*x])])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(165) = 330$.
time = 0.19, size = 480, normalized size = 2.54

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(315\sin(dx+c)\left(\cos^4(dx+c)\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{9}{2}}+1260\,\mathrm{s}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/5040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(315*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+1260*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+1890*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+1260*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+315*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*sin(d*x+c)-12256*cos(d*x+c)^5+11168*cos(d*x+c)^4+5312*cos(d*x+c)^3-4064*cos(d*x+c)^2-1280*cos(d*x+c)+1120)/sin(d*x+c)/cos(d*x+c)^4/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/630*(12*(105*sin(8*d*x + 8*c) + 280*sin(6*d*x + 6*c) + 546*sin(4*d*x + 4*c) + 312*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(315*cos(8*d*x + 8*c) + 840*cos(6*d*x + 6*c) + 1638*cos(4*d*x + 4*c) + 936*cos(2*d*x + 2*c) + 383)*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 315*((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d

```
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4
 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) + 2*(a*d*cos(2*d*x + 2*c)^4 + a*d*
sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 +
4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c)
 + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(14*d*x + 14*c)*cos(2*d*x
+ 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(
2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*c
os(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
+ sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*
c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x
 + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(
12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*
c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x +
2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x
+ 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(
8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*co
s(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((c
os(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c)
+ 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x +
8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x
+ 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d
*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*si
n(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*
sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (
cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c)
 + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x +
 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(
12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
 + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(11/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))/(a*cos(14*d*x + 14*c)^2 + 36*a*cos(12*d*x + 12*c)^2
```

+ 225*a*cos(10*d*x + 10*c)^2 + 400*a*cos(8*d*x + 8*c)^2 + 225*a*cos(6*d*x
+ 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 12*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ a*cos(2*d*x + 2*c)^2 + a*sin(14*d*x + 14*c)^2 + 36*a*sin(12*d*x + 12*c)^
2 + 225*a*sin(10*d*x + 10*c)^2 + 400*a*sin(8*d*x + 8*c)^2 + 225*a*sin(6*d*x
+ 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 12*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + a*sin(2*d*x + 2*c)^2 + 2*(6*a*cos(12*d*x + 12*c) + 15*a*cos(10*d*x + 10
*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c)
+ a*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a*cos(10*d*x + 10*c) + 20
*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(
2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d
*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) +
40*(15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos
(8*d*x + 8*c) + 30*(6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 2*(6*a*sin(12*d*x + 12*c) + 15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*x
+ 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x...

Fricas [A]
time = 2.32, size = 355, normalized size = 1.88

$$\left[-\frac{315\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}\right)\sqrt{-a}\log\left(\frac{2a\cos\left(dx+c\right)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)-2\left(383\cos\left(dx+c\right)^{4}+34\cos\left(dx+c\right)^{3}-132\cos\left(dx+c\right)^{2}-5\cos\left(dx+c\right)+35\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{315\left(ad\cos\left(dx+c\right)^{5}+ad\cos\left(dx+c\right)^{4}\right)},\ \frac{2\left(315\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+\left(383\cos\left(dx+c\right)^{4}+34\cos\left(dx+c\right)^{3}-132\cos\left(dx+c\right)^{2}-5\cos\left(dx+c\right)+35\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{315\left(ad\cos\left(dx+c\right)^{5}+ad\cos\left(dx+c\right)^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(383*cos(d*x + c)^4
+ 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x + c) + 35)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^5 + a*d*cos(d*
x + c)^4), 2/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(383*cos(d*x + c)^4 + 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x +
c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x
+ c)^5 + a*d*cos(d*x + c)^4)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{6}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral(tan(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)

Giac [A]
time = 2.81, size = 266, normalized size = 1.41

$$\frac{\sqrt{2}\left(\frac{315\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{4\left(315a^4-\left(1470a^4-\left(2772a^4+\left(257a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1314a^4\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)}{630\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/630*sqrt(2)*(315*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/abs(a) + 4*(315*a^4 - (1470*a^4 - (2772*a^4 + (257*
a^4*tan(1/2*d*x + 1/2*c)^2 - 1314*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

3.178 $\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*tan(d*x+c)/
d/(a+a*sec(d*x+c))^(1/2)+2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a^
2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3972, 470, 308, 209}

$$\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x]
)^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\ldots}{\sqrt{\ldots}}\right)}{d}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/\ldots}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+\ldots}{3d(a+a\sec(c+\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.88, size = 238, normalized size = 1.90

$$\frac{16\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{9/2}\left(-\frac{\cos(c+dx)(9+5\cos(c+dx))\csc^6\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(30\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\cos^2(c+dx)+(-29+22\cos(c+dx)-23\cos(2(c+dx)))\sqrt{1-\sec(c+dx)}\right)}{480\sqrt{1-\sec(c+dx)}}-\frac{4}{9}\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-2\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^5(c+dx)}{5d\sqrt{a(1+\sec(c+dx))}\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(9/2)*(-1/480*(Cos[c + d*x]*(9 + 5*Co
s[c + d*x])*Csc[(c + d*x)/2]^6*Sec[(c + d*x)/2]^2*(30*ArcTanh[Sqrt[1 - Sec[
c + d*x]]]*Cos[c + d*x]^2 + (-29 + 22*Cos[c + d*x] - 23*Cos[2*(c + d*x)])*S
qrt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2, 9/
2, 11/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^
2)/9)*Tan[c + d*x]^5)/(5*d*Sqrt[a*(1 + Sec[c + d*x])]*(1 - Tan[(c + d*x)/2]
^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(109) = 218$.
time = 0.17, size = 231, normalized size = 1.85

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\sin(dx+c)\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2}+15\sin(d\right.}{30d\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*sin(d*x+c)*cos(d*x+c)^2*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+15*sin(d*x+c)*cos(d*x+c)*2^(1
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+68*cos(d*x+c)^3-64*cos(d*x+c)
^2-16*cos(d*x+c)+12)/sin(d*x+c)/cos(d*x+c)^2/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(20*(3*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(15*cos(4*d*x + 4*c) + 20*cos(2*d*x +
2*c) + 17)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 15*((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
```

```
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - 1) + 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*
d*x + 2*c) + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8
*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(
4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin
(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*
cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*si
n(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)
*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c
)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*
c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x +
 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x
+ 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x
 + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x
+ 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(
8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))/(a*cos(10*d*x + 10*c)^2 + 16*a*cos(8*d*x + 8*c)^2 + 36*a*cos
(6*d*x + 6*c)^2 + 16*a*cos(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 16*a*sin(8*d*x + 8
*c)^2 + 36*a*sin(6*d*x + 6*c)^2 + 16*a*sin(4*d*x + 4*c)^2 + 8*a*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(8*d*x + 8*c) + 6
*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x
 + 10*c) + 8*(6*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2
*c))*cos(8*d*x + 8*c) + 12*(4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(
6*d*x + 6*c) + 2*(4*a*sin(8*d*x + 8*c) + 6*a*sin(6*d*x + 6*c) + 4*a*sin(4*d
*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 8*(6*a*sin(6*d*x + 6*c
) + 4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(4*a*s
in(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 6*(a*d*cos(2*
d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integ
rate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x +
 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8
*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(
```

```
4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*co
s(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*c
os(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*
cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4
*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (
(cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c)
+ 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
 - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c
) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*
c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10
*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2...
```

Fricas [A]
time = 3.91, size = 311, normalized size = 2.49

$$\left[-\frac{15\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\left(17\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)-3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{15\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)},\ -\frac{2\left(15\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+\left(17\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)-3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{15\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(17*cos(d*x + c)^2 + c
os(d*x + c) - 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d
*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -2/15*(15*(cos(d*x + c)^3 + cos(d*x
+ c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) + (17*cos(d*x + c)^2 + cos(d*x + c) - 3)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(
d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(109) = 218.

time = 1.65, size = 228, normalized size = 1.82

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{4\left(\left(13a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-40a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)}{30\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*(15*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*
sqrt(2)*abs(a) - 6*a))/abs(a) + 4*((13*a^2*tan(1/2*d*x + 1/2*c)^2 - 40*a^2)
*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/
2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^4}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

3.179 $\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 327, 209}

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.77, size = 119, normalized size = 1.89

$$\frac{16\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{5/2}\left(-\text{ArcSin}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\cos(c+dx)+\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}}\sin(c+dx)\right)}{d\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Cos[(c + d*x)/2]^6*Sec[c + d*x]^4*((1 + Sec[c + d*x])^(-1))^(5/2)*(-(Ar
cSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos[c + d*x]) + Sqrt[C
os[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sin[c + d*x]))/(d*Sqrt[a*(1 + Se
c[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(55) = 110$.
time = 0.13, size = 116, normalized size = 1.84

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)-2\cos(dx+c)+2\right)}{d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*cos(d*x+c)+2)/sin(d*x+c)/a

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*((2*a*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(6*d*x + 6*c)^2 + 4*a*cos(4*d*x + 4*c)^2 + 4*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 4*a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 2*a*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c

```
) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))/(a*cos(6*d*x + 6*c)^2 + 4*a*cos(4*d*x + 4*c)^2 + 4*a*
cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*
c)^2 + 4*a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*s
in(2*d*x + 2*c)^2 + 2*(2*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x
 + 6*c) + 2*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)),
x) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sqrt(a)*d)
```

Fricas [**A**]
time = 3.42, size = 235, normalized size = 3.73

$$\left[-\frac{\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad},\ \frac{2\left(\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{ad\cos\left(dx+c\right)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
 - a)/(cos(d*x + c) + 1)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d), 2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
 sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) +
a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(55) = 110.
time = 1.24, size = 187, normalized size = 2.97

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{4\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\right)}{2\,d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/abs(a) - 4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*
x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.180 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{7\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d}-\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx}}{4ad}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+7/8*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4
*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)*sec(1/2*d*
x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 483, 597, 536, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{7\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}\,d}-\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4ad}-\frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(4*Sq
rt[2]*Sqrt[a]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\text{Subst}\left(\int \frac{}{x^2(}\right.}{}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\,\sqrt{}}{4ad}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\,\sqrt{}}{4ad}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{7\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\,\sqrt{a}\,d} - \frac{\text{co}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.30, size = 5534, normalized size = 33.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(139) = 278$.
time = 0.19, size = 374, normalized size = 2.27

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(8\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)\sqrt{2}+7\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+7*sin(d*x+c)*cos(d*x+c)^2*ln(-
(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-8*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))*sin(d*x+c)-7*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-6*cos(d*x+c)^3+4*cos(d*x+c)^2+2*cos(d*x+c))/sin(d*x+c)^3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 3.63, size = 503, normalized size = 3.05

$$\left[-\frac{7 \sqrt{2} \sqrt{-a} (\cos(dx+c)+1) \log\left(\frac{2 \sqrt{2} \sqrt{-a} \sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c)\sin(dx+c) + 3a\cos(dx+c)^2 + 2a\cos(dx+c) - a}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)\sin(dx+c) + 8\sqrt{-a}(\cos(dx+c)+1)\log\left(-\frac{8a\cos(dx+c)^3 - 4(2\cos(dx+c)^2 - \cos(dx+c))\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c) - 7a\cos(dx+c) + a}{\cos(dx+c)+1}\right)\sin(dx+c) + 4(3\cos(dx+c)^2 + \cos(dx+c))\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{16(ad\cos(dx+c)+ad)\sin(dx+c)}, -\frac{7\sqrt{2}\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c) + 8\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2 + a\cos(dx+c) - a}\right)\sin(dx+c) + 2(3\cos(dx+c)^2+\cos(dx+c))\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{8(ad\cos(dx+c)+ad)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(7*sqrt(2)*sqrt(-a)*(cos(d*x + c) + 1)*log((2*sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x
+ c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d
*x + c) + 8*sqrt(-a)*(cos(d*x + c) + 1)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos
(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4
*(3*cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))
/((a*d*cos(d*x + c) + a*d)*sin(d*x + c)), -1/8*(7*sqrt(2)*sqrt(a)*(cos(d*x
+ c) + 1)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 8*sqrt(a)*(cos(d*x + c) + 1)*arct
an(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(3*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d
*x + c) + a*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.15, size = 269, normalized size = 1.63

$$\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{7\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}}+\frac{2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{8\sqrt{-a}}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a}\right)}{16\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) - 7*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) + 2*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*tan(1/2*d*x + 1/2*c)/a + 8*sqrt(-a)/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.181 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=251

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{107\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d}+\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad}$$

```
[Out] 43/96*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d-15/32*cos(d*x+c)*cot(d*x+c)
^3*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^2/d-1/16*cos(d*x+c)^2*cot(
d*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctan(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-107/128*arctan(1/2*a^(1/2)*ta
n(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+21/64*cot(d*x+c)
*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\frac{43\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{96a^2d}-\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{16a^2d}-\frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{32a^2d}+\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{107\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{64\sqrt{2}\sqrt{a}\,d}+\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
107*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(64*
Sqrt[2]*Sqrt[a]*d) + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(64*a*d) +
(43*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(96*a^2*d) - (15*Cos[c + d*x
]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(32*a^2*d)
- (Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(3
/2))/(16*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d}$$

$$= -\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+}{}$$

$$= \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+\right.}{32a^2d}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} -$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} -$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{107\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d} + \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.08, size = 5574, normalized size = 22.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(216) = 432.
time = 0.24, size = 722, normalized size = 2.88

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))(-1+\cos(dx+c))^2\left(384(\cos^3(dx+c))\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx}{1+\cos(d}}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))^
2*(384*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^
(1/2)+321*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-
(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
+384*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1
/2)+321*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3
84*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)-
321*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-384*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-410*cos(d*x+c)^4-
321*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-142*cos(d*x+c)^3+2
98*cos(d*x+c)^2+126*cos(d*x+c))/sin(d*x+c)^7/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 2.42, size = 666, normalized size = 2.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(321*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d
*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(
d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*
(205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3 + a*d*co
s(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c)), 1/384*(321*sqrt(2)*(c
os(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sq
rt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(
205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3 + a*d*cos
(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [**A**]
time = 1.23, size = 385, normalized size = 1.53

$$\frac{\sqrt{2}\left(6\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{21}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{384\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{321\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}}-\frac{64\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}-15\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}a+8\sqrt{-a}a^2\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^3}\right)}{768\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*sqrt(2)*(6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/2*c
)^2/a - 21/a)*tan(1/2*d*x + 1/2*c) - 384*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2
)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 321*log((sqrt(-a)*ta
```

```
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 64*
(9*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*
sqrt(-a) - 15*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*sqrt(-a)*a + 8*sqrt(-a)*a^2)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

3.182 $\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=335

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{835\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}\,d}-\frac{189\cot(c+dx)\sqrt{a+a\sec(c}{512ad}$$

```
[Out] -323/768*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d+579/640*cot(d*x+c)^5*(a+
a*sec(d*x+c))^(5/2)/a^3/d-101/128*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^2*(a+a*sec(d*x+c))^(5/2)/a^3/d-23/192*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d
*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+c)^5*sec
(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^3/d-2*arctan(a^(1/2)*tan(d*x+c)/
(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+835/1024*arctan(1/2*a^(1/2)*tan(d*x+c)*2^
(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-189/512*cot(d*x+c)*(a+a*sec
(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.21, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\frac{579\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{640a^3d}-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{48a^3d}-\frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{192a^3d}-\frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{128a^3d}-\frac{323\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{768a^2d}-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{835\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{512\sqrt{2}\sqrt{a}\,d}-\frac{189\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(835*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(51
2*Sqrt[2]*Sqrt[a]*d) - (189*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(512*a*d
) - (323*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(768*a^2*d) + (579*Cot[
c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(640*a^3*d) - (101*Cos[c + d*x]*Cot[
c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(128*a^3*d) - (23
*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2
))/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c + d*x)/2]^6*(a + a*S
ec[c + d*x])^(5/2))/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} - \frac{\text{Subst}\Big(\int$$

$$= -\frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} - \frac{\cos^3(c}{}$$

$$= -\frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} - \frac{23\cos^2}{}$$

$$= \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c\right.}{128a^3d}$$

$$= -\frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} + \frac{579\cot^5(c+dx)(a+a\sec(c+dx))}{640a^3d}$$

$$= -\frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d}$$

$$= -\frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{835\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}\,d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.00, size = 5618, normalized size = 16.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(291) = 582$.
time = 0.20, size = 1068, normalized size = 3.19

method	result	size
default	Expression too large to display	1068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15360/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+
c))^3*(-15360*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*
2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))-12525*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*
x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))-15360*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))*2^(1/2)-12525*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(
d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))+30720*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*2^(1/2)+19474*cos(d*x+c)^6+25050*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+30720*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+6902*cos(d*x+c)^5+25050*sin(d*x+c)*co
s(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-15360*sin(d*x+c)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)-28788*cos(d*x+c)^4-
12525*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-15360
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-12316*cos(d*x
+c)^3-12525*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+12130*cos(
d*x+c)^2+5670*cos(d*x+c))/sin(d*x+c)^11/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^6/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 3.18, size = 823, normalized size = 2.46

$$\left[-\frac{12525\sqrt{2}\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+15360\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{8a\cos(dx+c)^3-4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)+4\left(9737\cos(dx+c)^6+3451\cos(dx+c)^5-14394\cos(dx+c)^4-6158\cos(dx+c)^3+6065\cos(dx+c)^2+2835\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{30720\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2ad\cos(dx+c)^3-2ad\cos(dx+c)^2+ad\cos(dx+c)+ad\right)\sin(dx+c)},\ -\frac{12525\sqrt{2}\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+15360\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+2\left(9737\cos(dx+c)^6+3451\cos(dx+c)^5-14394\cos(dx+c)^4-6158\cos(dx+c)^3+6065\cos(dx+c)^2+2835\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{15360\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2ad\cos(dx+c)^3-2ad\cos(dx+c)^2+ad\cos(dx+c)+ad\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(
d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*s
in(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2
*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(
2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c
) + 4*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)^5 - 14394*cos(d*x + c)^4 - 6
158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 2835*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a
*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x
+ c)), -1/15360*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x
 + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin
(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*c
os(d*x + c) - a))*sin(d*x + c) + 2*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)
^5 - 14394*cos(d*x + c)^4 - 6158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 283
5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)
^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d
*cos(d*x + c) + a*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.32, size = 500, normalized size = 1.49

$$\frac{\sqrt{2}\left(10\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{43}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{567}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{15360\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{12525\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}}+\frac{192\left(145\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^8\sqrt{-a}-500\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6\sqrt{-a}a+710\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}a^2-460\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}a^3+121\sqrt{-a}a^4\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^5}\right)}{30720\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30720*sqrt(2)*(10*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*tan(1/2*d*x +
 1/2*c)^2/a - 43/a)*tan(1/2*d*x + 1/2*c)^2 + 567/a)*tan(1/2*d*x + 1/2*c) +
15360*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))/abs(a) - 12525*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) + 192*(145*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a) - 500*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 710*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt
(-a)*a^2 - 460*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*sqrt(-a)*a^3 + 121*sqrt(-a)*a^4)/((sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

3.183 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+a\sec(c+dx)}}{a^2d}-\frac{2(a+a\sec(c+dx))^{3/2}}{a^3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*(a+a*sec(d*x+c))^(3/
2)/a^3/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^4/d+2*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 90, 52, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^4d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{a^3d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a^2*d) - (2*(a + a*Sec[c + d*x])^(3/2))/(a^3*d) + (2*(a + a
*Sec[c + d*x])^(5/2))/(5*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2\sqrt{a+ax} + \frac{a^2\sqrt{a+ax}}{x} + a(a+ax)^{3/2}\right)\,dx, x, \sec(c+dx\right.}{a^4d}$$

$$= -\frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx\right.}{a^2d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d} +$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d} +$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2(a+a\sec(c}{a^3}$$

Mathematica [A]

time = 0.17, size = 79, normalized size = 0.79

$$\frac{2\left(1-2\sec(c+dx)-2\sec^2(c+dx)+\sec^3(c+dx)-5\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{5ad\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 - 2*Sec[c + d*x] - 2*Sec[c + d*x]^2 + Sec[c + d*x]^3 - 5*ArcTanh[Sqrt
[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(5*a*d*Sqrt[a*(1 + Sec[c + d*x
])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(86) = 172.
time = 0.16, size = 224, normalized size = 2.24

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\left(\cos^2(dx+c)\right)\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+10\cos(dx+c)\sqrt{2}\arctan\left(\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*cos(d*x+c)^2*2^(1/2)*arctan(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(5/2)+10*cos(d*x+c)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+5*2^(1/2)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)+8*cos(d*x+c)^2-24*cos(d*x+c)+8)/cos(d*x+c)^2/a^2
```

Maxima [A]
time = 0.49, size = 110, normalized size = 1.10

$$\frac{\frac{5\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^4}-\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^3}+\frac{10\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(3/2) + 2*(a + a/cos(d*x + c))^(5/2)/a^4 - 10*(a + a/cos(d*x +
 c))^(3/2)/a^3 + 10*sqrt(a + a/cos(d*x + c))/a^2)/d
```

Fricas [A]
time = 4.08, size = 261, normalized size = 2.61

$$\left[\frac{5\,\sqrt{a}\,\cos\left(dx+c\right)^{2}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\left(\cos\left(dx+c\right)^{2}-3\,\cos\left(dx+c\right)+1\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{10\,a^{2}d\cos\left(dx+c\right)^{2}},\ \frac{5\,\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos\left(dx+c\right)^{2}+2\left(\cos\left(dx+c\right)^{2}-3\,\cos\left(dx+c\right)+1\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{5\,a^{2}d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)
^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*co
s(d*x + c) - a) + 4*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2), 1/5*(5*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^2 + 2*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 2.43, size = 154, normalized size = 1.54

$$\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{asgn}(\cos(dx+c))}+\frac{\sqrt{2}\left(5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}+10\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a+4\,a^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,\mathrm{asgn}(\cos(dx+c))}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(5*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*(5*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2 +
 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 4*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 -
a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)
```

3.184 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+a\sec(c+dx)}}{a^2d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a+a*sec(d*x+c))^(1/2
)/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3965, 81, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*S
ec[c + d*x]])/(a^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-a+ax}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 1.04

$$\frac{2\left(1+\sec(c+dx)+\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{ad\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 + Sec[c + d*x] + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x
]]))/(a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.14, size = 81, normalized size = 1.50

method	result	size

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}-2\right)}{d\,a^{2}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-2)/a^2
```

Maxima [A]
time = 0.49, size = 71, normalized size = 1.31

$$-\frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt
(a)))/a^(3/2) - 2*sqrt(a + a/cos(d*x + c))/a^2)/d
```

Fricas [A]
time = 3.40, size = 191, normalized size = 3.54

$$\left[\frac{\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^{2}-4\left(2\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{2\,a^{2}d},\,-\frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+a}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{a^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d), -(sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.41, size = 85, normalized size = 1.57

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}-\frac{\sqrt{2}}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\mathrm{sgn}(\cos(dx+c))}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*sgn(cos(d*x + c))) - sqrt(2)/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(c
os(d*x + c))))/(a*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\tan(c+d\,x)}^3}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

3.185 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2}{ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+a*sec(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 53, 65, 213}

$$\frac{2}{ad\sqrt{a\sec(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 38, normalized size = 0.70

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\sec(c+dx)\right)}{ad\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(a*d*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+a\sec(dx+c)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	45
default	$\frac{\frac{2}{a\sqrt{a+a\sec(dx+c)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a/(a+a*sec(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^
(1/2)))
```

Maxima [A]
time = 0.49, size = 70, normalized size = 1.30

$$\frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(
a)))/a^(3/2) + 2/(sqrt(a + a/cos(d*x + c))*a))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.
time = 2.93, size = 244, normalized size = 4.52

$$\left[\frac{\sqrt{a}\,(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2+4\,(2\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,(a^2d\cos(dx+c)+a^2d)},\ \frac{\sqrt{-a}\,(\cos(dx+c)+1)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a^2d\cos(dx+c)+a^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^2*d*cos(d*x + c) + a^2*d), (sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^2*d*cos(d*
x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.07, size = 87, normalized size = 1.61

$$\frac{\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*
sgn(cos(d*x + c))))/d
```

Mupad [B]
time = 1.59, size = 50, normalized size = 0.93

$$\frac{2}{a\,d\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] 2/(a*d*(a + a/cos(c + d*x))^(1/2)) - (2*atanh((a + a/cos(c + d*x))^(1/2)/a^
(1/2)))/(a^(3/2)*d)
```

3.186 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{1}{3d(a+a\sec(c+dx))^{3/2}}-\frac{3}{2ad\sqrt{a+a\sec(c+dx)}}$$

[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/3/d/(a+a*sec(d*x+c))^(3/2)-1/4*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-3/2/a/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3965, 87, 157, 162, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{3}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{1}{3d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) - 1/(3*d*(a + a*Sec[c + d*x])^(3/2)) - 3/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-2a^4+3a}{x(-a+ax)\sqrt{a}}\right.}{}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a}}\right.}{}$$

$$= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{}{3d(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 60, normalized size = 0.50

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1+\sec(c+dx))\right) - 2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\sec(c+dx)\right)}{3d(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(95) = 190$.
time = 0.17, size = 376, normalized size = 3.13

method	result

default	$-\frac{(-1+\cos(dx+c))^2\left(12\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^2(dx+c))\sqrt{2}+3\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/d*(-1+cos(d*x+c))^2*(12*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+3*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))*cos(d*x+c)^2+24*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+6*cos(d*x+c)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2))+12*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+22*cos(d*x+c)^2+3*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+18*cos(d*x+
c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^4/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(95) = 190.
time = 3.03, size = 485, normalized size = 4.04

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+12\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)-4\left(11\cos(dx+c)^2+9\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-12\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)-2\left(11\cos(dx+c)^2+9\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{12\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt
(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(
d*x + c) - a)/(cos(d*x + c) - 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(1
```

```
1*cos(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/12*(3*sqrt(2)*(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 12*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(11*co
s(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.02, size = 164, normalized size = 1.37

$$\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(\cos\left(dx+c\right)\right)} - \frac{24\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(\cos\left(dx+c\right)\right)} - \frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}a^{6}+9\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,a^{7}\right)}{a^{9}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/12*(3*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(cos(d*x + c))) - 24*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - sqrt(2)*((-a*tan(1/2*d
*x + 1/2*c)^2 + a)^(3/2)*a^6 + 9*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^7)/(
a^9*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cot}\left(c+d\,x\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

3.187 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{3a}{20d(a+a\sec(c+dx))^{5/2}}+\frac{}{2d(1}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-3/20*a/d/(a+a*sec(d*x+
c))^(5/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+5/24/d/(a+a*sec(d*x
+c))^(3/2)+11/32*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2
)/d*2^(1/2)+21/16/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3965, 105, 157, 162, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{3a}{20d(a\sec(c+dx)+a)^{5/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}+\frac{5}{24d(a\sec(c+dx)+a)^{3/2}}+\frac{21}{16ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (11*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)*d) - (3*a)/(
20*d*(a + a*Sec[c + d*x])^(5/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(5/2)) + 5/(24*d*(a + a*Sec[c + d*x])^(3/2)) + 21/(16*a*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{7a^2x}{2}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x\right.}{2d}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + -$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{2}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{2}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{2}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{}{2}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{11\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 90, normalized size = 0.51

$$\frac{a\left(-10-11\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))\right)}{20d(-1+\sec(c+dx))(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*(-10 - 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(20*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(143) = 286.
time = 0.23, size = 514, normalized size = 2.92

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))(-1+\cos(dx+c))^{3}\left(480\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^{4}(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/480/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c))^
3*(480*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^4*2^(1/2)+165*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^4+96
0*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+330*cos(d*x+c)^3*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+898*cos
(d*x+c)^4-960*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+702*cos(d*x+c)^3-330*co
s(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))-480*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-730*cos(d*x+c)^2-165*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2))-630*cos(d*x+c))/sin(d*x+c)^8/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(141) = 282$.
time = 3.05, size = 592, normalized size = 3.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1
)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 480*(cos(d*x + c)^
4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2
+ 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(449*cos(d*x + c)^4 + 351*cos(d*x +
c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*
x + c) - a^2*d), -1/480*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2
*cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 480*(cos(d*x + c)^4 +
 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(4
49*cos(d*x + c)^4 + 351*cos(d*x + c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.20, size = 253, normalized size = 1.44

$$-\frac{\frac{165\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}-\frac{960\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}+\frac{15\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}-\frac{2\sqrt{2}\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{16}+20\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^{17}+165\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{18}\right)}{a^{20}\mathrm{sgn}(\cos(dx+c))}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(165*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a*sgn(cos(d*x + c))) - 960*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) + 15*sqrt(2)*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2
) - 2*sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)*a^16 + 20*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^17 + 165*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^18)/(a^20*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

3.188 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{203\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}+\frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}}-\frac{}{4d(1}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+139/224*a^2/d/(a+a*sec(
d*x+c))^(7/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(7/2)-19/16*a^2/d
/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+15/64*a/d/(a+a*sec(d*x+c))^(5/2)-53/
384/d/(a+a*sec(d*x+c))^(3/2)-203/512*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-309/256/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{203\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}+\frac{139a^2}{224d(a\sec(c+dx)+a)^{7/2}}-\frac{19a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/2}}+\frac{15a}{64d(a\sec(c+dx)+a)^{5/2}}-\frac{53}{384d(a\sec(c+dx)+a)^{3/2}}-\frac{309}{256ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (203*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/2)*d) + (139*a
^2)/(224*d*(a + a*Sec[c + d*x])^(7/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(7/2)) - (19*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(7/2)) + (15*a)/(64*d*(a + a*Sec[c + d*x])^(5/2)) - 53/(384*d*(a + a*
Sec[c + d*x])^(3/2)) - 309/(256*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{11a^2x}{2}}{x(-a+ax)^2(a+ax)^{9/2}}\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\,\mathrm{s}}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} -$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} -$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} -$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} -$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} -$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} -$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.31, size = 99, normalized size = 0.42

$$\frac{\cot^4(c+dx)\left(-322+203\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2-64\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))^2+266\sec(c+dx)\right)}{224d(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-322 + 203*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 266*Sec[c + d*x]))/(224*d*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(197) = 394$.
time = 0.20, size = 866, normalized size = 3.64

method	result	size
default	Expression too large to display	866

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10752/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x
+c))^4*(10752*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^6*2^(1/2)+21504*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2))*cos(d*x+c)^5*2^(1/2)+4263*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^6-10752*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*cos(d*x+c)^4*2^(1/2)+8526*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^5-43008*cos(d*x+c)^3*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2))*2^(1/2)-4263*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^4+20726*cos(d*x+c)^6-10752*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)-17052*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+16074*cos
(d*x+c)^5+21504*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-4263*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x
+c)^2-33076*cos(d*x+c)^4+10752*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+8526*cos(d*x+c)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))-28476*cos(d*x+c)^3+4263*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+14462*cos(d*x+c)^2+12978*cos(d*x+c
))/sin(d*x+c)^12/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

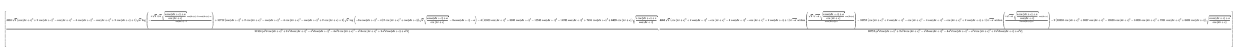
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(193) = 386$.
time = 3.19, size = 837, normalized size = 3.52



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/21504*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4
- 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*s
qrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*c
os(d*x + c) - a)/(cos(d*x + c) - 1)) + 10752*(cos(d*x + c)^6 + 2*cos(d*x +
c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4
*(10363*cos(d*x + c)^6 + 8037*cos(d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238
*cos(d*x + c)^3 + 7231*cos(d*x + c)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a
^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d), 1/10752*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x
 + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 10752*(cos(d*x + c)^6 + 2*cos(d*
x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(10363*cos(d*x + c)^6 + 8037*cos(
d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238*cos(d*x + c)^3 + 7231*cos(d*x + c
)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*co
s(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(
d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

[Out] Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)

Giac [A]
time = 1.11, size = 322, normalized size = 1.35

$$\frac{\frac{4263\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}-\frac{21504\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}-\frac{21\left(29\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}-27\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a\right)}{a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{8\sqrt{2}\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{30}-21\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{31}-112\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^{32}-882\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{33}\right)}{a^{35}\mathrm{sgn}(\cos(dx+c))}}{10752\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/10752*(4263*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - 21504*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - 21*(29*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 27*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4) + 8*sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^30 - 21*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^31 - 112*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^32 - 882*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^33)/(a^35*sgn(cos(d*x + c))))/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)

[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)

3.189 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}-\frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a\tan^5(c+d...}{5d(a+a\sec(c+...}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*tan(d*x+c)
/a/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a
*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^2*tan(d*x+c)^7/d/(a+a*sec(d*x+
c))^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3972, 470, 308, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2a^2\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x])
^(5/2)) + (2*a^2*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \frac{x^4}{a} - \frac{1}{a^3(1+ax^2)}\right) dx, x, -\right.}{d}$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c}{3d(a+a\sec(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.68, size = 248, normalized size = 1.58

$$\frac{32\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{11/2}\left(\frac{\cos(c+dx)(11+7\cos(c+dx))\csc^8\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(105\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\cos^3(c+dx)+(76-198\cos(c+dx)+61\cos(2(c+dx))-44\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\right)}{3360\sqrt{1-\sec(c+dx)}} - \frac{4}{11}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};-2\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^7(c+dx)}{7d(a(1+\sec(c+dx)))^{3/2}\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

[Out] (32*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(11/2)*((Cos[c + d*x]*(11 + 7*Cos[c + d*x])*Csc[(c + d*x)/2]^8*Sec[(c + d*x)/2]^2*(105*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x]^3 + (76 - 198*Cos[c + d*x] + 61*Cos[2*(c + d*x)] - 44*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]))/(3360*Sqrt[1 - Sec[c + d*x]]) - (4*Hypergeometric2F1[2, 11/2, 13/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^2)/11)*Tan[c + d*x]^7)/(7*d*(a*(1 + Sec[c + d*x]))^(3/2)*(1 - Tan[(c + d*x)/2]^2)^(9/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(137) = 274$.
time = 0.17, size = 391, normalized size = 2.49

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(105\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^3(dx+c))\sin(dx+c)\sqrt{2}+31}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/840/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(105*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+105*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)+2336*cos(d*x+c)^4-2848*cos(d*x+c)^3+128*cos(d*x+c)^2+624*cos(d*x+c)-240)/sin(d*x+c)/cos(d*x+c)^3/a^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/210*(105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2

```
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - 1) + 2*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)
^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(14*d*x + 14*c)*cos(2
*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)
*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6
*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*
c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x
 + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(
2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*
sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x
+ 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*
x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*
d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*
cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*
c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x
 + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d
*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(
2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)
*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*
c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*
c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x
 + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d
*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*si
n(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c)
+ 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*
c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)))/(a^2*cos(14*d*x + 14*c)^2 + 36*a^2*cos(12*d*x + 12
*c)^2 + 225*a^2*cos(10*d*x + 10*c)^2 + 400*a^2*cos(8*d*x + 8*c)^2 + 225*a^2
*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 12*a^2*cos(4*d*x + 4*c)*c
os(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(14*d*x + 14*c)^2 + 36*a^
2*sin(12*d*x + 12*c)^2 + 225*a^2*sin(10*d*x + 10*c)^2 + 400*a^2*sin(8*d*x +
 8*c)^2 + 225*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 12*a^2*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(6*a^2*cos(12
*d*x + 12*c) + 15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x + 8*c) + 15*a^2
```

```
*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(14*d
*x + 14*c) + 12*(15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x + 8*c) + 15*a
^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(12
*d*x + 12*c) + 30*(20*a^2*cos(8*d*x + 8*c) + 15*a^2*cos(6*d*x + 6*c) + 6*a^
2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a^2*
cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) + 30*(6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(6*d*x + 6*
c) + 2*(6*a^2*sin(12*d*x + 12*c) + 15*a^2*sin(10*d*x + 10*c) + 20*a^2*sin(8
*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*
d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^2*sin(10*d*x + 10*c) + 20*a^2*sin
(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(
2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^2*sin(8*d*x + 8*c) + 15*a^2*sin
(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x +
 10*c) + 40*(15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d
*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c)), x) - 12*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*
x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*...
```

Fricas [**A**]
time = 2.20, size = 343, normalized size = 2.18

$$\left[-\frac{105\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\sqrt{-a}\log\left(\frac{2a\cos\left(dx+c\right)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)-2\left(146\cos\left(dx+c\right)^{3}-32\cos\left(dx+c\right)^{2}-24\cos\left(dx+c\right)+15\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{105\left(a^{2}d\cos\left(dx+c\right)^{4}+a^{2}d\cos\left(dx+c\right)^{3}\right)},\ \frac{2\left(105\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+\left(146\cos\left(dx+c\right)^{3}-32\cos\left(dx+c\right)^{2}-24\cos\left(dx+c\right)+15\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{105\left(a^{2}d\cos\left(dx+c\right)^{4}+a^{2}d\cos\left(dx+c\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(146*cos(d*x + c)^3
- 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3), 2/105
*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (146*cos(d*x + c
)^3 - 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(137) = 274.
time = 3.61, size = 296, normalized size = 1.89

$$\frac{105\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)}{a^2\mathrm{sgn}(\cos(dx+c))}\right)+\frac{2\left(\left(\left(\frac{139\sqrt{2}a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}(\cos(dx+c))}-\frac{539\sqrt{2}a^2}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{385\sqrt{2}a^2}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{105\sqrt{2}a^2}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/105*(105*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(cos(d*x + c))) - log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(cos(d*x + c)))) + 2*(((139*sqrt(2)*a^2*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 539*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 385*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 - 105*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)

[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)

3.190 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-2*tan(d*x+c)/
a/d/(a+a*sec(d*x+c))^(1/2)+2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 308, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x,\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\tan^3(c+}{3d(a+a\sec(c+}$$

Mathematica [A]
time = 4.38, size = 162, normalized size = 1.71

$$\frac{64\cos^6\left(\frac{1}{2}(c+dx)\right)\cot^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{7/2}\left(3\text{ArcSin}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\cos^2(c+dx)+\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}}(\sin(c+dx)-2\sin(2(c+dx)))\right)}{3d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)^2(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (64*Cos[(c + d*x)/2]^6*Cot[(c + d*x)/2]^4*Sec[c + d*x]^5*((1 + Sec[c + d*x]
)^(-1))^(7/2)*(3*ArcSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos
[c + d*x]^2 + Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*(Sin[c + d*x
] - 2*Sin[2*(c + d*x)])))/(3*d*(-1 + Cot[(c + d*x)/2]^2)^2*(a*(1 + Sec[c +
d*x]))^(3/2))
```

Maple [A]
time = 0.16, size = 142, normalized size = 1.49

method	result

default	$-\frac{\left(3\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\cos(dx+c)-8\left(\cos^2(dx+c)\right)+10\cos(dx}{3d\sin(dx+c)\cos(dx+c)a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(
d*x+c)-8*cos(d*x+c)^2+10*cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/
sin(d*x+c)/cos(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(2*a^2*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(3/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*
d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4
*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(
2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin
(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*c
os(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin
(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*
sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)
*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c
)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x +
2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x
+ 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x
 + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x +
 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8
*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
+ 2*c) + 1)))/(a^2*cos(10*d*x + 10*c)^2 + 16*a^2*cos(8*d*x + 8*c)^2 + 36*a^
2*cos(6*d*x + 6*c)^2 + 16*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c)*c
os(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 16*a^
2*sin(8*d*x + 8*c)^2 + 36*a^2*sin(6*d*x + 6*c)^2 + 16*a^2*sin(4*d*x + 4*c)^
2 + 8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(4
*a^2*cos(8*d*x + 8*c) + 6*a^2*cos(6*d*x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a
^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(6*a^2*cos(6*d*x + 6*c) + 4*a^2
*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 12*(4*a^2*cos(
4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a^2*sin(8*d*x
+ 8*c) + 6*a^2*sin(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x +
2*c))*sin(10*d*x + 10*c) + 8*(6*a^2*sin(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*
c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(4*a^2*sin(4*d*x + 4*c) +
a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 4*a^2*d*integrate(-(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(10*d*
x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*
d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d
*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(
2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*s
in(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*
sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)
*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c
)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x +
2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x +
10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x +
2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x +
 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d
*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6
*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*
d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a^2*cos(10*d*x + 10*c)^
2 + 16*a^2*cos(8*d*x + 8*c)^2 + 36*a^2*cos(6*d*x + 6*c)^2 + 16*a^2*cos(4*d*
x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)
^2 + a^2*sin(10*d*x + 10*c)^2 + 16*a^2*sin(8*d*x + 8*c)^2 + 36*a^2*sin(6*d*
x + 6*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(4*a^2*cos(8*d*x + 8*c) + 6*a^2*cos(6*d*
x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c
) + 8*(6*a^2*cos(6*d*x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*
c))*cos(8*d*x + 8*c) + 12*(4*a^2*cos(4*d*x + 4*...
```

Fricas [A]
time = 2.84, size = 295, normalized size = 3.11

$$\left[-\frac{3\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(4\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)},\ -\frac{2\left(3\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(4\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)\right)}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + a^
2*d*cos(d*x + c)), -2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(83) = 166.
time = 2.36, size = 230, normalized size = 2.42

$$-\frac{3\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)}{a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+a\left(2\,\sqrt{2}-3\right)\right|\right)}{a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)+\frac{2\left(\frac{5\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{3\,\sqrt{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(cos(d*x + c))) - log(
abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
 + a*(2*sqrt(2) - 3)))/(a^2*sgn(cos(d*x + c)))) + 2*(5*sqrt(2)*tan(1/2*d*x
```

```
+ 1/2*c)^2/sgn(cos(d*x + c)) - 3*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x + 1
/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

3.191 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{2\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*arctan(1/2
*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 492, 209}

$$\frac{2\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(a^(3/2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} - \frac{4\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{}{\sqrt{}}\right.}{ad}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 23.94, size = 4739, normalized size = 55.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-32*Cos[(c + d*x)/4]^2*Cos[(c + d*x)/2]^2*(-2/Sqrt[Sec[c + d*x]] + 2*Sqrt[
Sec[c + d*x]])*Sec[c + d*x]^2*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqr
t[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]],
1/2] - (2 + Sqrt[2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt
[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2
+ Sqrt[2])*EllipticPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c
+ d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2]
 + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] +
 Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(
c + d*x)/4])^2*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c
 + d*x)/4])])/4 + EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3
 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*S
qrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]))/(d*(a*(1 + Se
c[c + d*x]))^(3/2)*(16*Cos[(c + d*x)/4]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/4]
*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x
)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*Elliptic
Pi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 +
```

```
(1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[
2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])
*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sq
rt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt
[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt
[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]])/4 + EllipticPi[
-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[
2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*
Sqrt[2] + Tan[(c + d*x)/4]^2)]) - 16*Cos[(c + d*x)/4]^2*Sec[c + d*x]^(3/2)*
Sin[c + d*x]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] +
Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[
2])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*
x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*Ellipt
icPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (
1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sqrt[(-1 - Sqrt[2] + Tan[(c + d*x)/
4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4]
)/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sq
rt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]])/4 +
 EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 1
7 - 12*Sqrt[2]]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[
2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) - 32*Cos[(c + d*x)/4]^2*Sqrt[Se
c[c + d*x]]*((((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + T
an[(c + d*x)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2
])*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x
)/4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*Ellipti
cPi[1/Sqrt[2], ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1
 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2])*Sec[(c + d*x)/4]^2*Sqrt[(-1 - Sqrt[2
] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*Sqrt[(1 - Sqrt[2]
+ Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[
(c + d*x)/4])*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c
+ d*x)/4])]])/8 + ((-3 + 2*Sqrt[2])*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(c
 + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sec[(c + d*x)/4]^2*Tan[(c
 + d*x)/4]*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2])/(4*Sqrt[(-3 + 2*Sqrt[2
])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)/4]^2)]) - (EllipticPi[-3 + 2*Sqrt[2], Ar
cSin[Tan[(c + d*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]*Sec[(c + d*x)/
4]^2*Tan[(c + d*x)/4]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqrt[2] + Tan[(c + d*x)
/4]^2)])/(4*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]) + (Sec[(c + d*x)/4]^2
*Sqrt[3 + 2*Sqrt[2] - Tan[(c + d*x)/4]^2]*Sqrt[(-3 + 2*Sqrt[2])*(-3 + 2*Sqr
t[2] + Tan[(c + d*x)/4]^2)])/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(c + d*x)/
4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(c + d*x)/4]^2)/(3 -
2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(c + d*x)/4]^2)/(3 - 2*Sqrt[2]))) +
(((2 + Sqrt[2])*EllipticF[ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/
4])/(1 + (1 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] - (2 + Sqrt[2])*EllipticPi
[-(1/Sqrt[2]), ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1
 + Sqrt[2])*Tan[(c + d*x)/4])]], 1/2] + (-2 + Sqrt[2])*EllipticPi[1/Sqrt[2]
```

```
, ArcSin[2^(1/4)*Sqrt[(1 + Sqrt[2] + Tan[(c + d*x)/4])/(1 + (1 + Sqrt[2])*T
an[(c + d*x)/4])]], 1/2])*Sqrt[(1 - Sqrt[2] + Tan[(c + d*x)/4])/(-1 + Sqrt[
2] + Tan[(c + d*x)/4])]*(-1 + Sqrt[2] + Tan[(c + d*x)/4])^2*Sqrt[(1 + Sqrt[
2] + Tan[(c + d*x)/4])/(-1 + Sqrt[2] + Tan[(c + d*x)/4])]*(-1/4*(Sec[(c + d
*x)/4]^2*(-1 - Sqrt[2] + Tan[(c + d*x)/4]))/(-1...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(70) = 140$.
time = 0.12, size = 142, normalized size = 1.67

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+2\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{\sin(dx}\right.\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))+2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c)))/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 2.72, size = 295, normalized size = 3.47

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\,\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{a^2d}, -\frac{2\left(\sqrt{2}\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\right)}{a^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c
```

```
) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a^2*d), -2*(sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [**A**]
time = 1.26, size = 72, normalized size = 0.85

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqr
t(a))/a^(3/2) - 2*arctan(sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/a^(3/
2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.192 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{71\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{2}\ \sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}\ a^{3/2}d}+\frac{7\cot(c+dx)\sqrt{a+a\sec(c+}{32a^2d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+71/64*arctan
(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+7
/32*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d-13/32*cos(d*x+c)*cot(d*x+c)*sec
(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)
*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A]

time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{71\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{2}\ \sqrt{a\sec(c+dx)+a}}\right)}{32\sqrt{2}\ a^{3/2}d}+\frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32a^2d}-\frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{16a^2d}-\frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(71*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(32*
Sqrt[2]*a^(3/2)*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32*a^2*d) -
 (13*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])
/(32*a^2*d) - (Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt[a + a*Se
c[c + d*x]])/(16*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d} - \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} - \frac{\cos^2(c}{}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+d\right.}{32a^2d}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+d\right.}{32a^2d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{71\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.03, size = 5578, normalized size = 25.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(184) = 368$.
time = 0.20, size = 542, normalized size = 2.52

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(-64\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^2(dx+c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-64*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-71*sin(d*
x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-128*sin(d*x+c)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)-142*sin(d*x+c)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-64*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+54*cos(d*x+c)^3-71*ln(-(-(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+24*cos(d*x+c)^2-14*cos(d*x+c))/sin(
d*x+c)^5/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 3.28, size = 603, normalized size = 2.80

$$\left[-\frac{71 \sqrt{2} \left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + 3a\cos(dx+c)^2 + 2a\cos(dx+c) - a}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)\sin(dx+c) + 64\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{-a}\log\left(-\frac{8a\cos(dx+c)^3 - 4\left(2\cos(dx+c)^2 - \cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c) - 7a\cos(dx+c) + a}{\cos(dx+c)+1}\right)\sin(dx+c) + 4\left(27\cos(dx+c)^3 + 12\cos(dx+c)^2 - 7\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{128\left(a^2 d\cos(dx+c)^2 + 2a^2 d\cos(dx+c) + a^2 d\right)\sin(dx+c)}, -\frac{71\sqrt{2}\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c) + 64\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2 + a\cos(dx+c) - a}\right)\sin(dx+c) + 2\left(27\cos(dx+c)^3 + 12\cos(dx+c)^2 - 7\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{64\left(a^2 d\cos(dx+c)^2 + 2a^2 d\cos(dx+c) + a^2 d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(71*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1))*sin(d*x + c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqr
t(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)
+ a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(27*cos(d*x + c)^3 + 12*cos(d*x +
```

```
c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*co
s(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/64*(71*sqrt(
2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x +
 c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x
 + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(27*cos(d*x + c)^3 + 12*cos
(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.15, size = 143, normalized size = 0.67

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{17\,\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,\sqrt{2}}{\left(\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\right)\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^2*sgn(cos(d*x + c))) - 17*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) + 16*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.193 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{533\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{21\cot(c+dx)\sqrt{a+a\sec(c+}}{256a^2d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+277/384*cot(d
*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d-81/128*cos(d*x+c)*cot(d*x+c)^3*sec(1/2
*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^3/d-7/64*cos(d*x+c)^2*cot(d*x+c)^3*s
ec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+
c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^3/d-533/512*arctan(1/2*a
^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-21/256*
cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3972, 483, 593, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{533\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{256\sqrt{2}\,a^{3/2}d}+\frac{277\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{384a^3d}-\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{48a^3d}-\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{64a^3d}-\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{128a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{256a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
533*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(256
*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(256*a^2*d
) + (277*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(384*a^3*d) - (81*Cos[c
 + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(128*
a^3*d) - (7*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c +
 d*x])^(3/2))/(64*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^3*Sec[(c + d*x)/2]^
6*(a + a*Sec[c + d*x])^(3/2))/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
```

```
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d} - \frac{\cos^3(c}{}$$

$$= -\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d} - \frac{7\cos^2(}{}$$

$$= \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c\right.}{128a^3d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{533\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.01, size = 5620, normalized size = 18.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(263) = 526.
time = 0.27, size = 732, normalized size = 2.42

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))(-1+\cos(dx+c))^3\left(1536\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^4(dx+c))\sin(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1536/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c)
)^3*(1536*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*2^(1/2)+3072*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*2^(1/2)+1599*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x
+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))+3198*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))-3072*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(
d*x+c)-1638*cos(d*x+c)^5-1536*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*sin(d*x+c)-3198*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*c
os(d*x+c)-984*cos(d*x+c)^4-1599*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+1380*cos(d*x+c)^3+856*cos(d*x+c)^2-126*cos(d*x+c))/sin(d*x+c)^9/a
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 3.60, size = 712, normalized size = 2.35

$$\left[-\frac{1599\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+1536\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{-a}\log\left(-\frac{8a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)-4\left(819\cos(dx+c)^5+492\cos(dx+c)^4-690\cos(dx+c)^3-428\cos(dx+c)^2+63\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{3072\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}, \frac{1599\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+1536\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+2\left(819\cos(dx+c)^5+492\cos(dx+c)^4-690\cos(dx+c)^3-428\cos(dx+c)^2+63\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{1536\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)
- 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 1536*(cos(d*x + c)^4
+ 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
+ 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*
x + c) - 4*(819*cos(d*x + c)^5 + 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 -
428*cos(d*x + c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)
- a^2*d)*sin(d*x + c)), 1/1536*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x +
c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 1536*(co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(819*cos(d*x + c)^5
+ 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 - 428*cos(d*x + c)^2 + 63*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^4 + 2*
a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.24, size = 261, normalized size = 0.86

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{2}\mathrm{sgn}(\cos(dx+c))}-\frac{37\sqrt{2}}{a^{2}\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{417\sqrt{2}}{a^{2}\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{32\sqrt{2}\left(21\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{4}-36\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}a+19\,a^{2}\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\right)^{3}\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/1536*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/
2*c)^2/(a^2*sgn(cos(d*x + c))) - 37*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/
2*d*x + 1/2*c)^2 + 417*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c
```

```
) - 32*sqrt(2)*(21*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2*a + 19*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

3.194 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{16363\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{8192\sqrt{2}\,a^{3/2}d}-\frac{21\cot(c+dx)\sqrt{a+a\sec(}{8192a^2d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-8171/12288*c
ot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d+12267/10240*cot(d*x+c)^5*(a+a*sec(
d*x+c))^(5/2)/a^4/d-2045/2048*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*
(a+a*sec(d*x+c))^(5/2)/a^4/d-511/3072*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x
+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^4/d-29/768*cos(d*x+c)^3*cot(d*x+c)^5*sec
(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^4/d-1/128*cos(d*x+c)^4*cot(d*x+c
)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^4/d+16363/16384*arctan(1/
2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-21/8
192*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{16363\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{8192\sqrt{2}\,a^{3/2}d}+\frac{12267\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{10240a^4d}-\frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{128a^4d}-\frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{768a^4d}-\frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{3072a^4d}-\frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{2048a^4d}-\frac{8171\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12288a^3d}-\frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8192a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(16363*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
8192*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8192*
a^2*d) - (8171*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12288*a^3*d) + (
12267*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(10240*a^4*d) - (2045*Cos[
c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(204
8*a^4*d) - (511*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec
[c + d*x])^(5/2))/(3072*a^4*d) - (29*Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c +
 d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(768*a^4*d) - (Cos[c + d*x]^4*Cot[c
+ d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(128*a^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4 d}$$

$$= -\frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\text{Subst}\Big(\int$$

$$= -\frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c}{}$$

$$= -\frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\,\mathrm{co}}{}$$

$$= -\frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511}{}$$

$$= \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2}{204}$$

$$= -\frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\cot^5(c+dx)(a+a\sec(c+}{10240a^4d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^3}{12288a^3d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^3}{12288a^3d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{16363\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right.}{8192\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.16, size = 5662, normalized size = 14.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

[Out] Result too large to show

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(338) = 676$.

time = 0.22, size = 1240, normalized size = 3.20

method	result	size
default	Expression too large to display	1240

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/245760/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*
x+c))^4*(-245760*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*2^(1/2)-245445*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))-491520*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(
d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))-490890*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))+245760*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*
sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*2^(1/2)+302082*cos(d*x+c)^7+245445*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+983040*cos(d*x+c)^3*sin(d*x+c)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+207048*cos(d*x+c)^6+981780*
cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+245760*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-457
998*cos(d*x+c)^5+245445*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)-491520*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcta
nh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*
2^(1/2)*cos(d*x+c)-362512*cos(d*x+c)^4-490890*sin(d*x+c)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+co
s(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-245760*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*sin(d*x+c)+195222*cos(d*x+c)^3-245445*ln(-(-(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+164680*cos(d*x+c)^2+630*cos(d*x+c))/sin(d*
x+c)^13/a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 3.70, size = 955, normalized size = 2.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/491520*(245445*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c
)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(
(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*si
n(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1))*sin(d*x + c) + 245760*(cos(d*x + c)^6 + 2*cos(d*x + c)^
5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x
+ c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(151041*cos(d*x + c)^7 + 103
524*cos(d*x + c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^4 + 97611*
cos(d*x + c)^3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a
^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/245760*(245445*sqrt(2)*(cos(d*x
+ c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 245760*(c
os(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*co
s(d*x + c) - a))*sin(d*x + c) + 2*(151041*cos(d*x + c)^7 + 103524*cos(d*x +
 c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^4 + 97611*cos(d*x + c)^
3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x
+ c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d)*sin(d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]

time = 1.42, size = 379, normalized size = 0.98

$$-\frac{5\left(2\left(4\left(\frac{6\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\operatorname{sgn}(\cos(dx+c))}-\frac{65\sqrt{2}}{a^2\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1451\sqrt{2}}{a^2\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{13503\sqrt{2}}{a^2\operatorname{sgn}(\cos(dx+c))}\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{256\sqrt{2}\left(555\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^8-1950\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6a+2780\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4a^2-1810\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a^3+473a^4\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^5\sqrt{-a}\operatorname{sgn}(\cos(dx+c))}}{245760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/245760*(5*(2*(4*(6*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(cos(d*x + c))
) - 65*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 1451*sqrt(
2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 13503*sqrt(2)/(a^2*sgn
(cos(d*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) +
 256*sqrt(2)*(555*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^8 - 1950*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^6*a + 2780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*a^2 - 1810*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^3 + 473*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*sgn(cos(d*x
+ c))))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

3.195 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3*(a+a*sec(d*x+c))^(
3/2)/a^4/d-6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3965, 90, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^4d}-\frac{6\sqrt{a\sec(c+dx)+a}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (6*Sqrt[a + a*
Sec[c + d*x]])/(a^3*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^4*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3a^2}{\sqrt{a+ax}}+\frac{a^2}{x\sqrt{a+ax}}+a\sqrt{a+ax}\right)dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx\right.}{a^2d}$$

$$= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}+\frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x,\right.}{a^3}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c}{3a^4}$$

Mathematica [A]
time = 0.15, size = 69, normalized size = 0.88

$$\frac{2\left(-8-7\sec(c+dx)+\sec^2(c+dx)-3\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{3a^2d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(-8 - 7*Sec[c + d*x] + Sec[c + d*x]^2 - 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]
]*Sqrt[1 + Sec[c + d*x]]))/(3*a^2*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(66) = 132.
time = 0.16, size = 155, normalized size = 1.99

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos(dx+c)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{\ldots}\right.\right.}{6d\cos(dx+c)a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*2^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+32*cos(d*x+c)-4)/cos(d*x+c)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(66) = 132$.
time = 0.49, size = 163, normalized size = 2.09

$$\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^4}-\frac{18\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^3}+\frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}-\frac{6}{\sqrt{a+\frac{a}{\cos(dx+c)}}\ a^2}-\frac{2}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(5/2) + 2*(a + a/cos(d*x + c))^(3/2)/a^4 - 18*sqrt(a + a/cos(d
*x + c))/a^3 + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)*a^2)
- 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a))/d
```

Fricas [A]
time = 3.84, size = 241, normalized size = 3.09

$$\left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(8\cos(dx+c)-1\right)}{6a^3d\cos(dx+c)},\ \frac{3\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)\cos(dx+c)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(8\cos(dx+c)-1\right)}{3a^3d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
```

```
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) -
1))/(a^3*d*cos(d*x + c)), 1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
- 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) - 1))/(a^3*d*co
s(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 2.76, size = 126, normalized size = 1.62

$$\frac{2\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\operatorname{sgn}(\cos(dx+c))}-\frac{\sqrt{2}\left(9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-7a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{2}\operatorname{sgn}(\cos(dx+c))}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 2/3*(3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*a^2*sgn(cos(d*x + c))) - sqrt(2)*(9*a*tan(1/2*d*x + 1/2*c)^2 - 7*a)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*sgn
(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^{5}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

3.196 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-4/a^2/d/(a+a*sec(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3965, 79, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{4}{a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - 4/(a^2*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{-a+ax}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\ &= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\ &= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 50, normalized size = 0.93

$$\frac{2\left(-2+\tanh^{-1}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{a^2d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
[Out] (2*(-2 + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(a^2*d*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(46) = 92.
time = 0.13, size = 154, normalized size = 2.85

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\cos(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sqrt{2}+\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\right.}{d(1+\cos(dx+c))a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(
1/2)+(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+4*cos(d*x+c))/(1+cos(d*x+c))/a^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(46) = 92$.
time = 0.51, size = 125, normalized size = 2.31

$$-\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}+\frac{6}{\sqrt{a+\frac{a}{\cos(dx+c)}}\,a^2}-\frac{2}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2
)*a^2) + 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a
))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(46) = 92$.
time = 3.13, size = 245, normalized size = 4.54

$$\left[\frac{\sqrt{a}\,(\cos(dx+c)+1)\log\left(-8a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)-8\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\left(a^3d\cos(dx+c)+a^3d\right)},\ -\frac{\sqrt{-a}\,(\cos(dx+c)+1)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a^3d\cos(dx+c)+a^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 8*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^3*d*cos(d*x + c) + a^3*d), -(sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^3*d*cos(d
*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.77, size = 90, normalized size = 1.67

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\operatorname{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\operatorname{sgn}(\cos(dx+c))}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2
*sgn(cos(d*x + c))))/(a*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

3.197 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2}{3ad(a+a\sec(c+dx))^{3/2}}+\frac{2}{a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3/a/d/(a+a*sec(d*x+c
))^(3/2)+2/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 53, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{2}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2}{3ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + 2/(3*a*d*(a +
a*Sec[c + d*x])^(3/2)) + 2/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,\ldots\right.}{a^2\ldots}$$

$$= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, \ldots\right.}{\ldots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{\ldots}{a^2d\sqrt{a+\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 40, normalized size = 0.51

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\sec(c+dx)\right)}{3ad(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*a*d*(a*(1 + Sec[c
 + d*x]))^(3/2))
```

Maple [A]
time = 0.04, size = 62, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{2}{a^{2}\sqrt{a+a\sec\left(dx+c\right)}}+\frac{2}{3a\left(a+a\sec\left(dx+c\right)\right)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{d}$	62
default	$\frac{\frac{2}{a^{2}\sqrt{a+a\sec\left(dx+c\right)}}+\frac{2}{3a\left(a+a\sec\left(dx+c\right)\right)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{d}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^2/(a+a*sec(d*x+c))^(1/2)+2/3/a/(a+a*sec(d*x+c))^(3/2)-2/a^(5/2)*ar
ctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 87, normalized size = 1.12

$$\frac{\frac{3\,\log\left(\frac{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,a}{\cos\left(dx+c\right)}\right)}{\left(a+\frac{a}{\cos\left(dx+c\right)}\right)^{\frac{3}{2}}a^{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)
*a^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(66) = 132.
time = 3.56, size = 321, normalized size = 4.12

$$\left[\frac{3\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}-8\,a\cos\left(dx+c\right)-a\right)+4\left(4\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{6\left(a^{3}d\cos\left(dx+c\right)^{2}+2\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\ \frac{3\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+a}\right)+2\left(4\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}}{3\left(a^{3}d\cos\left(dx+c\right)^{2}+2\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(4*cos(d*x + c)^2 + 3*cos(d*x + c
))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + a^3*d), 1/3*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
2*a*cos(d*x + c) + a)) + 2*(4*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) +
 a^3*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]

time = 1.30, size = 116, normalized size = 1.49

$$\frac{\frac{12\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^8+6\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^9\right)}{a^{12}\mathrm{sgn}(\cos(dx+c))}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(12*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a^2*sgn(cos(d*x + c))) + sqrt(2)*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(
3/2)*a^8 + 6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^9)/(a^12*sgn(cos(d*x + c
))))/d
```

Mupad [B]

time = 1.91, size = 69, normalized size = 0.88

$$\frac{\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)}{a^2}+\frac{2}{3\,a}}{d\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] ((2*(a + a/cos(c + d*x)))/a^2 + 2/(3*a))/(d*(a + a/cos(c + d*x))^(3/2)) - (
2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(5/2)*d)
```

3.198 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{1}{5d(a+a\sec(c+dx))^{5/2}}-\frac{}{2ad(a+a\,\mathrm{se}}$$

[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-1/5/d/(a+a*sec(d*x+c))^(5/2)-1/2/a/d/(a+a*sec(d*x+c))^(3/2)-1/8*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-7/4/a^2/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3965, 87, 157, 162, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{7}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{1}{2ad(a\sec(c+dx)+a)^{3/2}}-\frac{1}{5d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) - 1/(5*d*(a + a*
Sec[c + d*x])^(5/2)) - 1/(2*a*d*(a + a*Sec[c + d*x])^(3/2)) - 7/(4*a^2*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-6a^4+9}{x(-a+ax)(a}\right.}{}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+}}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+}}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{}{5d(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 60, normalized size = 0.42

$$\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1+\sec(c+dx))\right) - 2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1+\sec(c+dx)\right)}{5d(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]])/(5*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(115) = 230.
time = 0.17, size = 496, normalized size = 3.44

method	result

default	$\frac{(-1+\cos(dx+c))^3\left(40(\cos^3(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sqrt{2}+5(\cos^3(dx+c))\sqrt{-\frac{2\cos(}{1+\cos}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40/d*(-1+cos(d*x+c))^3*(40*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+5*cos(
d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))+120*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)+15*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*c
os(d*x+c)^2+120*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+98*cos(d*x+c)^3+15*co
s(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))+40*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+160*cos(d*x+c)^2+5*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
+70*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^6/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(115) = 230$.
time = 2.99, size = 573, normalized size = 3.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 40*(cos(d*x + c)^3 +
```

```
 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4
*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)) - 8*a*cos(d*x + c) - a) - 4*(49*cos(d*x + c)^3 + 80*cos(d*x + c)^2
+ 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/40*(5*sq
rt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
a*cos(d*x + c) + a)) - 40*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(49*cos(d*x + c)^3 + 80*cos(d*x +
c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (c+dx)}{(a (\sec (c+dx)+1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.03, size = 206, normalized size = 1.43

$$\frac{\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}-\frac{80\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}-\frac{\sqrt{2}\left(\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{20}+5\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^{21}+35\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{22}\right)}{a^{25}\mathrm{sgn}(\cos(dx+c))}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/40*(5*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a^2*sgn(cos(d*x + c))) - 80*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - sqrt(2)*((a*tan(1/
2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^20 + 5*(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^21 + 35*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a)*a^22)/(a^25*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cot}(c+d\,x)}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

3.199 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}-\frac{5a}{28d(a+a\sec(c+dx))^{7/2}}+\frac{}{2d(1-}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-5/28*a/d/(a+a*sec(d*x+
c))^(7/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+3/40/d/(a+a*sec(d*x
+c))^(5/2)+19/48/a/d/(a+a*sec(d*x+c))^(3/2)+13/64*arctanh(1/2*(a+a*sec(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)+51/32/a^2/d/(a+a*sec(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3965, 105, 157, 162, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}+\frac{51}{32a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5a}{28d(a\sec(c+dx)+a)^{7/2}}+\frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}+\frac{3}{40d(a\sec(c+dx)+a)^{5/2}}+\frac{19}{48ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + (13*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)*d) - (5*a)/(
28*d*(a + a*Sec[c + d*x])^(7/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(7/2)) + 3/(40*d*(a + a*Sec[c + d*x])^(5/2)) + 19/(48*a*d*(a + a*Sec
[c + d*x])^(3/2)) + 51/(32*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{9a^2x}{2}}{x(-a+ax)(a+ax)^{9/2}}\,dx, x, \right.}{2d}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{\text{Su}}{}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40}$$

$$= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{}{40}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{13\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d} - \frac{}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 90, normalized size = 0.45

$$\frac{a\left(-14-13\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))\right)}{28d(-1+\sec(c+dx))(a(1+\sec(c+dx)))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
[Out] (a*(-14 - 13*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(28*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(163) = 326$.
time = 0.24, size = 744, normalized size = 3.72

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(1+\cos(dx+c))(-1+\cos(dx+c))^4\left(6720\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^5(\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6720/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c)
)^4*(6720*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^5*2^(1/2)+20160*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
cos(d*x+c)^4*2^(1/2)+1365*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^5+13440*cos(d*x+c)^3*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*2^(1/2))*2^(1/2)+4095*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^4-13440*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(
d*x+c)^2*2^(1/2)+2730*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arc
tan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+16034*cos(d*x+c)^5-20160*cos(d*
x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-2730*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+25280*cos(d*x+c)
^4-6720*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-4095*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-3164*cos(d*x+c)
^3-1365*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))-24080*cos(d*x+c)^2-10710*cos(d*x+c))/sin(d*x+c)^10/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(161) = 322$.
time = 3.32, size = 748, normalized size = 3.74

$$\left[\frac{1365\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3a\cos(dx+c)+a}{\cos(dx+c)-1}\right)+6720\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\sqrt{a}\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\cos(dx+c)-a\right)+4\left(8017\cos(dx+c)^5+12640\cos(dx+c)^4-1582\cos(dx+c)^3-12040\cos(dx+c)^2-5355\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{13440\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+2a^3d\cos(dx+c)^3-2a^3d\cos(dx+c)^2-3a^3d\cos(dx+c)-a^3d\right)}, -\frac{1365\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-6720\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+a}\right)-2\left(8017\cos(dx+c)^5+12640\cos(dx+c)^4-1582\cos(dx+c)^3-12040\cos(dx+c)^2-5355\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{6720\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+2a^3d\cos(dx+c)^3-2a^3d\cos(dx+c)^2-3a^3d\cos(dx+c)-a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/13440*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)
/(cos(d*x + c) - 1)) + 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x
+ c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x +
c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(8017*cos(d*x + c)^5 + 12640*co
s(d*x + c)^4 - 1582*cos(d*x + c)^3 - 12040*cos(d*x + c)^2 - 5355*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*
d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*
d*cos(d*x + c) - a^3*d), -1/6720*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x
+ c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(a*cos(d*x + c) + a)) - 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(
d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
+ a)) - 2*(8017*cos(d*x + c)^5 + 12640*cos(d*x + c)^4 - 1582*cos(d*x + c)^3
 - 12040*cos(d*x + c)^2 - 5355*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x
 + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.18, size = 295, normalized size = 1.48

$$-\frac{\frac{1365\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}-\frac{13440\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}+\frac{105\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{2\sqrt{2}\left(15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{36}-84\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{37}-385\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^{38}-2730\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^{39}\right)}{a^{42}\mathrm{sgn}(\cos(dx+c))}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6720*(1365*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/
(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 13440*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) + 105*sqrt(
2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(cos(d*x + c))*tan(1/2*d*x +
 1/2*c)^2) + 2*sqrt(2)*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a^36 - 84*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*a^37 - 385*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a
^38 - 2730*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^39)/(a^42*sgn(cos(d*x + c)
)))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

3.200 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{263\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d}+\frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}}-\frac{}{4d(1-}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+199/288*a^2/d/(a+a*sec(
d*x+c))^(9/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(9/2)-21/16*a^2/d
/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(9/2)+135/448*a/d/(a+a*sec(d*x+c))^(7/2)+7
/640/d/(a+a*sec(d*x+c))^(5/2)-83/256/a/d/(a+a*sec(d*x+c))^(3/2)-263/1024*ar
ctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-761/512
/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{263\tanh^{-1}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d}+\frac{199a^2}{288d(a\sec(c+dx)+a)^{9/2}}-\frac{21a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}-\frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}}-\frac{761}{512a^2d\sqrt{a\sec(c+dx)+a}}+\frac{135a}{448d(a\sec(c+dx)+a)^{7/2}}+\frac{7}{640d(a\sec(c+dx)+a)^{5/2}}-\frac{83}{256ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (263*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(512*Sqrt[2]*a^(5/2)*d) + (199*a
^2)/(288*d*(a + a*Sec[c + d*x])^(9/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(9/2)) - (21*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(9/2)) + (135*a)/(448*d*(a + a*Sec[c + d*x])^(7/2)) + 7/(640*d*(a + a
*Sec[c + d*x])^(5/2)) - 83/(256*a*d*(a + a*Sec[c + d*x])^(3/2)) - 761/(512*
a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{11/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{13a^2x}{2}}{x(-a+ax)^2(a+ax)^{11/2}}\right.}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{}{16}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{263\tanh^{-1}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d} + \frac{}{28}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.33, size = 99, normalized size = 0.38

$$\frac{\cot^4(c+dx)\left(-450+263\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2-64\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};1+\sec(c+dx)\right)(-1+\sec(c+dx))^2+378\sec(c+dx)\right)}{288d(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-450 + 263*Hypergeometric2F1[-9/2, 1, -7/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-9/2, 1, -7/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 378*Sec[c + d*x]))/(288*d*(a*(1 + Sec[c
+ d*x]))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(217) = 434$.
time = 0.23, size = 986, normalized size = 3.76

method	result	size
default	Expression too large to display	986

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/322560/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x
+c))^5*(322560*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^7+82845*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d
*x+c)^7+967680*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^6*2^(1/2)+248535*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(
d*x+c)^6+322560*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^5*2^(1/2)+82845*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(
d*x+c)^5-1612800*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^4*2^(1/2)+764402*cos(d*x+c)^
7-414225*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2))*cos(d*x+c)^4-1612800*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1
/2)+1183040*cos(d*x+c)^6-414225*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+322560*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))*cos(d*x+c)^2*2^(1/2)-807214*cos(d*x+c)^5+82845*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+9
67680*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-2224080*cos(d*x+c)^4+248535*cos
```

(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))+322560*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-378378*cos(d*x+c)^3+8284
5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2))+1063440*cos(d*x+c)^2+479430*cos(d*x+c))/sin(d*x+c)^14/a^3

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(213) = 426.
time = 3.21, size = 905, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/645120*(82845*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^
5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 322560*(cos(d*x
+ c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x
+ c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(382201*cos(d*x + c)^7 + 591520*c
os(d*x + c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 189189*cos
(d*x + c)^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a
^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/322560*(82845*sqrt(2)*
(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*
cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
+ c) + a)) - 322560*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 -
5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(2*a*cos(d*x + c) + a)) - 2*(382201*cos(d*x + c)^7 + 591520*cos(d*x +
c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 189189*cos(d*x + c)

```
^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(
d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.35, size = 364, normalized size = 1.39

$$\frac{\frac{82845\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{645120\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{315\left(33\sqrt{2}\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}-31\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a\right)}{a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}}-\frac{8\sqrt{2}\left(35\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{4}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{56}-225\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{57}+1008\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{58}+4410\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}a^{59}+31185\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,a^{60}\right)}{a^{63}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/322560*(82845*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a)
)/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 645120*arctan(1/2*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 315*(3
3*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 31*sqrt(2)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*a)/(a^4*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4) - 8
*sqrt(2)*(35*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)*a^56 - 225*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)*a^57 + 1008*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)*a^58 + 4410*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^59 +
31185*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^60)/(a^63*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cot}\left(c+d\,x\right)}^5}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

3.201 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}-\frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}}+\frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+2*tan(d*x+c)
/a^2/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/a/d/(a+a*sec(d*x+c))^(3/2)+2
/5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 308, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\tan^3(c+dx)}{3ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*a
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx)}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3}{3ad(a+a\,\text{s}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 447 vs. 2(127) = 254.
time = 6.07, size = 447, normalized size = 3.52

$$\frac{\sqrt{2}\cot^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{9/2}\sqrt{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(-1+\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^3\left(\frac{\sqrt{2}\,\text{ArcSin}\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}\right)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{1-\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}}+\frac{8\tan^6\left(\frac{1}{2}(c+dx)\right)}{5\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-1+\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^3}+\frac{4\tan^4\left(\frac{1}{2}(c+dx)\right)}{3\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1+\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^2}+\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(-1+\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)}\right)\tan^7(c+dx)}{d(a(1+\sec(c+dx)))^{5/2}\left(1-\frac{2\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*Cot[(c + d*x)/2]^8*((1 + Sec[c + d*x])^(-1))^(9/2)*Sqrt[1 + Tan[(c
 + d*x)/2]^2]*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3*((Sq
rt[2]*ArcSin[(Sqrt[2]*Tan[(c + d*x)/2])/Sqrt[1 + Tan[(c + d*x)/2]^2]]*Tan[(
c + d*x)/2])/(Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 - (2*Tan[(c + d*x)/2]^2)/
(1 + Tan[(c + d*x)/2]^2)]) + (8*Tan[(c + d*x)/2]^6)/(5*(1 + Tan[(c + d*x)/2
]^2)^3*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3) + (4*Tan[(
c + d*x)/2]^4)/(3*(1 + Tan[(c + d*x)/2]^2)^2*(-1 + (2*Tan[(c + d*x)/2]^2)/(
1 + Tan[(c + d*x)/2]^2))^2) + (2*Tan[(c + d*x)/2]^2)/((1 + Tan[(c + d*x)/2]
^2)*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))))*Tan[c + d*x]^7
)/(d*(a*(1 + Sec[c + d*x]))^(5/2)*(1 - (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(111) = 222$.
time = 0.17, size = 302, normalized size = 2.38

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\left(\cos^2(dx+c)\right)\sin(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}+30\cos(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*cos(d*x+c)^2*sin(d*x+c)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+30*cos(d*x+c)*sin(d*x+c)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+15*2^(1/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-184*cos(d*x+c)^3+272*cos(d*x+c)^2-112*co
s(d*x+c)+24)/sin(d*x+c)/cos(d*x+c)^2/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/30*(20*(9*sin(4*d*x + 4*c) + 16*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(45*cos(4*d*x + 4*c) + 80*cos(2*d*x
+ 2*c) + 23)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 15*
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1) + 2*(a^3*d*cos(2*d*x + 2*c)^2 + a^3*d*sin(2*d*x + 2*c)^2 + 2*a^3*
d*cos(2*d*x + 2*c) + a^3*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*
```

```
c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(c
os(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2
*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*c
os(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(cos
(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^
2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x
+ 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*
d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(sin(2*
d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*(sin(2*d*x +
 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 +
(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x +
 4*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - ((sin(2*d*x +
 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*c
os(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^
3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(10*
d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (co
s(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*d*x + 6*
c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)
*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(14*d*x +
 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(cos(2*d*x + 2
*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*
d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(8*d*x + 8
*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*d*x + 2*c)^3
 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
 2*c))*sin(4*d*x + 4*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (((sin(2*d*x
 + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))
*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c
)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(1
```

```
0*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (
cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*d*x +
6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*
c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(14*d*x
 + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + ...
```

Fricas [A]
time = 2.67, size = 323, normalized size = 2.54

$$\left[-\frac{15\,\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)-2\,\left(23\,\cos\left(dx+c\right)^{2}-11\,\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+a^{3}d\cos\left(dx+c\right)^{2}\right)},\ \frac{2\,\left(15\,\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)}{\sqrt{a}\,\sin\left(dx+c\right)}\right)+\left(23\,\cos\left(dx+c\right)^{2}-11\,\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+a^{3}d\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(23*cos(d*x + c)^2 - 1
1*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2), 2/15*(15*(cos(d*x + c)^3 + co
s(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c))) + (23*cos(d*x + c)^2 - 11*cos(d*x + c) + 3)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + a^3*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(111) = 222.
time = 4.18, size = 257, normalized size = 2.02

$$\frac{15\,\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)}{a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{\log\left(\left|\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+a\left(2\,\sqrt{2}-3\right)\right|\right)}{a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)+\frac{2\,\left(\left(\frac{37\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{40\,\sqrt{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{15\,\sqrt{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^3*sgn(cos(d*x + c))) - log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 + a*(2*sqrt(2) - 3)))/(a^3*sgn(cos(d*x + c)))) + 2*((37*sqrt(2)*tan(1/2*d
*x + 1/2*c)^2/sgn(cos(d*x + c)) - 40*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x
 + 1/2*c)^2 + 15*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/
2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

3.202 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-4*arctan(1/2*
a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+2*tan(
d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3972, 490, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
4*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(a^(5/2)*d) + (2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan(c+dx)}{a^2 d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d}$$

$$= \frac{2\tan(c+dx)}{a^2 d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d} -$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 23.72, size = 5491, normalized size = 48.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
[Out] Result too large to show
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(96) = 192.
time = 0.16, size = 327, normalized size = 2.89

method	result
default	$\frac{\left(-\cos(dx+c)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-4\cos(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\ln\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-4*cos(d*
x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-2^(1/2)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+2*sin(d*x
+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1+cos(d*x+c))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 3.29, size = 414, normalized size = 3.66

$$\left[\frac{2\sqrt{2}\left(a\cos\left(dx+c\right)+a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{a^3d\cos\left(dx+c\right)+a^3d},-\frac{2\left(\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}-\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{a^3d\cos\left(dx+c\right)+a^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x +
c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a
)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos
```

```
(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c) + a^3*d), -2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) - sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 2.07, size = 66, normalized size = 0.58

$$-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a^2d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*tan
(1/2*d*x + 1/2*c)^2 - a)*a^2*d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

3.203 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{3\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+3/2*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/2
*sec(1/2*d*x+1/2*c)^2*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3972, 482, 536, 209}

$$-\frac{2\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{3\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt
[2]*a^(5/2)*d) + (Sec[(c + d*x)/2]^2*Sin[c + d*x])/(2*a^2*d*Sqrt[a + a*Sec[
c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c}}\right.}{a^2d}$$

$$= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{a^2d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 23.97, size = 5521, normalized size = 43.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(107) = 214$.
time = 0.12, size = 370, normalized size = 2.91

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\cos(dx+c)+3\sin(dx-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+3*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)

Fricas [A]
time = 2.50, size = 492, normalized size = 3.87

$$\left[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{4\left(a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+a^3d\right)}, -\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-4\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\left(a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*

```
x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*co
s(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + a^3*d), -1/2*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 4*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [**A**]
time = 1.56, size = 47, normalized size = 0.37

$$\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,a^{3}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^3*d
*sgn(cos(d*x + c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

3.204 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=265

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{319\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d}+\frac{63\cot(c+dx)\sqrt{a+a\sec(c+}{128a^3d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+319/256*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
+63/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d-191/384*cos(d*x+c)*cot(d*x+
c)*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^3/d-19/192*cos(d*x+c)^2*co
t(d*x+c)*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^3/d-1/48*cos(d*x+c)^
3*cot(d*x+c)*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{319\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}\,a^{5/2}d}+\frac{63\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128a^3d}-\frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{192a^3d}-\frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{384a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(319*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(12
8*Sqrt[2]*a^(5/2)*d) + (63*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*a^3*
d) - (191*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d
*x]])/(384*a^3*d) - (19*Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt
[a + a*Sec[c + d*x]])/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]*Sec[(c + d
*x)/2]^6*Sqrt[a + a*Sec[c + d*x]])/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} - \frac{\text{Subst}\left(\int \ldots\right.}{}$$

$$= -\frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+\ldots}{}$$

$$= -\frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} - \frac{19\cos^2(\ldots}{}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+\ldots\right.}{384a^3d}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+\ldots\right.}{384a^3d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{319\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.07, size = 5604, normalized size = 21.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(229) = 458.
time = 0.21, size = 714, normalized size = 2.69

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^3\left(-768\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+}{2\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^3*(-768*cos(d*x
+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-957*cos(
d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-2304*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-2871*sin(
d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2304*sin(d*x+
c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+818*cos(d*x
+c)^4-2871*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-
768*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+698*cos(d*
x+c)^3-957*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-370*cos(d*x
+c)^2-378*cos(d*x+c))/sin(d*x+c)^7/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 4.00, size = 691, normalized size = 2.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1536*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 768*(cos(d*x + c)^3 + 3
*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4
*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x +
 c) + 4*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 185*cos(d*x + c)^2 - 189
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)),
 -1/768*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 768*(cos(d*x + c)^3 + 3*cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*
x + c) - a))*sin(d*x + c) + 2*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 18
5*cos(d*x + c)^2 - 189*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
))/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.18, size = 177, normalized size = 0.67

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{31\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{291\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{96\sqrt{2}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/768*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2*
c)^2/(a^3*sgn(cos(d*x + c))) - 31*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*
d*x + 1/2*c)^2 + 291*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)
- 96*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a)*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

3.205 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=355

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{9683\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{2}\ \sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}\ a^{5/2}d}-\frac{1491\cot(c+dx)\sqrt{a+a\sec(c}{4096a^3d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+5587/6144*cot
(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d-1527/2048*cos(d*x+c)*cot(d*x+c)^3*se
c(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^4/d-145/1024*cos(d*x+c)^2*cot(d
*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^4/d-9/256*cos(d*x+c)^
3*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^4/d-1/128*cos(
d*x+c)^4*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(3/2)/a^4/d-968
3/8192*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)-1491/4096*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{9683\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{2}\ \sqrt{a\sec(c+dx)+a}}\right)}{4096\sqrt{2}\ a^{5/2}d}+\frac{5587\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{6144a^4d}-\frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{128a^4d}-\frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{256a^4d}-\frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{1024a^4d}-\frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{2048a^4d}-\frac{1491\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4096a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
9683*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(40
96*Sqrt[2]*a^(5/2)*d) - (1491*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4096*
a^3*d) + (5587*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(6144*a^4*d) - (1
527*Cos[c + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/
2))/(2048*a^4*d) - (145*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a
 + a*Sec[c + d*x])^(3/2))/(1024*a^4*d) - (9*Cos[c + d*x]^3*Cot[c + d*x]^3*S
ec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(3/2))/(256*a^4*d) - (Cos[c + d*x]^4
*Cot[c + d*x]^3*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(3/2))/(128*a^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4 d}$$

$$= -\frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4 d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4 d} - \frac{\cos^4(c}{}$$

$$= -\frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4 d} - \frac{9\cos}{}$$

$$= -\frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4 d} - \frac{145}{}$$

$$= \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4 d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\right.}{2048}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3 d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx)}{6144a^4 d}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3 d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx)}{6144a^4 d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{9683\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}\,a^{5/2}d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.24, size = 5646, normalized size = 15.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. 2(310) = 620.
time = 0.30, size = 1066, normalized size = 3.00

method	result	size
default	Expression too large to display	1066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(d*x+c)
)^4*(24576*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*2^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))+29049*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*sin(d*x+c
)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))+73728*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))*2^(1/2)+87147*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x
+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))+49152*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(
1/2))*2^(1/2)+58098*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))-49152*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*2^(1/2)-29258*cos(d*x+c)^6-58098*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-73728*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)-28466*cos(d*x+c)^5-87147*sin(d*x+c)*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-24576*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+28116*cos(d*x+c)^4-29049*ln(-(-(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+34852*cos(d*x+c)^3-4490*cos(d*x
+c)^2-8946*cos(d*x+c))/sin(d*x+c)^11/a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.70, size = 868, normalized size = 2.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/49152*(29049*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c
)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a
*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c
)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c
)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*si
n(d*x + c) - 4*(14629*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(d*x
 + c)^4 - 17426*cos(d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*co
s(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*co
s(d*x + c) - a^3*d)*sin(d*x + c)), 1/24576*(29049*sqrt(2)*(cos(d*x + c)^5 +
 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) -
1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*
x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a
)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(1462
9*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(d*x + c)^4 - 17426*cos(
d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3
*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)*
sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.48, size = 296, normalized size = 0.83

$$\frac{3\left(2\left(4\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{19\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{369\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{2989\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{512\sqrt{2}\left(12\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-21\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+11a^2\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^3\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}}{24576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*(3*(2*(4*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c)))
- 19*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 369*sqrt(2)/
(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 2989*sqrt(2)/(a^3*sgn(cos
(d*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 512
*sqrt(2)*(12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 21*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*a + 11*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a)^3*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

3.206 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=439

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{74461\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}\,a^{5/2}d}+\frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-41693/49152*
cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d+58077/40960*cot(d*x+c)^5*(a+a*sec
(d*x+c))^(5/2)/a^5/d-9467/8192*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2
*(a+a*sec(d*x+c))^(5/2)/a^5/d-2473/12288*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*
d*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^5/d-155/3072*cos(d*x+c)^3*cot(d*x+c)^
5*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^5/d-7/512*cos(d*x+c)^4*cot(
d*x+c)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^5/d-1/320*cos(d*x+c)
^5*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^10*(a+a*sec(d*x+c))^(5/2)/a^5/d+74461/65
536*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d
*2^(1/2)+8925/32768*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.28, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{74461\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32768\sqrt{2}\,a^{5/2}d}+\frac{58077\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{40960a^5d}-\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{320a^5d}-\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{512a^5d}-\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{3072a^5d}-\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{12288a^5d}-\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{8192a^5d}-\frac{41693\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{49152a^4d}+\frac{8925\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32768a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(74461*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
32768*Sqrt[2]*a^(5/2)*d) + (8925*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32
768*a^3*d) - (41693*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(49152*a^4*d
) + (58077*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(40960*a^5*d) - (9467
*Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))
/(8192*a^5*d) - (2473*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a +
 a*Sec[c + d*x])^(5/2))/(12288*a^5*d) - (155*Cos[c + d*x]^3*Cot[c + d*x]^5*
Sec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(3072*a^5*d) - (7*Cos[c + d*
x]^4*Cot[c + d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(512*a^5
*d) - (Cos[c + d*x]^5*Cot[c + d*x]^5*Sec[(c + d*x)/2]^10*(a + a*Sec[c + d*x
])^(5/2))/(320*a^5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

`tegerQ[n - 1/2]`

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^6}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^5 d}$$

$$= -\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5 d} - \frac{\text{Subst}\Big(}{}$$

$$= -\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5 d} - \frac{\cos^5(c}{}$$

$$= -\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5 d} - \frac{7\cos}{}$$

$$= -\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5 d} - \frac{155}{}$$

$$= -\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5 d} - \frac{2473}{}$$

$$= \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5 d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2}{819}$$

$$= -\frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4 d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+}{40960a^5 d}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3 d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx)}{49152a^4 d}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3 d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx)}{49152a^4 d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{74461\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 24.13, size = 5688, normalized size = 12.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. 2(385) = 770.
time = 0.27, size = 1412, normalized size = 3.22

method	result	size
default	Expression too large to display	1412

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/983040/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*
x+c))^5*(983040*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c
)^7*sin(d*x+c)+2949120*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*2^(1/2)+1116915*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos
(d*x+c)^7*sin(d*x+c)+983040*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
^5*sin(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))+3350745*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+
cos(d*x+c)-1)/sin(d*x+c))-4915200*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)^4*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+1116915*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^5*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))-1278126*cos(d*x+c)^8-4915200*cos(d*x+c)^3*s
in(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-5584575*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-1363110*cos(d*x+
c)^7+983040*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*2^(1/2)-5584575*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))+1972170*cos(d*x+c)^6+2949120*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
```

```
(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+1116915*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2720050*cos(d*x+c)^5+983040*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+3350745*sin(d*x+c)*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)-810890*cos(d*x+c)^4+1116915*l
n(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-1673842*cos(d*x+c)^3-30
610*cos(d*x+c)^2+267750*cos(d*x+c))/sin(d*x+c)^15/a^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.29, size = 1023, normalized size = 2.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1966080*(1116915*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x +
 c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x +
c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 983040*(cos(d*x + c
)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)
^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d
*x + c) + 4*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 986085*cos(d*x
 + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 836921*cos(d*x +
 c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*c
os(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos
(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), -1/983040*(11169
15*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x
+ c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*ar
```

```
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c)))*sin(d*x + c) + 983040*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + c
os(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*co
s(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*
sin(d*x + c) + 2*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 986085*co
s(d*x + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 836921*cos(
d*x + c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^
3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.64, size = 412, normalized size = 0.94

$$\frac{\left(2\left(4\left(6\left(\frac{8\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{91\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3043\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{47185\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{349965\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{1024\sqrt{2}\left(345\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^8-1230\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6a+1760\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4a^2-1150\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a^3+299a^4\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)^5\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}}{983040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/983040*((2*(4*(6*(8*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c))
) - 91*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 3043*sqrt(
2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 47185*sqrt(2)/(a^3*sgn
(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 349965*sqrt(2)/(a^3*sgn(cos(d*x +
 c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) - 1024*sqrt
(2)*(345*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^8 - 1230*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*a + 1760*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4*a^2 - 1150*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*a^3 + 299*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*a*sgn(cos(d*x + c))))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

3.207 $\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2}f}+\frac{91\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}\,a^{9/2}f}+\frac{\tan(e+fx)}{3af(a+a\sec(e+fx))^{7/2}}+$$

```
[Out] -2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f+91/64*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f*2^(1/2)+1
/3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(7/2)+11/24*tan(f*x+e)/a^2/f/(a+a*sec(f*
x+e))^(5/2)+27/32*tan(f*x+e)/a^3/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.14, antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 482, 541, 536, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}f}+\frac{91\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}\,a^{9/2}f}+\frac{27\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{64a^4f\sqrt{a\sec(e+fx)+a}}+\frac{\sin(e+fx)\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)}{24a^4f\sqrt{a\sec(e+fx)+a}}+\frac{11\sin(e+fx)\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{96a^4f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(9/2)*f) +
(91*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32*
Sqrt[2]*a^(9/2)*f) + (27*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(64*a^4*f*Sqrt[a
+ a*Sec[e + f*x]]) + (11*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(96*
a^4*f*Sqrt[a + a*Sec[e + f*x]]) + (Cos[e + f*x]^2*Sec[(e + f*x)/2]^6*Sin[e
+ f*x])/(24*a^4*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = -\frac{2\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3 f}$$

$$= \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1-5ax^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\right.}{3a^4 f}$$

$$= \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)}{24a^4 f\sqrt{a+a\sec(e+}}$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+f}{96a^4 f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+f}{96a^4 f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2}f} + \frac{91\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}\,a^{9/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.18, size = 5584, normalized size = 31.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(148) = 296.
time = 0.95, size = 724, normalized size = 4.09

method	result

default	$-\frac{\left(192\sin(fx+e)\left(\cos^4(fx+e)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+273\sin(fx+e)\left(\cos^4(f\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/f*(192*sin(f*x+e)*cos(f*x+e)^4*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)+273*sin(f*x+e)*cos(f*x+e)^4*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)+384*sin(f*x+e)*cos(f*x+e)^3*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)+546*sin(f*x+e)*cos(f*x+e)^3*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-384*sin(f*x+e)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-314*cos(f*x+e)^5-546*sin(f*x+e)*cos(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)-192*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
+216*cos(f*x+e)^4-273*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+34
8*cos(f*x+e)^3-88*cos(f*x+e)^2-162*cos(f*x+e))*(a*(cos(f*x+e)+1)/cos(f*x+e)
)^(1/2)/sin(f*x+e)^3/(cos(f*x+e)+1)^2/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sec(f*x + e) + a)^(9/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(158) = 316$.
time = 3.46, size = 733, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(273*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2
+ 4*cos(f*x + e) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*co
s(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 384*(cos(f*x + e)^
4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(157*
cos(f*x + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x +
 e)^3 + 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f), -1/192*(273
*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x
+ e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 384*(cos(f*x + e)^4 + 4*cos(f*x + e)^3
 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(157*cos(f*x
 + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x + e)^3 +
 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sec(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**A**]
time = 1.38, size = 109, normalized size = 0.62

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\left(2\left(\frac{4\,\sqrt{2}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^5\mathrm{sgn}(\cos(fx+e))}-\frac{19\,\sqrt{2}}{a^5\mathrm{sgn}(\cos(fx+e))}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\frac{111\,\sqrt{2}}{a^5\mathrm{sgn}(\cos(fx+e))}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*(2*(4*sqrt(2)*tan(1/2*f*x + 1/2*e
)^2/(a^5*sgn(cos(f*x + e))) - 19*sqrt(2)/(a^5*sgn(cos(f*x + e))))*tan(1/2*f
*x + 1/2*e)^2 + 111*sqrt(2)/(a^5*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2),x)
[Out] int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2), x)
```

3.208 $\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=125

$$\frac{2^{1+m+n}F_1\left(\frac{1+m}{2};m+n,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{1+m+n}(a+a\sec(c+dx))^n(e\tan(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] 2^(1+m+n)*AppellF1(1/2+1/2*m,n+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d*x
+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1+m+n)*(a+a*sec
(d*x+c))^n*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3974}

$$\frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}F_1\left(\frac{m+1}{2};m+n,1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(1 + m + n)*AppellF1[(1 + m)/2, m + n, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(1 + m + n)*(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \frac{2^{1+m+n}F_1\left(\frac{1+m}{2};m+n,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{1+m+n}(a+a\sec(c+dx))^n(e\tan(c+dx))^{1+m}}{de(1+m)}$$

Mathematica [F]
time = 1.27, size = 0, normalized size = 0.00

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a+a\sec \left(dx+c\right)\right)^{n} \left(e\tan \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec \left(c+dx\right)+1\right)\right)^{n} \left(e\tan \left(c+dx\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*(e*tan(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

3.209 $\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=243

$$\frac{3a^3(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1-}{}\right.}{}$$

```
[Out] 3*a^3*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*
m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+3*a^3*(cos(d*x+c)^2)^(1+1/
2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e
*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*(cos(d*x+c)^2)^(2+1/2*m)*hypergeom([2+1/2*
m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*tan(d*x+c))^(1+m)/d
/e/(1+m)
```

Rubi [A]
time = 0.17, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3971, 3557, 371, 2697, 2687, 32}

$$\frac{a^3(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+4}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{3a^3\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+2}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{3a^3(e\tan(c+dx))^{m+1}}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (3*a^3*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 +
m)) + (3*a^3*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 +
 m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d
*e*(1 + m)) + (a^3*(Cos[c + d*x]^2)^((4 + m)/2)*Hypergeometric2F1[(1 + m)/2
, (4 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(e*Tan[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx &= \int \left(a^3(e\tan(c+dx))^m+3a^3\sec(c+dx)(e\tan(c+dx))^m+3a\right. \\ &= a^3\int (e\tan(c+dx))^m\,dx+a^3\int \sec^3(c+dx)(e\tan(c+dx))^m \\ &= \frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+}{de(1+m)} \\ &= \frac{3a^3(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)}{de(1+m)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.26, size = 391, normalized size = 1.61

$$\frac{a^3e\left(9\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\sec^2(c+dx)\right)+{}_2F_1\left(\frac{3}{2},\frac{1-m}{2};\frac{5}{2};\sec^2(c+dx)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3(e\tan(c+dx))^{-1+m}\left(-\tan^2(c+dx)\right)^{\frac{1-m}{2}}}{24d}+\frac{2^{-4-m}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tan^{-m}(c+dx)(e\tan(c+dx))^m\left(i2^m\left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^m(1+m)\cos(c+dx)\,{}_2F_1\left(1,m;1+m;-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)-i\left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m(1+m)\cos(c+dx)\,{}_2F_1\left(m,m;1+m;\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+3\,2^{1+m}m\sin(c+dx)\tan^m(c+dx)\right)}{dm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^3*e*(9*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x
]^2] + Hypergeometric2F1[3/2, (1 - m)/2, 5/2, Sec[c + d*x]^2])*Sec[(c + d*x
)/2]^6*(1 + Sec[c + d*x])^3*(e*Tan[c + d*x])^(-1 + m)*(-Tan[c + d*x]^2)^((1
 - m)/2))/(24*d) + (2^(-4 - m)*a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*
Sec[c + d*x]*(e*Tan[c + d*x])^m*(I*2^m*(((-I)*(-1 + E^((2*I)*(c + d*x))))/(
1 + E^((2*I)*(c + d*x))))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[1, m, 1
+ m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] - I*(((-I)*(-
1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d
*x)))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(
c + d*x)))/2] + 3*2^(1 + m)*m*Sin[c + d*x]*Tan[c + d*x]^m))/(d*m*(1 + m)*Ta
n[c + d*x]^m)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a + a\sec\left(dx + c\right)\right)^{3} \left(e\tan\left(dx + c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*tan(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int (e\tan(c+dx))^m\,dx+\int 3(e\tan(c+dx))^m\sec(c+dx)\,dx+\int 3(e\tan(c+dx))^m\sec^2(c+dx)\,dx+\int (e\tan(c+dx))^m\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(e*tan(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*tan(c + d*x))**m, x) + Integral(3*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*tan(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*tan(c + d*x))**m*sec(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

3.210 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=161

$$\frac{a^2(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{2a^2\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2}\right.}{}$$

```
[Out] a^2*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m]
,-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+2*a^2*(cos(d*x+c)^2)^(1+1/2*
m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*t
an(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {3971, 3557, 371, 2697, 2687, 32}

$$\frac{a^2(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+2}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^2(e\tan(c+dx))^{m+1}}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^2*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^2*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)
) + (2*a^2*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 + m
)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d*e
*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx=\int \left(a^2(e\tan(c+dx))^m+2a^2\sec(c+dx)(e\tan(c+dx))^m+a^2\right.$$

$$=a^2\int (e\tan(c+dx))^m\,dx+a^2\int \sec^2(c+dx)(e\tan(c+dx))^m$$

$$=\frac{2a^2\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+}{de(1+m)}$$

$$=\frac{a^2(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e}{de(1+m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.37, size = 358, normalized size = 2.22

$$\frac{a^2(1+\cos(c+dx))^2\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\sec^2(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(e\tan(c+dx))^m\left(-\tan^2(c+dx)\right)^{\frac{1-m}{2}}}{2d}+\frac{2^{-3-m}a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tan^{-m}(c+dx)(e\tan(c+dx))^m\left(i2^m\left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^m(1+m)\cos(c+dx)\,{}_2F_1\left(1,m;1+m;-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)-i\left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m(1+m)\cos(c+dx)\,{}_2F_1\left(m,m;1+m;\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)+2^{1+m}m\sin(c+dx)\tan^m(c+dx)\right)}{dm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Csc[c + d*x]*Hypergeometric2F1[1/2, (1 - m)/2, 3/
2, Sec[c + d*x]^2]*Sec[(c + d*x)/2]^4*(e*Tan[c + d*x])^m*(-Tan[c + d*x]^2)^
((1 - m)/2))/(2*d) + (2^(-3 - m)*a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^
```

```
4*Sec[c + d*x]*(e*Tan[c + d*x])^m*(I*2^m*(((-I)*(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x))))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[1, m,
1 + m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] - I*(((-I)*
(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c +
 d*x)))^m*(1 + m)*Cos[c + d*x]*Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)
*(c + d*x)))/2] + 2^(1 + m)*m*Sin[c + d*x]*Tan[c + d*x]^m))/(d*m*(1 + m)*Ta
n[c + d*x]^m)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(a + a\sec\left(dx + c\right)\right)^{2} \left(e\tan\left(dx + c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*tan(d*x + c))^m
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \left(e\tan\left(c + dx\right)\right)^{m}\,dx + \int 2\left(e\tan\left(c + dx\right)\right)^{m}\sec\left(c + dx\right)dx + \int \left(e\tan\left(c + dx\right)\right)^{m}\sec^{2}\left(c + dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**m,x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**m, x) + Integral(2*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral((e*tan(c + d*x))**m*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

3.211 $\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)}+\frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)}{de(1+m)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/
d/e/(1+m)+a*(cos(d*x+c)^2)^(1+1/2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3969, 3557, 371, 2697}

$$\frac{a(e\tan(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{de(m+1)}+\frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{m+2}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (a*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c
 + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = a\int (e\tan(c+dx))^m\,dx + a\int \sec(c+dx)(e\tan(c+dx))^m\,dx$$
$$= \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{2+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sec(c+dx)}{de(1+m)}$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a\cos^2}{}$$

Mathematica [A]
time = 0.76, size = 105, normalized size = 0.81

$$\frac{a(e\tan(c+dx))^m\left(\frac{{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan(c+dx)}{1+m}+\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\sec^2(c+dx)\right)\left(-\tan^2(c+dx)\right)^{\frac{1-m}{2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
[Out] (a*(e*Tan[c + d*x])^m*((Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c +
 d*x]^2]*Tan[c + d*x])/(1 + m) + Csc[c + d*x]*Hypergeometric2F1[1/2, (1 - m
)/2, 3/2, Sec[c + d*x]^2]*(-Tan[c + d*x]^2)^((1 - m)/2)))/d
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))(e\tan(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\tan(c+dx))^m\,dx + \int (e\tan(c+dx))^m\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**m,x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*sec(c +
d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

3.212 $\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{e\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)}-\frac{e\cos^2(c+dx)^{m/2}\,{}_2F_1\left(\frac{1}{2}(-1+m),\frac{m}{2};\frac{1+m}{2};\right.}{ad(}$$

```
[Out] e*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-1+m
)/a/d/(1-m)-e*(cos(d*x+c)^2)^(1/2*m)*hypergeom([1/2*m, -1/2+1/2*m],[1/2+1/2
*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(-1+m)/a/d/(1-m)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3973, 3969, 3557, 371, 2697}

$$\frac{e(e\tan(c+dx))^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\tan^2(c+dx)\right)}{ad(1-m)}-\frac{e\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,{}_2F_1\left(\frac{m-1}{2},\frac{m}{2};\frac{m+1}{2};\sin^2(c+dx)\right)}{ad(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c +
d*x])^(-1 + m))/(a*d*(1 - m)) - (e*(Cos[c + d*x]^2)^(m/2)*Hypergeometric2F1
[(-1 + m)/2, m/2, (1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^
(-1 + m))/(a*d*(1 - m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx &= \frac{e^2\int(-a+a\sec(c+dx))(e\tan(c+dx))^{-2+m}\,dx}{a^2}\\ &= -\frac{e^2\int(e\tan(c+dx))^{-2+m}\,dx}{a}+\frac{e^2\int\sec(c+dx)(e\tan(c+dx))^{-2+m}\,dx}{a}\\ &= -\frac{e\cos^2(c+dx)^{m/2}\,{}_2F_1\left(\frac{1}{2}(-1+m),\frac{m}{2};\frac{1+m}{2};\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx}{ad(1-m)}\\ &= \frac{e\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)}-\frac{e\cos^2(c+dx)^{m/}}{}\end{aligned}$$

Mathematica [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\tan(c+d\,x)\right)}^m}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

3.213 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=169

$$-\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\tan^2(c+dx)\right)(e\tan(c+dx))^{-3+m}}{a^2d(3-m)}+\frac{2e^3\cos^2}{}$$

```
[Out] -e^3*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)-e^3*hypergeom([1, -3/2+1/2*m],[-1/2+
1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)+2*e^3*(cos(d*x+c)^2
)^(-1+1/2*m)*hypergeom([-1+1/2*m, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*se
c(d*x+c)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)
```

Rubi [A]
time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3973, 3971, 3557, 371, 2697, 2687, 32}

$$-\frac{e^3(e\tan(c+dx))^{m-3}\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\tan^2(c+dx)\right)}{a^2d(3-m)}+\frac{2e^3\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-3}\,{}_2F_1\left(\frac{m-3}{2},\frac{m-2}{2};\frac{m-1}{2};\sin^2(c+dx)\right)}{a^2d(3-m)}-\frac{e^3(e\tan(c+dx))^{m-3}}{a^2d(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^3*(e*Tan[c + d*x])^(-3 + m))/(a^2*d*(3 - m))) - (e^3*Hypergeometric2F1
[1, (-3 + m)/2, (-1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-3 + m))/(a^
2*d*(3 - m)) + (2*e^3*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeometric2F1[(-3 +
 m)/2, (-2 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x]
)^(-3 + m))/(a^2*d*(3 - m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{e^4\int(-a+a\sec(c+dx))^2(e\tan(c+dx))^{-4+m}\,dx}{a^4}$$

$$= \frac{e^4\int\left(a^2(e\tan(c+dx))^{-4+m}-2a^2\sec(c+dx)(e\tan(c+dx))^{-4+m}+a^2\sec^2(c+\right.}{a^4}$$

$$= \frac{e^4\int(e\tan(c+dx))^{-4+m}\,dx}{a^2}+\frac{e^4\int\sec^2(c+dx)(e\tan(c+dx))^{-4+m}\,dx}{a^2}-\frac{(2e^4)}{}$$

$$= \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,{}_2F_1\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)}{a^2d(3-m)}$$

$$= -\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\tan^2(c+dx)\right)(e}{a^2d(3-m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.25, size = 329, normalized size = 1.95

$$\frac{\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^m\tan\left(\frac{1}{2}(c+dx)\right)\left(-3(3+m)\,{}_2F_1\left(m,\frac{1+m}{2};\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1+m)\,{}_2F_1\left(m,\frac{3+m}{2};\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(e\tan(c+dx))^m}{2a^2d(1+m)(3+m)}+\frac{i2^{1-m}\left(-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}\right)^m\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2^m\,{}_2F_1\left(1,m;1+m;-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)-\left(1+e^{2i(c+dx)}\right)^m\,{}_2F_1\left(m,m;1+m;\frac{1}{2}\left(1-e^{2i(c+dx)}\right)\right)\right)\sec^2(c+dx)\tan^{-m}(c+dx)(e\tan(c+dx))^m}{dm(a+a\sec(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(-3*(3 + m)*Hypergeom
etric2F1[m, (1 + m)/2, (3 + m)/2, Tan[(c + d*x)/2]^2] + (1 + m)*Hypergeomet
ric2F1[m, (3 + m)/2, (5 + m)/2, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)*(e*
Tan[c + d*x])^m)/(2*a^2*d*(1 + m)*(3 + m)) + (I*2^(1 - m)*(((-I)*(-1 + E^((
2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c/2 + (d*x)/2]^4*(2^m*Hy
pergeometric2F1[1, m, 1 + m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c
+ d*x))))] - (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1
- E^((2*I)*(c + d*x)))/2])*Sec[c + d*x]^2*(e*Tan[c + d*x])^m)/(d*m*(a + a*S
ec[c + d*x])^2*Tan[c + d*x]^m)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(e\tan\left(dx+c\right)\right)^{m}}{\left(a+a\sec\left(dx+c\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,(e\tan(c+d\,x))^m}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.214 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=252

$$\frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\,{}_2F_1\left(1,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\tan^2(c+dx)\right)(e\tan(c+dx))^{-5+m}}{a^3d(5-m)}-\frac{3e^5\cos^2(}{}$$

```
[Out] 3*e^5*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)+e^5*hypergeom([1, -5/2+1/2*m],[-3/2
+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-3*e^5*(cos(d*x+c)^
2)^(-2+1/2*m)*hypergeom([-2+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*s
ec(d*x+c)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-e^5*(cos(d*x+c)^2)^(-1+1/2*m)*h
ypergeom([-1+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*
tan(d*x+c))^(-5+m)/a^3/d/(5-m)
```

Rubi [A]

time = 0.25, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3973, 3971, 3557, 371, 2697, 2687, 32}

$$\frac{e^5(e\tan(c+dx))^{m-5}\,{}_2F_1\left(1,\frac{m-5}{2};\frac{m-3}{2};-\tan^2(c+dx)\right)}{a^3d(5-m)}-\frac{e^5\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-5}\,{}_2F_1\left(\frac{m-5}{2},\frac{m-2}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)}-\frac{3e^5\sec(c+dx)\cos^2(c+dx)^{\frac{m-4}{2}}(e\tan(c+dx))^{m-5}\,{}_2F_1\left(\frac{m-5}{2},\frac{m-4}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)}+\frac{3e^5(e\tan(c+dx))^{m-5}}{a^3d(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*e^5*(e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Hypergeometric2F1[
1, (-5 + m)/2, (-3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-5 + m))/(a^3
*d*(5 - m)) - (3*e^5*(Cos[c + d*x]^2)^((-4 + m)/2)*Hypergeometric2F1[(-5 +
m)/2, (-4 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])
^(-5 + m))/(a^3*d*(5 - m)) - (e^5*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeomet
ric2F1[(-5 + m)/2, (-2 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(
e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{e^6\int(-a+a\sec(c+dx))^3(e\tan(c+dx))^{-6+m}\,dx}{a^6}$$

$$= \frac{e^6\int\left(-a^3(e\tan(c+dx))^{-6+m}+3a^3\sec(c+dx)(e\tan(c+dx))^{-6+m}-3a^3\sec\right.}{a^6}$$

$$= -\frac{e^6\int(e\tan(c+dx))^{-6+m}\,dx}{a^3}+\frac{e^6\int\sec^3(c+dx)(e\tan(c+dx))^{-6+m}\,dx}{a^3}+\frac{(3}{}$$

$$= -\frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,{}_2F_1\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m);\frac{1}{2}(-3+m);\sin^2(c+dx\right.}{a^3d(5-m)}$$

$$= \frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\,{}_2F_1\left(1,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\tan^2(c+dx)\right)(}{a^3d(5-m)}$$

Mathematica [F]
time = 10.78, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\tan(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,(e\tan(c+d\,x))^m}{a^3\,(\cos(c+d\,x)+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^3*(e*tan(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

3.215 $\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=131

$$\frac{2^{\frac{5}{2}+m}F_1\left(\frac{1+m}{2};\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+m}(a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] 2^(5/2+m)*AppellF1(1/2+1/2*m,3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d
*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+m)*(a+a*s
ec(d*x+c))^(3/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3974}

$$\frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(5/2 + m)*AppellF1[(1 + m)/2, 3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(5/2 + m)*(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^
(1 + m))/(d*e*(1 + m))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \frac{2^{\frac{5}{2}+m}F_1\left(\frac{1+m}{2};\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+m}(a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^{1+m}}{de(1+m)}$$

Mathematica [F]
time = 20.04, size = 0, normalized size = 0.00

$$\int (a + a\sec(c + dx))^{3/2}(e\tan(c + dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^{\frac{3}{2}} (e\tan(dx + c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^{\frac{3}{2}} (e\tan(c + dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(e*tan(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

3.216 $\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=131

$$\frac{2^{\frac{3}{2}+m}F_1\left(\frac{1+m}{2};\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+m}\sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx}{de(1+m)}$$

```
[Out] 2^(3/2+m)*AppellF1(1/2+1/2*m,1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d
*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+m)*(a+a*s
ec(d*x+c))^(1/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{3974\}$

$$\frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m+\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(3/2 + m)*AppellF1[(1 + m)/2, 1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(3/2 + m)*Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx = \frac{2^{\frac{3}{2}+m}F_1\left(\frac{1+m}{2};\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{}{1+\sec}\right.}{de(1+m)}$$

Mathematica [F]
time = 53.24, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sec\left(dx+c\right)}\,\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(e\tan\left(c+dx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*tan(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\tan(c+d\,x))^m\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

3.217 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m}(e\tan(c+dx))^{1+m}}{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2+1/2*m,-1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(
d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+m)*(e*ta
n(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3974}

$$\frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{1}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2^(1/2 + m)*AppellF1[(1 + m)/2, -1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
+ Sec[c + d*x])^(-1))^(1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{1}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m}(}{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [F]
time = 62.30, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]], x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((e*tan(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\tan(c+d\,x)\right)}^m}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

3.218 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2^{-\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+m}(e\tan(c+dx))^{1+m}}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2^(-1/2+m)*AppellF1(1/2+1/2*m,-3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec
(d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+m)*(e*
tan(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3974}

$$\frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}F_1\left(\frac{m+1}{2};m-\frac{3}{2},1;\frac{m+3}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2^(-1/2 + m)*AppellF1[(1 + m)/2, -3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(-1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2^{-\frac{1}{2}+m}F_1\left(\frac{1+m}{2};-\frac{3}{2}+m,1;\frac{3+m}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+m}}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

Mathematica [F]
time = 48.99, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2), x]
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\tan(c+d\,x))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

3.219 $\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{7(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}+\frac{{}_2F_1(1,4+n;5+n;1+\sec(c+dx))(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}-\frac{5(a+a\sec(c+dx))}{a^5d(5+n)}$$

```
[Out] 7*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)+hypergeom([1, 4+n],[5+n],1+sec(d*x+c))
*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)-5*(a+a*sec(d*x+c))^(5+n)/a^5/d/(5+n)+(a
+a*sec(d*x+c))^(6+n)/a^6/d/(6+n)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {3965, 90, 67}

$$\frac{(a\sec(c+dx)+a)^{n+6}}{a^6d(n+6)}-\frac{5(a\sec(c+dx)+a)^{n+5}}{a^5d(n+5)}+\frac{(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(1,n+4;n+5;\sec(c+dx)+1)}{a^4d(n+4)}+\frac{7(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] (7*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n)) + (Hypergeometric2F1[1, 4
+ n, 5 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n))
 - (5*(a + a*Sec[c + d*x])^(5 + n))/(a^5*d*(5 + n)) + (a + a*Sec[c + d*x])^
(6 + n)/(a^6*d*(6 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^7(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^3(a+ax)^{3+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^6d}$$

$$= \frac{\text{Subst}\left(\int \left(7a^3(a+ax)^{3+n} - \frac{a^3(a+ax)^{3+n}}{x} - 5a^2(a+ax)^{4+n} + a(a+\right.}{a^6d}$$

$$= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)} - \frac{5(a+a\sec(c+dx))^{5+n}}{a^5d(5+n)} + \frac{(a+a\sec(c+}{a^6d(6+}$$

$$= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)} + \frac{{}_2F_1(1,4+n;5+n;1+\sec(c+dx))(a-}{a^4d(4+n)}$$

Mathematica [A]
time = 0.48, size = 87, normalized size = 0.71

$$\frac{(1+\sec(c+dx))^4(a(1+\sec(c+dx)))^n\left(\frac{7}{4+n}+\frac{{}_2F_1(1,4+n;5+n;1+\sec(c+dx))}{4+n}-\frac{5(1+\sec(c+dx))}{5+n}+\frac{(1+\sec(c+dx))^2}{6+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] ((1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n*(7/(4 + n) + Hypergeometric2
F1[1, 4 + n, 5 + n, 1 + Sec[c + d*x]]/(4 + n) - (5*(1 + Sec[c + d*x]))/(5 +
 n) + (1 + Sec[c + d*x])^2/(6 + n)))/d
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)^{n}\left(\tan^{7}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^n \tan^7 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**7,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^7\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

3.220 $\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal. Leaf size=97

$$-\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}-\frac{{}_2F_1(1,3+n;4+n;1+\sec(c+dx))(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}+\frac{(a+a\sec(c+dx))^4}{a^4d(4+n)}$$

```
[Out] -3*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)-hypergeom([1, 3+n],[4+n],1+sec(d*x+c)
)*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)+(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3965, 90, 67}

$$\frac{(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}-\frac{(a\sec(c+dx)+a)^{n+3}\,{}_2F_1(1,n+3;n+4;\sec(c+dx)+1)}{a^3d(n+3)}-\frac{3(a\sec(c+dx)+a)^{n+3}}{a^3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)) - (Hypergeometric2F1[1, 3
 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)
) + (a + a*Sec[c + d*x])^(4 + n)/(a^4*d*(4 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{2+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{2+n}+\frac{a^2(a+ax)^{2+n}}{x}+a(a+ax)^{3+n}\right)\,dx, x,\right.}{a^4d}$$

$$= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}+\frac{(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}+\frac{\text{Subst}\left(\int \frac{(a\right.}{}$$

$$= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}-\frac{{}_2F_1(1,3+n;4+n;1+\sec(c+dx))}{a^3d(3+n)}$$

Mathematica [A]
time = 0.18, size = 72, normalized size = 0.74

$$\frac{(1+\sec(c+dx))^3(a(1+\sec(c+dx)))^n(-9-2n-(4+n)\,{}_2F_1(1,3+n;4+n;1+\sec(c+dx))+(3+n)\sec(c+dx))}{d(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] ((1 + Sec[c + d*x])^3*(a*(1 + Sec[c + d*x]))^n*(-9 - 2*n - (4 + n)*Hypergeo
metric2F1[1, 3 + n, 4 + n, 1 + Sec[c + d*x]] + (3 + n)*Sec[c + d*x]))/(d*(3
 + n)*(4 + n))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\tan^5(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^n \tan^5 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**5,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\,{\left(a+\frac{a}{\cos (c+d\,x)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

3.221 $\int (a+a\sec(c+dx))^n \tan^3(c+dx)\,dx$

Optimal. Leaf size=69

$$\frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}+\frac{{}_2F_1(1,2+n;3+n;1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}$$

```
[Out] (a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)+hypergeom([1, 2+n],[3+n],1+sec(d*x+c))*(
a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

Rubi [A]

time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3965, 81, 67}

$$\frac{(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(1,n+2;n+3;\sec(c+dx)+1)}{a^2d(n+2)}+\frac{(a\sec(c+dx)+a)^{n+2}}{a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] (a + a*Sec[c + d*x])^(2 + n)/(a^2*d*(2 + n)) + (Hypergeometric2F1[1, 2 + n,
 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(a^2*d*(2 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^3(c+dx)\,dx=\frac{\text{Subst}\left(\int\frac{(-a+ax)(a+ax)^{1+n}}{x}\,dx,x,\sec(c+dx)\right)}{a^2d}$$
$$=\frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}-\frac{\text{Subst}\left(\int\frac{(a+ax)^{1+n}}{x}\,dx,x,\sec(c+dx)\right)}{ad}$$
$$=\frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}+\frac{{}_2F_1(1,2+n;3+n;1+\sec(c+dx))(a+}{a^2d(2+n)}$$

Mathematica [**A**]
time = 0.06, size = 49, normalized size = 0.71

$$\frac{(1+{}_2F_1(1,2+n;3+n;1+\sec(c+dx)))(1+\sec(c+dx))^2(a(1+\sec(c+dx)))^n}{d(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((1 + Hypergeometric2F1[1, 2 + n, 3 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*
x])^2*(a*(1 + Sec[c + d*x]))^n)/(d*(2 + n))
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\tan^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec{(c+dx)}+1))^n\tan^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**3,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.222 $\int (a+a\sec(c+dx))^n \tan(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{{}_2F_1(1,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3965, 67}

$$-\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(1,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x]
)^(1 + n))/(a*d*(1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^n \tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+ax)^n}{x}\,dx, x, \sec(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1(1,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 1.00

$$-\frac{{}_2F_1(1,1+n;2+n;1+\sec(c+dx))(a(1+\sec(c+dx)))^{1+n}}{ad(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(1 + n))/(a*d*(1 + n)))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^n \tan(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^n \tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(c+d\,x)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

3.223 $\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=74

$$-\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}+\frac{{}_2F_1(1,n;1+n;1+\sec(c+dx))(a+a\sec(c+dx))^n}{dn}$$

```
[Out] -1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n+hype
rgeom([1, n],[1+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

Rubi [A]

time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3965, 88, 67, 70}

$$\frac{(a\sec(c+dx)+a)^n\,{}_2F_1(1,n;n+1;\sec(c+dx)+1)}{dn}-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d
*x])^n)/(d*n) + (Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]]*(a + a*Se
c[c + d*x])^n)/(d*n)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{a\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{-a+ax}\,d\right.}{d}$$
$$= -\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn} + \frac{{}_2F_1(1,}{}$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 0.77

$$-\frac{\left({}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)-2\,{}_2F_1(1,n;1+n;1+\sec(c+dx))\right)(a(1+\sec(c+dx)))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*((Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2] - 2*Hypergeomet
ric2F1[1, n, 1 + n, 1 + Sec[c + d*x]])*(a*(1 + Sec[c + d*x]))^n)/(d*n)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \cot(dx+c)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^n \cot (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

3.224 $\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{a(4-n)\,{}_2F_1\left(1,-1+n;n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)}+\frac{a\,{}_2F_1(1,-1+n;n;1+\sec(c+dx)}{d(1-n)}$$

```
[Out] -1/4*a*(4-n)*hypergeom([1, -1+n],[n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^(
-1+n)/d/(1-n)+a*hypergeom([1, -1+n],[n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(-1+
n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/d/(1-sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3965, 105, 162, 67, 70}

$$-\frac{a(4-n)(a\sec(c+dx)+a)^{n-1}\,{}_2F_1\left(1,n-1;n;\frac{1}{2}(\sec(c+dx)+1)\right)}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}\,{}_2F_1(1,n-1;n;\sec(c+dx)+1)}{d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}}{2d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(4 - n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])/2]*(a +
a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*Hypergeometric2F1[1, -1 + n, n,
1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Se
c[c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x]))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx &= \frac{a^4\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x(-a+ax)^2}\,dx,x,\sec(c+dx)\right)}{d}\\ &= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}-\frac{a\text{Subst}\left(\int \frac{(a+ax)^{-2+n}(2a^2+a^2(2-n)x)}{x(-a+ax)}\,dx,\right.}{2d}\\ &= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}+\frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x}\,dx,x,\sec(c+d\right.}{d}\\ &= -\frac{a(4-n)\,{}_2F_1\left(1,-1+n;n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx}{4d(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 96, normalized size = 0.76

$$-\frac{a\left(-2+2n+(-4+n)\,{}_2F_1\left(1,-1+n;n;\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+4\,{}_2F_1(1,-1+n;n;1+\sec(c+dx))(-1+\sec(c+dx))\right)(a(1+\sec(c+dx)))^{-1+n}}{4d(-1+n)(-1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(-2 + 2*n + (-4 + n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x]) + 4*Hypergeometric2F1[1, -1 + n, n, 1 + Sec[c +
```

```
 d*x]]*(-1 + Sec[c + d*x]))*(a*(1 + Sec[c + d*x]))^(-1 + n))/(d*(-1 + n)*(-
1 + Sec[c + d*x]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{n}\cot^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.225 $\int (a+a\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{2^{5+n}F_1\left(\frac{5}{2};4+n,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(a+a\sec(c+dx))^n\tan^5(c+dx)}{5d}$$

```
[Out] 1/5*2^(5+n)*AppellF1(5/2,4+n,1,7/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5+n)*(a+a*sec(d*x+c))^n*t
an(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{3974\}$

$$\frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^nF_1\left(\frac{5}{2};n+4,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] (2^(5 + n)*AppellF1[5/2, 4 + n, 1, 7/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(5 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5)/(5*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\frac{2^{5+n}F_1\left(\frac{5}{2};4+n,1;\frac{7}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(}{5d}$$

Mathematica [F]
time = 1.40, size = 0, normalized size = 0.00

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+a\sec \left(dx+c\right)\right)^{n} \left(\tan^{4}\left(dx+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{n} \tan^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**4,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.226 $\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{2^{3+n}F_1\left(\frac{3}{2};2+n,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}(a+a\sec(c+dx))^n\tan^3(c+dx)}{3d}$$

```
[Out] 1/3*2^(3+n)*AppellF1(3/2,2+n,1,5/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3+n)*(a+a*sec(d*x+c))^n*t
an(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3974}

$$\frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^nF_1\left(\frac{3}{2};n+2,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*AppellF1[3/2, 2 + n, 1, 5/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(3 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3)/(3*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^2(c+dx)\,dx = \frac{2^{3+n}F_1\left(\frac{3}{2};2+n,1;\frac{5}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 910 vs. 2(106) = 212.
time = 9.74, size = 910, normalized size = 8.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*((-4*Hypergeometric2F1[-1 - n, n, -n, (1 - Tan[(c
 + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 + Tan[(c + d*x)/2])^n
)/((1 + n)*(1 + Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])) - (Hypergeometric2
F1[1 - n, 2 + n, 2 - n, (1 - Tan[(c + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(-1 + Tan[(c + d*x)/2])*(1 + Tan[(c + d*x)/2])^n)/((-1 + n)*(1 +
 Sec[c + d*x])^n) - (120*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/
2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n
, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45
*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c
 + d*x)/2]^2*(1 + 2*n - 2*n*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1
/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*
(-5*AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 +
2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[
c + d*x] + Cos[2*(c + d*x)]) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x
)/2]^4) + 40*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)))/(4*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^n \left(\tan^2(dx + c)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n \tan^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**2,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.227 $\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=102

$$-\frac{2^{-1+n}F_1\left(-\frac{1}{2};-2+n,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-1+n}(a+a\sec(c+dx))^n}{d}$$

```
[Out] -2^(-1+n)*AppellF1(-1/2,-2+n,1,1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)*(1/(1+sec(d*x+c)))^(-1+n)*(a+a*sec
(d*x+c))^n/d
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3974}

$$-\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^nF_1\left(-\frac{1}{2};n-2,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*AppellF1[-1/2, -2 + n, 1, 1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]*((1
 + Sec[c + d*x])^(-1))^(-1 + n)*(a + a*Sec[c + d*x])^n)/d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-1+n}F_1\left(-\frac{1}{2};-2+n,1;\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 893 vs. 2(102) = 204.
time = 3.91, size = 893, normalized size = 8.75

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]

[Out] ((a*(1 + Sec[c + d*x]))^n*(-((2^n*Cot[(c + d*x)/2]*Hypergeometric2F1[-1/2, n, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n)/(1 + Sec[c + d*x])^n) + (2^n*Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*Tan[(c + d*x)/2])/(1 + Sec[c + d*x])^n - (60*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^2*(1 + 2*n - 2*n*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*(-5*AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x)/2]^4) + 40*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)))/(2*d)

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}\left(d x +c \right)\right) \left(a +a \sec \left(d x +c \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)

[Out] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n \cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.228 $\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=106

$$-\frac{2^{-3+n}F_1\left(-\frac{3}{2};-4+n,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-3+n}(a+a\sec(c+dx)}{3d}$$

```
[Out] -1/3*2^(-3+n)*AppellF1(-3/2,-4+n,1,-1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),
(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)^3*(1/(1+sec(d*x+c)))^(-3+n)*(
a+a*sec(d*x+c))^n/d
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3974}

$$-\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^nF_1\left(-\frac{3}{2};n-4,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/3*(2^(-3 + n)*AppellF1[-3/2, -4 + n, 1, -1/2, -((a - a*Sec[c + d*x])/(a
+ a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]
^3*((1 + Sec[c + d*x])^(-1))^(-3 + n)*(a + a*Sec[c + d*x])^n)/d
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-3+n}F_1\left(-\frac{3}{2};-4+n,1;-\frac{1}{2};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c-}{3d}$$

Mathematica [F]
time = 1.76, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n, x]
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\cot^{4}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.229 $\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{2^{\frac{7}{2}+n}F_1\left(\frac{5}{4};\frac{3}{2}+n,1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n}(a+a\sec(c+dx))^n\tan^{\frac{5}{2}}(c+dx)}{5d}$$

```
[Out] 1/5*2^(7/2+n)*AppellF1(5/4,3/2+n,1,9/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+n)*(a+a*sec(d*x+c
))^n*tan(d*x+c)^(5/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{3974\}$

$$\frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{5}{4};n+\frac{3}{2},1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] (2^(7/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(5/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(5/2))/(5*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx = \frac{2^{\frac{7}{2}+n}F_1\left(\frac{5}{4};\frac{3}{2}+n,1;\frac{9}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n}}{5d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2072 vs. $2(114) = 228$.

time = 19.77, size = 2072, normalized size = 18.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] (2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(-1
 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n
)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2,
 Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2
, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-
1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Tan[c + d*x]^2)/(d*((2^n*Sec[c + d*x]^2*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*
AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)
+ (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]
] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2
]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n)))/
Sqrt[Tan[c + d*x]] + 2^n*(-1/2 - n)*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-3/2 - n)*(-2*AppellF1[1/4, 1/2 +
n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2
 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/
2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*
x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]] +
2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2
 - n)*(-((1/2 + n)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 +
 n)*(-1 + Tan[(c + d*x)/2])^(-1/2 + n)) - 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)*(-1/5*(AppellF1[5/4, 1/2 + n
, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]) + ((1/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*(1/2 + n
)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 +
n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]) + (1/2 + n)*(AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c +
d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c +
 d*x)/2], -Tan[(c + d*x)/2]])*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(1/
2 + n)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)^(-1/2 + n) - ((1/2 + n)*(
AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] +
 AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])
*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(-1/2 + n)*(-1 + Tan[(c + d*x)/2
```

```
]^2)^(1/2 + n))/2 + (-1/6*((3/2 + n)*AppellF1[3/2, 1/2 + n, 5/2 + n, 5/2, T
an[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2) + ((3/2 + n)*Appell
F1[3/2, 5/2 + n, 1/2 + n, 5/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c
+ d*x)/2]^2)/6)*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^
(1/2 + n))*Sqrt[Tan[c + d*x]] + 2^(1 + n)*n*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(-1 + n)*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1
, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n
, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 +
n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2
])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]]*(-(Cos
[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d
*x]*Tan[c + d*x])))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^n \left(\tan^{\frac{3}{2}}(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

3.230 $\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{2^{\frac{5}{2}+n}F_1\left(\frac{3}{4};\frac{1}{2}+n,1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n}(a+a\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)}{3d}$$

```
[Out] 1/3*2^(5/2+n)*AppellF1(3/4,1/2+n,1,7/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+n)*(a+a*sec(d*x+c
))^n*tan(d*x+c)^(3/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3974}

$$\frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{3}{4};n+\frac{1}{2},1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(5/2 + n)*AppellF1[3/4, 1/2 + n, 1, 7/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(3/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2))/(3*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\frac{2^{\frac{5}{2}+n}F_1\left(\frac{3}{4};\frac{1}{2}+n,1;\frac{7}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n}}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(114) = 228.

time = 3.09, size = 238, normalized size = 2.09

$$\frac{56F_1\left(\frac{3}{4};\frac{1}{2}+n,1;\frac{7}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^n\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\tan(c+dx)}}{d\left(6\left(2F_1\left(\frac{7}{4};\frac{1}{2}+n,2;\frac{11}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-(1+2n)F_1\left(\frac{7}{4};\frac{3}{2}+n,1;\frac{11}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+21F_1\left(\frac{3}{4};\frac{1}{2}+n,1;\frac{7}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (56*AppellF1[3/4, 1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)/2]*Sqrt[Tan[c +
d*x]])/(d*(6*(2*AppellF1[7/4, 1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] - (1 + 2*n)*AppellF1[7/4, 3/2 + n, 1, 11/4, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 21*AppellF1[3/4, 1/2 + n, 1,
 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\sqrt{\tan}\,(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^n \sqrt{\tan (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan (c+d\,x)}\,\left(a+\frac{a}{\cos (c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=111

$$\frac{2^{\frac{3}{2}+n}F_1\left(\frac{1}{4};-\frac{1}{2}+n,1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}}{d}$$

```
[Out] 2^(3/2+n)*AppellF1(1/4,-1/2+n,1,5/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a
*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+n)*(a+a*sec(d*x+c))^
n*tan(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3974}

$$\frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{1}{4};n-\frac{1}{2},1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(3/2 + n)*AppellF1[1/4, -1/2 + n, 1, 5/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*
x])^(-1))^(1/2 + n)*(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]])/d
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \frac{2^{\frac{3}{2}+n}F_1\left(\frac{1}{4};-\frac{1}{2}+n,1;\frac{5}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\,\mathrm{se}}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(111) = 222.
time = 2.42, size = 229, normalized size = 2.06

$$\frac{10F_1\left(\frac{1}{4};-\frac{1}{2}+n,1;\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)(1+\cos(c+dx))(a(1+\sec(c+dx)))^n\sqrt{\tan(c+dx)}}{d\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2}+n,2;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1-2n)F_1\left(\frac{5}{4};\frac{1}{2}+n,1;\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+5F_1\left(\frac{1}{4};-\frac{1}{2}+n,1;\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, -1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x]*(1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Tan[c + d*x
]])/(d*(2*(2*AppellF1[5/4, -1/2 + n, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + (1 - 2*n)*AppellF1[5/4, 1/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*AppellF1[1/4, -1/2 + n, 1, 5/4
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sec(dx + c))^n}{\sqrt{\tan(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec{(c+dx)}+1))^n}{\sqrt{\tan{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(tan(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan{(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.232 $\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2^{\frac{1}{2}+n}F_1\left(-\frac{1}{4};-\frac{3}{2}+n,1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+a\sec(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

```
[Out] -2^(1/2+n)*AppellF1(-1/4,-3/2+n,1,3/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a
-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+n)*(a+a*sec(d*x+c
))^n/d/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{3974\}$

$$-\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(-\frac{1}{4};n-\frac{3}{2},1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[-1/4, -3/2 + n, 1, 3/4, -((a - a*Sec[c + d*x])/(a +
 a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c +
 d*x])^(-1))^(-1/2 + n)*(a + a*Sec[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2^{\frac{1}{2}+n}F_1\left(-\frac{1}{4};-\frac{3}{2}+n,1;\frac{3}{4};-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+}{d\sqrt{\tan(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2164 vs. $2(112) = 224$.
time = 17.86, size = 2164, normalized size = 19.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -1/21*(2^(1/2 + n)*Cot[c + d*x]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(21*Hypergeometric2
F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1,
7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hyperg
eometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*
AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2]^4))/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((2^(-1/2 + n)
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*Sec[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^n*(21*Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2]
 + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*
x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Tan[c + d*x]^(3/2)) + (2^(-1/2 + n)*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*(21*Hypergeom
etric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 +
n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*
Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^
2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Tan[(c + d*x)/2]^4))/(21*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(1 + n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(21*Hypergeometric2F1[-1/4,
 -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric
2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1
[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c +
d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Tan[c + d*x]]) -
 (2^(1/2 + n)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 7*Hypergeometric2F1[3/4,
-1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 6*
AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3 + 7*Tan[(c + d*x)/2]^2*((-3*AppellF1[7
/4, -1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
```

```
x)/2]^2*Tan[(c + d*x)/2])/7 + (3*(-1/2 + n)*AppellF1[7/4, 1/2 + n, 1, 11/4,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/7) - 3*Tan[(c + d*x)/2]^4*((-7*AppellF1[11/4, -1/2 + n, 2, 15/4, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11
+ (7*(-1/2 + n)*AppellF1[11/4, 1/2 + n, 1, 15/4, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11) + (21*Csc[(c + d*x)
/2]*Sec[(c + d*x)/2]*(Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/
2]^2] - (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4 + (21*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]*(-Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2] +
 (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(21*Hypergeometric2F1[-1/
4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometr
ic2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*Appell
F1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c
+ d*x)/2]^4)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[Tan[c + d*x]])))
```

Maple [**F**]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a + a \sec \left(dx + c\right)\right)^{n}}{\tan \left(dx + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c+dx)+1))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/tan(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^n}{{\tan(c+d\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

3.233 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=320

$$-\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d}-\frac{a(e\cot(c+dx))^{5/2}F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(}}{3d}$$

```
[Out] -2/3*(e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+1/3*a*(e*cot(d*x+c)
)^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*P
i+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/
2)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)
^(5/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3985, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{a\tan^{\frac{5}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}d}-\frac{a\tan^{\frac{5}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}d}+\frac{a\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{a\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)(e\cot(c+dx))^{5/2}}{3d}-\frac{a\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/(3*d) - (a*(e
*Cot[c + d*x])^(5/2)*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c
 + 2*d*x]]*Tan[c + d*x]^2)/(3*d) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[
2]*d) + (a*(e*Cot[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d) - (a*(e*Cot[c + d*x])^(5/2)*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt
[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} + \frac{1}{3}\Big(2(e\,c$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{1}{3}\Big(a(e\,c$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{\Big(a(e\,co}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{1}{3}\Big(a(e\,c$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(}{}$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} - \frac{a(e\cot(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.07, size = 185, normalized size = 0.58

$$-\frac{a(e\cot(c+dx))^{5/2}\sec(c+dx)\left(\sqrt{\cot(c+dx)}\left(4(1+\cos(c+dx))\cot(c+dx)-3\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}+3\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)+2\sqrt[4]{-1}\sqrt{\csc^2(c+dx)}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\cot(c+dx)}\right)\middle|-1\right)\sin(2(c+dx))\right)}{6d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6*(a*(e*Cot[c + d*x])^(5/2)*Sec[c + d*x]*(Sqrt[Cot[c + d*x]]*(4*(1 + Cos
[c + d*x])*Cot[c + d*x] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*
(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*S
qrt[Sin[2*(c + d*x)]]) + 2*(-1)^(1/4)*Sqrt[Csc[c + d*x]^2]*EllipticF[I*ArcS
inh[(-1)^(1/4)*Sqrt[Cot[c + d*x]]], -1]*Sin[2*(c + d*x)]))/(d*Cot[c + d*x]^
(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 1.21, size = 658, normalized size = 2.06

method	result
default	$\frac{a(-1+\cos(dx+c))\left(3i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a/d*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2)-3*I*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)-4*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)+2*2^(1/2)*cos(d*x+c))*(e*cos(d*x+c)/sin(d*x+c))^(5/2)*(1+cos(d*x+c))^2/c
os(d*x+c)^3/sin(d*x+c)*2^(1/2)
```

Maxima [A]
time = 0.25, size = 126, normalized size = 0.39

$$\frac{\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-3\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8}{\tan(dx+c)^{\frac{3}{2}}}\right)ae^{\frac{5}{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8/tan(d*x + c)^(3/2))*a*e^(5/2)/
d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{5/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.234 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=346

$$-\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d}-\frac{2a(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}}$$

```
[Out] -2*(e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+2*a*(e*cot(d*x+c))^(3
/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*
x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+2*c)^(1/2)-1/2*a*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2
*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)
/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+2*a*(e*cot(d*x+c))^(3/
2)*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.21, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3985, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{a\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}+\frac{a\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}+\frac{2a\sin(c+dx)\tan^2(c+dx)(e\cot(c+dx))^{3/2}}{d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)(e\cot(c+dx))^{3/2}}{d}-\frac{2a\sin(c+dx)\tan(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{3/2}}{d\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/d - (2*a*(e*C
ot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/
(d*Sqrt[Sin[2*c + 2*d*x]]) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d)
 - (a*(e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d) + (a*(e*Cot[c + d*x])^(3/2)*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
 + (2*a*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \Big(2(e\,\mathrm{co}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \Big(a(e\,\mathrm{co}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\,\mathrm{co}}{}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\,\mathrm{co}}{}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} + \frac{2a(e\,\mathrm{co}}{}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\,\mathrm{co}}{}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\,\mathrm{co}}{}$$
$$= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\,\mathrm{co}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.21, size = 191, normalized size = 0.55

$$\frac{ae(1+\cos(c+dx))\sqrt{e\cot(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^2(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\cot^2(c+dx)\right)+3\sqrt{\csc^2(c+dx)}\left(-4\cos(c+dx)-4\cos^2(c+dx)+\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}+\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)\right)}{12d\sqrt{\csc^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*e*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x
]*(8*Cot[c + d*x]^2*Hypergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] + 3*S
qrt[Csc[c + d*x]^2]*(-4*Cos[c + d*x] - 4*Cos[c + d*x]^2 + ArcSin[Cos[c + d*
x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x]
```

```
 + Sqrt[Sin[2*(c + d*x)]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[Csc[c + d*x
]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 1414, normalized size = 4.09

method	result	size
default	Expression too large to display	1414

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-c
os(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*cos(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-(-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))+2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
```

```
+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(
1/2))+4*2^(1/2)*cos(d*x+c))*(e*cos(d*x+c)/sin(d*x+c))^(3/2)*sin(d*x+c)/cos(
d*x+c)^2*2^(1/2)
```

Maxima [A]
time = 0.24, size = 125, normalized size = 0.36

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8}{\sqrt{\tan\left(dx+c\right)}}\right)ae^{\frac{3}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*log(sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt(
tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8/sqrt(tan(d*x + c)))*a*e^(3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\,dx+\int\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(3/2), x) + Integral((e*cot(c + d*x))**(3/2)*
sec(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{3/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.235 $\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}-\frac{a\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)\sqrt{e\,c}}{\sqrt{2}\,d}$$

```
[Out] -a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x
),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^
(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x
+c)^(1/2)/d*2^(1/2)-1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(
d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3985, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$-\frac{a\sqrt{\tan(c+dx)}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}\,d}+\frac{a\sqrt{\tan(c+dx)}\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}\,d}-\frac{a\sqrt{\tan(c+dx)}\,\sqrt{e\cot(c+dx)}\,\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{a\sqrt{\tan(c+dx)}\,\sqrt{e\cot(c+dx)}\,\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{a\sqrt{\sin(2c+2dx)}\,\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[
2*c + 2*d*x]])/d - (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a*Sqrt[e*Co
t[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c +
 d*x]])/(2*Sqrt[2]*d) + (a*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))\,dx &= \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx \\
&= \left(a\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx + \left(a\sqrt{}\right. \\
&= \frac{\left(a\sqrt{e\cot(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{\sin(c+d}}}{\sqrt{\cos(c+dx)}} \\
&= \left(a\sqrt{e\cot(c+dx)}\,\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2}} \\
&= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&= \frac{a\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.92, size = 169, normalized size = 0.62

$$
\frac{a(1+\cos(c+dx))\sqrt{e\cot(c+dx)}\,\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4\sqrt[4]{-1}\,\sqrt{\cot(c+dx)}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,\sqrt{\cot(c+dx)}\right)\middle|-1\right)+\sqrt{\csc^2(c+dx)}\left(-\text{ArcSin}(\cos(c+dx)-\sin(c+dx))+\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\right)\sqrt{\sin(2(c+dx))}\right)}{4d\sqrt{\csc^2(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x]*
(4*(-1)^(1/4)*Sqrt[Cot[c + d*x]]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Cot[c
+ d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(-ArcSin[Cos[c + d*x] - Sin[c + d*x]]
+ Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sqrt[Sin[2*(c
+ d*x)]]))/(4*d*Sqrt[Csc[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 289, normalized size = 1.05

method	result

default	$-\frac{a\sqrt{\frac{e\cos(dx+c)}{\sin(dx+c)}}\,(-1+\cos(dx+c))\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\Big(i\,\mathrm{E}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+c)
^2/cos(d*x+c)*(1+cos(d*x+c))^2*2^(1/2)
```

Maxima [A]
time = 0.24, size = 115, normalized size = 0.42

$$-\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)ae^{\frac{1}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a*e^(1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\cot\left(c+dx\right)}\,dx+\int\sqrt{e\cot\left(c+dx\right)}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*sec(c
+ d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\cot\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.236 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}-\frac{a\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a\text{ArcTan}\left(\right.}{\sqrt{2}d\sqrt{e\,\text{cc}}}$$

```
[Out] 2*a*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^
(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d*x+c
))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2
^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a*ln(1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(
1/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c
))^(1/2)/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3985, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{a\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{a\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{2a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a*Arc
Tan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[T
an[c + d*x]]) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[
e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - (
a*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{a\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \right.}{d\sqrt{e\cot(c+dx)}\,\sqrt{}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a}{}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a\cos(c+dx))\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, \right.}{d\sqrt{e\cot(c+dx)}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,d\right.}{2d\sqrt{e\cot(c+dx)}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c\right.}{2\sqrt{2}\,d\sqrt{e\cot(c+}}$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} - \frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\mathrm{ta}\right.}{\sqrt{2}\,d\sqrt{e\cot(c+dx)}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.70, size = 189, normalized size = 0.63

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^3(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\cot^2(c+dx)\right)-3\cot(c+dx)\sqrt{\csc^2(c+dx)}\left(-2+2\cos(2(c+dx))+\mathrm{ArcSin}(\cos(c+dx)-\sin(c+dx))\sqrt{\sin(2(c+dx))}+\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)\right)}{12d\sqrt{e\cot(c+dx)}\sqrt{\csc^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(8*Cot[c + d*x]^3*Hyp
ergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Sqrt[Csc[c
+ d*x]^2]*(-2 + 2*Cos[2*(c + d*x)] + ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sq
rt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*
x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Csc[c + d*x
]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.62, size = 1433, normalized size = 4.79

method	result	size
default	Expression too large to display	1433

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1
/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-
2*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*cos(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(d*x+c)+2*2^(1/2))/sin
(d*x+c)^5/(e*cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)
```

Maxima [A]

time = 0.24, size = 115, normalized size = 0.38

$$-\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)ae^{\left(-\frac{1}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a*e^(-1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{e\cot(c+dx)}}\,dx+\int\frac{\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*cot(c
 + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2), x)
```

3.237 $\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=320

$$\frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}}-\frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}}+\frac{a\mathrm{ArcTan}\left(\right.}{\sqrt{2}\,d(e\cot}$$

```
[Out] 2/3*cot(d*x+c)*(3*a+a*sec(d*x+c))/d/(e*cot(d*x+c))^(3/2)+1/3*a*cot(d*x+c)*c
sc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4
*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(d*x+c))^(3/2)-1/2*a*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2
)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/t
an(d*x+c)^(3/2)+1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*
x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A]

time = 0.19, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3985, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{a\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2\cot(c+dx)(a\sec(c+dx)+3a)}{3d(e\cot(c+dx))^{3/2}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3d(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*(3*a + a*Sec[c + d*x]))/(3*d*(e*Cot[c + d*x])^(3/2)) - (a*C
ot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]
])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3
/2)) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*
(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(
3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{\int (a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\int}{(e\cot(c} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+}}}{3(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right.}{3(e\cot(c+dx))^{3/2}} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{3d(e\cot(c+dx))^{3/2}} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{3d(e\cot(c+dx))^{3/2}} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{3d(e\cot(c+dx))^{3/2}} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)}{3d(e\cot(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.66, size = 224, normalized size = 0.70

$$
\frac{a(1+\cos(c+dx))\cos(2(c+dx))\csc(c+dx)\sqrt{\csc^2(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-4\sqrt[4]{-1}\cot^{\frac{3}{2}}(c+dx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\cot(c+dx)}\right)\middle|-1\right)+\sqrt{\csc^2(c+dx)}\left(4+12\cos(c+dx)+3\text{ArcSin}(\cos(c+dx)-\sin(c+dx))\cot(c+dx)\sqrt{\sin(2(c+dx))}-3\cot(c+dx)\log\left(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}\right)\sqrt{\sin(2(c+dx))}\right)\right)}{12d(e\cot(c+dx))^{3/2}\left(-1+\cot^2(c+dx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Csc[c + d*x]^2]*Se
c[(c + d*x)/2]^2*(-4*(-1)^(1/4)*Cot[c + d*x]^(3/2)*EllipticF[I*ArcSinh[(-1)
^(1/4)*Sqrt[Cot[c + d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(4 + 12*Cos[c + d*x]
 + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*x]*Sqrt[Sin[2*(c + d*x)]
] - 3*Cot[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]
```

```
]*Sqrt[Sin[2*(c + d*x)]]])))/(12*d*(e*Cot[c + d*x])^(3/2)*(-1 + Cot[c + d*x]
^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 698, normalized size = 2.18

method	result
default	$-\frac{a(-1+\cos(dx+c))\left(-3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/d*(-1+cos(d*x+c))*(-3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+3*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+4*cos(d*x+c)*sin(d*x+c)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*cos(d*x+c)*sin(d*x+c)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*cos(d*x+c)*sin(d*x+c)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*cos(d*x+c)^2*
2^(1/2)+4*2^(1/2)*cos(d*x+c)+2*2^(1/2))*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(
d*x+c))^(3/2)/sin(d*x+c)^5*2^(1/2)
```

Maxima [A]
time = 0.24, size = 125, normalized size = 0.39

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8\sqrt{\tan(dx+c)}\right)ae^{\left(-\frac{3}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log(sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sqrt(
tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*sqrt(tan(d*x + c)))*a*e^(-3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(3/2),x)
[Out] a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*cot(c +
 d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2), x)
```

3.238 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=357

$$-\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d}-\frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}F(\ldots$$

```
[Out] -4/3*a^2*(e*cot(d*x+c))^(5/2)*tan(d*x+c)/d-4/3*a^2*(e*cot(d*x+c))^(5/2)*sec
(d*x+c)*tan(d*x+c)/d+2/3*a^2*(e*cot(d*x+c))^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/
2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*
d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(
e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)+1/4*a^2*(e*c
ot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)
/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3985, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 30}

$$\frac{a^2\tan^{\frac{5}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}\,d}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}\,d}-\frac{4a^2\tan(c+dx)(e\cot(c+dx))^{5/2}}{3d}+\frac{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}-\frac{4a^2\tan(c+dx)\sec(c+dx)(e\cot(c+dx))^{5/2}}{3d}-\frac{2a^2\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x])/(3*d) - (4*a^2*(e*Cot[c + d*x]
)^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)*El
lipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]]*Tan[c + d*x]
^2)/(3*d) + (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) + (a^2*(e*Cot
[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(5/2))/(2*Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(5/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{(a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2\,\mathrm{se}}{\tan^{\frac{5}{2}}}\right.$$

$$= \left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx + \left(a\right.$$

$$= -\frac{2a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\,\mathrm{se}}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 32.48, size = 93, normalized size = 0.26

$$-\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\left(2+2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\tan^2(c+dx)\right)-{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2 + 2*Hypergeometric2F
1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Cot[c
+ d*x]^2])*Sec[ArcCot[Cot[c + d*x]]/2]^4)/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 660, normalized size = 1.85

method	result
default	$\frac{a^2(-1+\cos(dx+c))\left(3i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c}{\sin(dx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2/d*(-1+cos(d*x+c))*(3*I*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)-3*I*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)-2*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2)+4*2^(1/2)*cos(d*x+c))*(e*cos(d*x+c)/sin(d*x+c))^(5/2)*(1+cos(d*x+c))^2
/cos(d*x+c)^3/sin(d*x+c)*2^(1/2)
```

Maxima [A]
time = 0.25, size = 143, normalized size = 0.40

$$\frac{\left(\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-3\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8}{\tan(dx+c)^{\frac{3}{2}}}\right)a^2-\frac{8a^2}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\frac{5}{2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*lo
g(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*log(-sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8/tan(d*x + c)^(3/2))*a^2 - 8*a
^2/tan(d*x + c)^(3/2))*e^(5/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.239 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=343

$$-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right.}{d\sqrt{\sin(2c+2d}}$$

```
[Out] -4*a^2*(e*cot(d*x+c))^(3/2)*sin(d*x+c)/d-4*a^2*(e*cot(d*x+c))^(3/2)*tan(d*x
+c)/d+4*a^2*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+
2*c)^(1/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)
*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*c
ot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(3/2)*ln
(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a^2*
(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3985, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 30}

$$\frac{a^2\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{4a^2\sin(c+dx)(e\cot(c+dx))^{3/2}}{d}-\frac{4a^2\tan(c+dx)(e\cot(c+dx))^{3/2}}{d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}+\frac{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{4a^2\sin(c+dx)\tan(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{3/2}}{d\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3
/2)*Tan[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*
x, 2]*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (a^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(
Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(
3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d
) + (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{(a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2\,\mathrm{se}}{\tan^{\frac{3}{2}}}\right.$$

$$= \left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \left(a\right.$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\,\mathrm{ta}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 15.26, size = 220, normalized size = 0.64

$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+16\sqrt{\cot(c+dx)}+16\sqrt{\cot(c+dx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\tan^2(c+dx)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)}{4d\cot^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]] + 16*Sqrt[Cot[c + d*x]] + 16*Sqrt[Cot[c + d*x]]*Hypergeometric2F1[-1/4,
1/2, 3/4, -Tan[c + d*x]^2] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*
Sec[ArcCot[Cot[c + d*x]]/2]^4)/(d*Cot[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 1416, normalized size = 4.13

method	result	size
default	Expression too large to display	1416

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*co
s(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-8*cos(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
```

```
(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))-8*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))+8*2^(1/2)*cos(d*x+c))*sin(d*x+c)*(e*cos(d*x+c)/sin(d*x+c))^(3/2)/co
s(d*x+c)^2*2^(1/2)
```

Maxima [A]
time = 0.25, size = 142, normalized size = 0.41

$$\frac{\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8}{\sqrt{\tan\left(dx+c\right)}}\right)a^2-\frac{8a^2}{\sqrt{\tan\left(dx+c\right)}}\right)e^{\frac{3}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8/sqrt(tan(d*x + c)))*a^2 - 8*a^2/sq
rt(tan(d*x + c)))*e^(3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cot(c+d\,x))^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.240 $\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=311

$$\frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}-\frac{a^2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{}}{\sqrt{2}\,d}$$

```
[Out] -2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi
+d*x),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a
^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2
)/d*2^(1/2)+1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)
*tan(d*x+c)^(1/2)/d*2^(1/2)-1/4*a^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a^2*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+2
*a^2*(e*cot(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.21, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3985, 3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 30}

$$-\frac{a^2\sqrt{\tan(c+dx)}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}\,d}+\frac{a^2\sqrt{\tan(c+dx)}\,\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}\,d}+\frac{2a^2\tan(c+dx)\sqrt{e\cot(c+dx)}}{d}-\frac{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d}+\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[
Sin[2*c + 2*d*x]])/d - (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a^2
*Sqrt[e*Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sq
rt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*
a^2*Sqrt[e*Cot[c + d*x]]*Tan[c + d*x])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}} + \frac{2a^2\,\mathrm{se}}{\sqrt{\mathrm{tan}}}\right.$$

$$= \left(a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx + \Big(a$$

$$= \frac{\left(2a^2\sqrt{e\cot(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{\sin(c-}}}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,\tan(c+dx)}{d} + \left(2a^2\sqrt{e\cot(c+dx)}\,\sec(c+\right.$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2d}}{d}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2d}}{d}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2d}}{d}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+dx)\sqrt{\sin(2c+2d}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 32.01, size = 118, normalized size = 0.38

$$\frac{a^2e(1+\cos(c+dx))^2\left(3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+6\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)-2\cot^2(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)}{6d\sqrt{e\cot(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*e*(1 + Cos[c + d*x])^2*(3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x
]^2] + 6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2] - 2*Cot[c + d*x]
^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*Sec[ArcCot[Cot[c + d*x]
]/2]^4)/(6*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 663, normalized size = 2.13

method	result
default	$-\frac{a^{2}(1+\cos(dx+c))^{2}\sqrt{\frac{e\cos(dx+c)}{\sin(dx+c)}}\,(-1+\cos(dx+c))\left(i\sin(dx+c)\,\text{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\vphantom{1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/d*(1+cos(d*x+c))^2*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)-I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+2*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-2*2^(1/2)*cos(d*x+c)+2*2^(1/2))/sin(d*x+c)^3/cos(d*x+c)*2^(1/2)

Maxima [A]
time = 0.25, size = 132, normalized size = 0.42

$$-\frac{\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^{2}-8a^{2}\sqrt{\tan(dx+c)}\right)e^{\frac{1}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2 - 8*a^2*sqrt(tan(d*x + c)))*e^(1/2)/d

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{e\cot(c+dx)}\,dx+\int 2\sqrt{e\cot(c+dx)}\sec(c+dx)\,dx+\int\sqrt{e\cot(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{e\cot(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.241 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=339

$$\frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}-\frac{a^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a^2\text{ArcTan}(}{\sqrt{2}d\sqrt{e\,\text{co}}}$$

```
[Out] 4*a^2*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)
^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d
*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a^2*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a^2*
ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/ta
n(d*x+c)^(1/2)-1/4*a^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/
(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+2/3*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 30}

$$-\frac{a^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a^2\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}+\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{a^2\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (4*a^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (4*a^2*Cos[c + d*x]*Ellipti
cE[c - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a
^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*
Sqrt[Tan[c + d*x]]) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]
*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c +
d*x]]) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2
]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (2*a^2*Tan[c + d*x])/(3*d*Sq
rt[e*Cot[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{\int (a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}} \\
&= \frac{\int \left(a^2\sqrt{\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{\tan(c\right.}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}} \\
&= \frac{a^2\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}+\frac{a^2\int \sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}+\frac{(2a^2)}{\sqrt{e}} \\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{(4a^2)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}+\frac{a^2\text{Subst}\left(\int \sqrt{x}\,dx\right.}{d\sqrt{e\cot(c+dx)}} \\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}-\frac{\left(4a^2\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx}}{\sqrt{e\cot(c+dx)}\,\sqrt{\sin(c}} \\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}-\frac{(4a^2\cos(c+dx))\int \sqrt{\sin(2c+2dx)}}{\sqrt{e\cot(c+dx)}\,\sqrt{\sin(2c+2dx)}} \\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\,\sqrt{\sin(2c+2dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} \\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\,\sqrt{\sin(2c+2dx)}}+\frac{a^2\log\left(1-\sqrt{2}\sqrt{\text{ta}}\right.}{2\sqrt{2}\,d\sqrt{e\cot(c}} \\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\,\sqrt{\sin(2c+2dx)}}-\frac{a^2\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}\,d\sqrt{e\cot(c+dx}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 23.94, size = 221, normalized size = 0.65

$$
\frac{a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\left(4\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\sin\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{e\cot(c+dx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^5*(4*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]
 + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c +
d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] -
 Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*Sec[c + d*x]*Sec[ArcC
ot[Cot[c + d*x]]/2]^4*Sin[(c + d*x)/2])/(3*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 1504, normalized size = 4.44

method	result	size
default	Expression too large to display	1504

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*EllipticPi((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2+3*I*cos(d*x+c)*(-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*((-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+c
os(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2)*cos(d*x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))-3*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-
1/2*I,1/2*2^(1/2))-3*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
```

```
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-12*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))+24*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-14*cos(d*x
+c)^2*2^(1/2)+12*2^(1/2)*cos(d*x+c)+2*2^(1/2))/sin(d*x+c)^5/cos(d*x+c)/(e*c
os(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)
```

Maxima [A]
time = 0.24, size = 133, normalized size = 0.39

$$\frac{\left(8\,a^{2}\tan\left(dx+c\right)^{\frac{3}{2}}-3\left(2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^{2}\right)e^{\left(-\frac{1}{2}\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*a^2*tan(d*x + c)^(3/2) - 3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(t
an(d*x + c)))) + sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) - sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2)*e^
(-1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{1}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{2\sec\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{\sec^{2}\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\cot\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2), x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=375

$$\frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}}-\frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}}+$$

```
[Out] 2*a^2*cot(d*x+c)/d/(e*cot(d*x+c))^(3/2)+4/3*a^2*csc(d*x+c)/d/(e*cot(d*x+c))
^(3/2)+2/3*a^2*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*
Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(
d*x+c))^(3/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^
(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d
/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*a^2*ln(1-2^(1/2)*tan(d*x
+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a
^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)
/tan(d*x+c)^(3/2)+2/5*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(3/2)
```

Rubi [A]
time = 0.24, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3985, 3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 30}

$$\frac{a^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a^2\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}+\frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}}+\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{a^2\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{2a^2\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3d(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Cot[c + d*x])/(d*(e*Cot[c + d*x])^(3/2)) + (4*a^2*Csc[c + d*x])/(3*d
*(e*Cot[c + d*x])^(3/2)) - (2*a^2*Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - P
i/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a^2*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Ta
n[c + d*x]^(3/2)) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d
*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x
]^(3/2)) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt
[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (2*a^2*Tan[c + d*x])/(5*
d*(e*Cot[c + d*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(a^2\tan^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\tan^{\frac{3}{2}}(c+\right.}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2\int \tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\int \sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{(2a^2)\int}{(e\cot}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{(2a^2)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\cos^{\frac{3}{2}}\right.}{}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{\left(2a^2\cot(\right.}{}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\right.}{3d(e\cot(c+}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\right.}{3d(e\cot(c+}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\right.}{3d(e\cot(c+}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} - \frac{2a^2\cot(c+dx)\csc(c+dx)F\left(c-\right.}{3d(e\cot(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 38.60, size = 127, normalized size = 0.34

$$\frac{a^2\left({}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+2\left(5\cot^2(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-\tan^2(c+dx)\right)\right)\right)(1+\sec(c+dx))^2\sec^4\left(\frac{1}{2}\cot^{-1}(\cot(c+dx))\right)\sin^2(c+dx)}{10de\sqrt{e\cot(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + 2*(5*Cot[c + d*x]
^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Hypergeometric2F1[1/2
, 5/4, 9/4, -Tan[c + d*x]^2]))*(1 + Sec[c + d*x])^2*Sec[ArcCot[Cot[c + d*x]
]/2]^4*Sin[c + d*x]^2)/(10*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 731, normalized size = 1.95

method	result
default	$-\frac{a^2(-1+\cos(dx+c))\left(15i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,(\cos^2(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*a^2/d*(-1+cos(d*x+c))*(15*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*I*EllipticPi((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+10*sin(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-15*sin(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-15*sin(d*
x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-24*cos(d*x+c)^3*2^(1/2)+4*cos(d*x+c)^2*2^(1/2)+14*2^(1/2)*cos(d*x+c)+6*2^
(1/2))*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)/sin(d*x+
c)^5*2^(1/2)
```

Maxima [A]
time = 0.24, size = 143, normalized size = 0.38

$$\frac{\left(8\,a^{2}\tan\left(dx+c\right)^{\frac{5}{2}}+5\left(2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+8\,\sqrt{\tan\left(dx+c\right)}\right)a^{2}\right)e^{\left(-\frac{3}{2}\right)}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(8*a^2*tan(d*x + c)^(5/2) + 5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(t
an(d*x + c)))) - sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) + sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*sqrt
(tan(d*x + c)))*a^2)*e^(-3/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{2\sec\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{\sec^{2}\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*cot(c + d*x))**(3/2),
x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\cot\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2), x)
```

3.243 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=405

$$\frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad}-\frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad}+\frac{6(e\,\text{c}}{}$$

```
[Out] 2/5*cot(d*x+c)*(e*cot(d*x+c))^(3/2)*(1-sec(d*x+c))/a/d-2/5*(e*cot(d*x+c))^(
3/2)*(5-3*sec(d*x+c))*tan(d*x+c)/a/d-6/5*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi
+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d
*x+c)*tan(d*x+c)/a/d/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^
(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/4*
(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/a/d*2^(1/2)+1/4*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))*tan(d*x+c)^(3/2)/a/d*2^(1/2)-6/5*(e*cot(d*x+c))^(3/2)*sin(d*x+c)*t
an(d*x+c)^2/a/d
```

Rubi [A]
time = 0.30, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}\,ad}-\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}\,ad}+\frac{2\cot(c+dx)(1-\sec(c+dx))(e\cot(c+dx))^{3/2}}{5ad}-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad}+\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad}-\frac{6\sin(c+dx)\tan^2(c+dx)(e\cot(c+dx))^{3/2}}{5ad}-\frac{2\tan(c+dx)(5-3\sec(c+dx))(e\cot(c+dx))^{3/2}}{5ad}+\frac{6\sin(c+dx)\tan(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{3/2}}{5ad\sqrt{\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(e*Cot[c + d*x])^(3/2)*(1 - Sec[c + d*x]))/(5*a*d) - (2*(e*
Cot[c + d*x])^(3/2)*(5 - 3*Sec[c + d*x])*Tan[c + d*x])/(5*a*d) + (6*(e*Cot[
c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/(5*
a*d*Sqrt[Sin[2*c + 2*d*x]]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Co
t[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d) - (ArcTan[1 + Sqrt[2]*S
qrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d)
 - ((e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d) + ((e*Cot[c + d*x])^(3/2)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d
) - (6*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/(5*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} + \frac{\left(2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+\right.}{5a^2} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad} \\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(}{5ad}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 14.39, size = 316, normalized size = 0.78

$$-\frac{e\sqrt{e\cot(c+dx)}\left(30\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)-30\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)+120\cot^2(c+dx)-24\cot^4(c+dx)+24\cot^4(c+dx)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\tan^2(c+dx)\right)-120\cot^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\tan^2(c+dx)\right)-40\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+15\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-15\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{30ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/30*(e*Sqrt[e*Cot[c + d*x]]*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]]]*Cot[c + d*x]^(3/2) - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) + 120*Cot[c + d*x]^2 - 24*Cot[c + d*x]^4 + 24*Cot[c +
d*x]^4*Hypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 120*Cot[c + d
*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 40*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 15*Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 15*Sqrt[2]*Cot[c + d*x]^(3/
2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*x]*(1 + Sq
rt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 2149, normalized size = 5.31

method	result	size
default	Expression too large to display	2149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a/d*(-1+cos(d*x+c))*(5*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-5*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)-5*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)
^2+5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)+5*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)
^2-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ell
ipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+10*cos(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-
```

```
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*I*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+10*cos(d*x+c)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*I*cos(d*x+c)*(-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-24*cos(d*x+c)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))+5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))+5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticP
i((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)-4*2^(1/2)*cos(d*x+c))*(e*cos(
d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2/sin(d*x+c)*2^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e \cot{(c+dx)})^{\frac{3}{2}}}{\sec{(c+dx)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(c+d\,x)}\,\left(e\cot(c+d\,x)\right)^{3/2}}{a\,\left(\cos{(c+d\,x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*cot(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.244 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad}-\frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2}}{3ad}$$

```
[Out] 2/3*cot(d*x+c)*(1-sec(d*x+c))*(e*cot(d*x+c))^(1/2)/a/d+1/3*(sin(c+1/4*Pi+d*
x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+
c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/a/d+1/2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)+1/2*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/
2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d
*x+c)^(1/2)/a/d*2^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*co
t(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3985, 3973, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$-\frac{\sqrt{\tan(c+dx)}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}\,ad}+\frac{\sqrt{\tan(c+dx)}\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}\,ad}+\frac{2\cot(c+dx)(1-\sec(c+dx))\sqrt{e\cot(c+dx)}}{3ad}-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad}+\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad}-\frac{\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\cot(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Cot[c + d*x]]*(1 - Sec[c + d*x]))/(3*a*d) - (Sqrt[e*
Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*
x]])/(3*a*d) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) - (Sqrt[e*Cot[c +
d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]]
)/(2*Sqrt[2]*a*d) + (Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} + \frac{\left(2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx \cdots}{3a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)} \cdots}{3a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\,\sqrt{\sin(c+dx)} \cdots}{3a\sqrt{c \cdots}}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\,\sec(c+dx)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\right| \cdots}{3a \cdots}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\right| \cdots}{3a \cdots}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\right| \cdots}{3a \cdots}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}\,(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,F\left(c-\frac{\pi}{4}+dx\right| \cdots}{3a \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 31.86, size = 135, normalized size = 0.42

$$-\frac{4\sqrt{e\cot(c+dx)}\,\csc(c+dx)\left(\cot^2(c+dx)\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\tan^2(c+dx)\right)+3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)+\cot^2(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)\right)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{3ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cot[c + d*x]^2*Hypergeometric2F1[-3/
4, -1/2, 1/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c
```

```
+ d*x]^2] + Cot[c + d*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*
x]^2]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 1289, normalized size = 3.97

method	result	size
default	Expression too large to display	1289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a/d*(e*cos(d*x+c)/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*
(3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*
x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/
2+1/2*I,1/2*2^(1/2))+3*I*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2)-3*I*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(
d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)-8*sin(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2)+2*cos(d*x+c)^2*2^(1/2)-2*2^(1/2)*cos(d*x+c))/sin(d*x+c)^5/cos(d
*x+c)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cot(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(sqrt(e*cot(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\sqrt{e\cot(c+d\,x)}}{a\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*cot(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.245 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=347

$$\frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}}+\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}-\frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}-\frac{\mathrm{ArcTan}\Big(1-}{\sqrt{2}\,ad\sqrt{e\cot(}}$$

```
[Out] 2*cot(d*x+c)*(1-sec(d*x+c))/a/d/(e*cot(d*x+c))^(1/2)+2*sin(d*x+c)/a/d/(e*co
t(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*
EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a/d/(e*cot(d*x+c))^(1/2)/sin(2*d*x+2*c
)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(e*cot(d*x+c))^
(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(
e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.26, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{ad\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x]*(1 - Sec[c + d*x]))/(a*d*Sqrt[e*Cot[c + d*x]]) + (2*Sin[c +
 d*x])/(a*d*Sqrt[e*Cot[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*
x, 2])/(a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]
]*Sqrt[Tan[c + d*x]]) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/
(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[
Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
```

```
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\tan(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{\tan\cdots}}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(\cdots}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{\int \sqrt{\tan(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+d\cdots}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\int\cos(\cdots}{a\sqrt{e\cot\cdots}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{\left(2\sqrt{\cos\cdots}\right.}{\cdots}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{(2\cos(c\cdots}{a\sqrt{e\,c\cdots}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(\cdots}{ad\sqrt{e\,c\cdots}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(\cdots}{ad\sqrt{e\,c\cdots}}$$

$$= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(\cdots}{ad\sqrt{e\,c\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 13.87, size = 249, normalized size = 0.72

$$-\frac{\csc(c+dx)\left(24\cot^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\tan^2(c+dx)\right)+8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-3\cot^{\frac{3}{2}}(c+dx)\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{6ad\sqrt{e\cot(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Csc[c + d*x]*(24*Cot[c + d*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -T
an[c + d*x]^2] + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 3*Co
t[c + d*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt
[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]
*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c +
 d*x)/2]^2)/(a*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 359, normalized size = 1.03

method	result
default	$-\frac{\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(i\,\text{Elli}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(1+cos(d*x+c)
)^2*(-1+cos(d*x+c))*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+c)^3/(e*cos(d*x+c)/sin(
d*x+c))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)*sqrt(cot(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\sec(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,\sqrt{e\cot(c+d\,x)}\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.246 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=290

$$\frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}}+\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\text{ArcT}}{\sqrt{2}\,ad}$$

```
[Out] -cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipt
icF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(3/2
)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/
tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^
(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.21, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3985, 3973, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{ad(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*
x]])/(a*d*(e*Cot[c + d*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2
)) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{-a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,d\ldots}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(\ldots}$$

$$= \frac{\cos^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} - \frac{\text{Subst}\Big(\int \ldots}{ad(e\cot\ldots}$$

$$= \frac{\left(\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\ldots}{a(e\cot(c+dx))^{3/2}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} - \ldots$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} - \ldots$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \ldots$$

$$= \frac{\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 57.99, size = 112, normalized size = 0.39

$$\frac{4\cot^2(c+dx)\csc(c+dx)\left(3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)+\cot^2(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{3ad(e\cot(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*Cot[c + d*x]^2*Csc[c + d*x]*(3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c +
 d*x]^2] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*
```

```
(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*(e*Cot[c + d*x])^(3/2
))
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 325, normalized size = 1.12

method	result
default	$\frac{(1+\cos(dx+c))^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/d*(1+cos(d*x+c))^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF((-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)/(e*cos(d*x+c)/sin(
d*x+c))^(3/2)/sin(d*x+c)^4*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((a*sec(d*x + c) + a)*cot(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\cot(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.247 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=325

$$\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}-\frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}}+\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+}$$

```
[Out] 2*cos(d*x+c)*cot(d*x+c)/a/d/(e*cot(d*x+c))^(5/2)+2*cos(d*x+c)*cot(d*x+c)^2*
(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2
^(1/2))/a/d/(e*cot(d*x+c))^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c
)^(5/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(5
/2)*2^(1/2)/tan(d*x+c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

Rubi [A]

time = 0.23, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3985, 3973, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}+\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}+\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{2\cos(c+dx)\cot^2(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{ad\sqrt{\sin(2c+2dx)}(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*Cot[c + d*x])/(a*d*(e*Cot[c + d*x])^(5/2)) - (2*Cos[c + d*x
]*Cot[c + d*x]^2*EllipticE[c - Pi/4 + d*x, 2])/(a*d*(e*Cot[c + d*x])^(5/2)*
Sqrt[Sin[2*c + 2*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a
*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x
])^(5/2)*Tan[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{\int \sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int \sec(c+dx)\sqrt{\tan(c+a}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos^{\frac{5}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}}{a(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{(2\cos(c+dx)\cot^2(c+dx))\int\sqrt{\mathrm{si}}}{a(e\cot(c+dx))^{5/2}\sqrt{\sin(2c}}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}\right.}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+}}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}\right.}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+}}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}\right.}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 70.86, size = 194, normalized size = 0.60

$$-\frac{\sqrt{e\cot(c+dx)}\left(-8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{6ade^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Sqrt[e*Cot[c + d*x]]*(-8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*
x]^2] + 3*Sqrt[2]*Cot[c + d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]))*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d*e^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 1443, normalized size = 4.44

method	result	size
default	Expression too large to display	1443

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))+2*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*
cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*(-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))
^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d
*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-4*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cos(d*x+c)-2*2^(1/2))*cos(d*x+c)^2*(1+cos
(d*x+c))^2/sin(d*x+c)^7/(e*cos(d*x+c)/sin(d*x+c))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(-5/2)*integrate(1/((a*sec(d*x + c) + a)*cot(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{5/2}\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.248 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=335

$$-\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}}-\frac{\text{ArcTan}(}{\sqrt{2}\,ad(e\,c}$$

```
[Out] -2/3*cot(d*x+c)^3*(3-sec(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)+1/3*cot(d*x+c)^3*
csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/
4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(7/2)+1/2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(
7/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2
)/tan(d*x+c)^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot
(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

Rubi [A]
time = 0.25, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3985, 3973, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{\sqrt{\sin(2c+2dx)}\cot^3(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3ad(e\cot(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-2*Cot[c + d*x]^3*(3 - Sec[c + d*x]))/(3*a*d*(e*Cot[c + d*x])^(7/2)) - (Co
t[c + d*x]^3*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x
]])/(3*a*d*(e*Cot[c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/
2)) - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*
Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,d}{3a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cos^{\frac{7}{2}}(c+dx)\int \frac{}{\sqrt{\cos(c+}}}{3a(e\cot(c+dx))^{7/}}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\left(\cot^3(c+dx)\csc(c+dx)\right.}{3a}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F}{3ad(e\,c}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F}{3ad(e\,c}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F}{3ad(e\,c}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)F}{3ad(e\,c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 43.96, size = 130, normalized size = 0.39

$$-\frac{4\sqrt{e\cot(c+dx)}\csc(c+dx)\left(3-3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\tan^2(c+dx)\right)+3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(c+dx)\right)+\cot^2(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{3ade^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(3 - 3*Hypergeometric2F1[-1/2, 1/4, 5
/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]
+ Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*(1 + Sqrt
[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*e^4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 708, normalized size = 2.11

method	result
default	$-\frac{(-1+\cos(dx+c))\left(3i\cos(dx+c)\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/d*(-1+cos(d*x+c))*(3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+
c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(d*x+c)*
sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^2*2
^(1/2)-8*2^(1/2)*cos(d*x+c)+2*2^(1/2))*cos(d*x+c)^2*(1+cos(d*x+c))^2/(e*cos
(d*x+c)/sin(d*x+c))^(7/2)/sin(d*x+c)^7*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate(1/((a*sec(d*x + c) + a)*cot(d*x + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\cot(c+d\,x))^{7/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

3.249 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=371

$$-\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}+\frac{6\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\right|2}{5ad(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}}$$

```
[Out] -6/5*cos(d*x+c)*cot(d*x+c)^3/a/d/(e*cot(d*x+c))^(9/2)-2/15*cot(d*x+c)^3*(5-
3*sec(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)-6/5*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1
/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/
a/d/(e*cot(d*x+c))^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x
+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+
1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)*2^(1
/2)/tan(d*x+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*c
ot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

Rubi [A]

time = 0.26, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3973, 3966, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}-\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{6\cos(c+dx)\cot^4(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{5ad\sqrt{\sin(2c+2dx)}\,(e\cot(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-6*Cos[c + d*x]*Cot[c + d*x]^3)/(5*a*d*(e*Cot[c + d*x])^(9/2)) - (2*Cot[c
+ d*x]^3*(5 - 3*Sec[c + d*x]))/(15*a*d*(e*Cot[c + d*x])^(9/2)) + (6*Cos[c +
 d*x]*Cot[c + d*x]^4*EllipticE[c - Pi/4 + d*x, 2])/(5*a*d*(e*Cot[c + d*x])^
(9/2)*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c
 + d*x])^(9/2)*Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{5}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{2\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx\right.}{5a^2(e\cot(c+dx))^{9}}$$

$$= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{3\int\sec(c+dx)\sqrt{\tan(c+}}{5a(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx}{15ad(e\cot(c+dx))^{9/2}}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx}{15ad(e\cot(c+dx))^{9/2}}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx}{15ad(e\cot(c+dx))^{9/2}}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx}{15ad(e\cot(c+dx))^{9/2}}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx}{15ad(e\cot(c+dx))^{9/2}}$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx}{15ad(e\cot(c+dx))^{9/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 28.18, size = 261, normalized size = 0.70

$$\frac{\sqrt{e\cot(c+dx)}\left(-8+6\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)-6\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)+8\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)-8\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2(c+dx)\right)+3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\sin^2\left(\frac{1}{2}(c+dx)\right)}{6ade^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*(-8 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot
[c + d*x]^(3/2) + 8*Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 8*
Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c + d*x]^
(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*Cot[c
+ d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*
x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(6*a*d*e^5)
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 1529, normalized size = 4.12

method	result	size
default	Expression too large to display	1529

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/a/d*(-1+cos(d*x+c))^2*(15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))+15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-15*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^3*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2)*cos(d*x+c)^3*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
```

```
,1/2*2^(1/2))-15*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))-36*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))+18*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Ell
ipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-15*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*((-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+28*cos(d
*x+c)^3*2^(1/2)-24*cos(d*x+c)^2*2^(1/2)-10*2^(1/2)*cos(d*x+c)+6*2^(1/2))*co
s(d*x+c)^2*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(9/2)/sin(d*x+c)^9*2^
(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(-9/2)*integrate(1/((a*sec(d*x + c) + a)*cot(d*x + c)^(9/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{a\,(e\cot(c+d\,x))^{9/2}\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)), x)
```

3.250 $\int \frac{1}{\sqrt{e \cot(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=413

$$\frac{2\cot(c+dx)}{a^2 d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2 d\sqrt{e\cot(c+dx)}}-\frac{4\cot^3(c+dx)}{5a^2 d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2 d\sqrt{e\cot(c+dx)}}-\frac{12\,\mathrm{c}}{5a^2 d\sqrt{}}$$

```
[Out] 2*cot(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)-12/5*cos(d*x+c)*cot(d*x+c)/a^2/d/(e
*cot(d*x+c))^(1/2)-4/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(1/2)+4/5*cot(d*x+
c)^2*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*
x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e
*cot(d*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/
4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1
/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^
(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 19, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3985, 3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2689, 2688, 2695, 2652, 2719, 2687, 30}

$$-\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2 d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,a^2 d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{4\cot^3(c+dx)}{5a^2 d\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)}{a^2 d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2 d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2 d\sqrt{e\cot(c+dx)}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2 d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2 d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{5a^2 d\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x])/(a^2*d*Sqrt[e*Cot[c + d*x]]) - (12*Cos[c + d*x]*Cot[c + d*
x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]) - (4*Cot[c + d*x]^3)/(5*a^2*d*Sqrt[e*Cot
[c + d*x]]) + (4*Cot[c + d*x]^2*Csc[c + d*x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]
) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2])/(5*a^2*d*Sqrt[e*Cot[c +
d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e
*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{7}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}}$$

$$= \frac{\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\,\sqrt{\tan(}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{}{a^2\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(}{5a^2d\sqrt{e\,c}}$$

Mathematica [F]

time = 14.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 2153, normalized size = 5.21

method	result	size
default	Expression too large to display	2153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(-5*I*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))+5*I*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))-5*I*cos(d*x+c)^2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))-5*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2)*cos(d*x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2*2^(1/2))-12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))-10*I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+
c)+10*I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))-10*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x
```

```
+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
-10*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4
8*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-24*cos(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-5*(-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))+24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*2^(1/2)-2*2^(
1/2)*cos(d*x+c))/sin(d*x+c)^7/(e*cos(d*x+c)/sin(d*x+c))^(1/2)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cot(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e \cot{\left(c + dx \right)}} \sec^{2}{\left(c + dx \right)} + 2 \sqrt{e \cot{\left(c + dx \right)}} \sec{\left(c + dx \right)} + \sqrt{e \cot{\left(c + dx \right)}}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*s
ec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.251 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=359

$$-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{2\cot(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+}}{3a^2d(e\cot(c+dx))^{3/2}}$$

```
[Out] -4/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(3/2)+4/3*cot(d*x+c)^2*csc(d*x+c)/a^
2/d/(e*cot(d*x+c))^(3/2)-2/3*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1
/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(
1/2)/a^2/d/(e*cot(d*x+c))^(3/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2
/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x
+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.28, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3985, 3973, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 30}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}+\frac{2\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3a^2d(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (4*Cot[c + d*x]^2*Cs
c[c + d*x])/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (2*Cot[c + d*x]*Csc[c + d*x]
*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d
*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[
c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/
2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(}$$

$$= -\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{}{3a^2(e}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{(2\cot}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{(2\cot}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(}{}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(}{}$$

Mathematica [F]
time = 70.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 1287, normalized size = 3.58

method	result	size
default	Expression too large to display	1287

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(d*x+c)*(
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(d*x+c)*sin(
d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(d*
x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*I*sin(d*x
+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-3*I*sin(d*x+
c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*
EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+3*sin(d*x+c)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)-10*sin(d*x+c)*Ellip
ticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+4*cos(d*x+c)^2*2^(1/2)-4*2^(1/2)
*cos(d*x+c))*cos(d*x+c)*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(3/2)/si
n(d*x+c)^7*2^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec^2(c+dx)+2(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*cot(c + d*x))**(
3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{a^2\,{\left(e\,\cot(c+d\,x)\right)}^{3/2}\,{\left(\cos(c+d\,x)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.252 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=355

$$-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}}+\frac{\text{ArcTa}}{\sqrt{2}\,a^2d(}$$

```
[Out] -4*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(5/2)+4*cos(d*x+c)*cot(d*x+c)^3/a^2/d/
(e*cot(d*x+c))^(5/2)-4*cos(d*x+c)*cot(d*x+c)^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/
sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+c))
^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(
e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/4*ln(1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+
c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))
^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

Rubi [A]
time = 0.29, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3985, 3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 30}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}+\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^2(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{a^2d\sqrt{\sin(2c+2dx)}\,(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c
+ d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^2*E
llipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(5/2)*Sqrt[Sin[2*c + 2
*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(S
qrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1 - Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*T
an[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan}$$

$$= -\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} - \frac{}{a^2(e}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{(4\,\mathrm{co}}{}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{(4\cos}{}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos}{a^2}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos}{a^2}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos}{a^2}$$

$$= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos}{a^2}$$

Mathematica [F]
time = 5.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 367, normalized size = 1.03

method	result
default	$(1+\cos(dx+c))^2 \left(i \,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2}-\frac{i}{2}, \frac{\sqrt{2}}{2}\right) - i \,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2/d*(1+cos(d*x+c))^2*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*EllipticF((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+8*EllipticE((-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2/(e*cos(d*x+c)/sin
(d*x+c))^(5/2)/sin(d*x+c)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate(1/((a*sec(d*x + c) + a)^2*cot(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.253 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=321

$$\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{\sin(2c+2dx)}}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{}\right.}{\sqrt{2}\,a^2d(e\cot(c+dx))}$$

```
[Out] 2*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(7/2)+2*cot(d*x+c)^3*csc(d*x+c)*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))
*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(7/2)+1/2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)
^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(
7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

Rubi [A]
time = 0.26, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3973, 3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 30}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}-\frac{2\sqrt{\sin(2c+2dx)}\,\cot^3(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{a^2d(e\cot(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(7/2)) - (2*Cot[c + d*x]^3*Csc[c
 + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*(e*Cot[
c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e
*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) - Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x]
)^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}} - \frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\right) d}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= \frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\int \frac{\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}}{a^2(e\cot(c+dx))^{7/2}\tan}$$

$$= -\frac{\left(2\cos^{\frac{7}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+dx)} + \frac{\text{Su}}{a^2}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{\left(2\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c}\right.}{a^2(e\cot(c+}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+\right.}{a^2d(e\cot(c+dx}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+\right.}{a^2d(e\cot(c+dx}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+\right.}{a^2d(e\cot(c+dx}$$

$$= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)F\left(c-\frac{\pi}{4}+\right.}{a^2d(e\cot(c+dx}$$

Mathematica [F]
time = 65.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]

[Out] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2), x]

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 665, normalized size = 2.07

method	result
default	$\left(-i\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/2/a^2/d*(-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*si
n(d*x+c)+I*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d
*x+c)-sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
-sin(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+6*si
n(d*x+c)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)+2*2^(1/2)*cos(d*x+
c)-2*2^(1/2))*(-1+cos(d*x+c))*cos(d*x+c)^3*(1+cos(d*x+c))^2/sin(d*x+c)^7/(e
*cos(d*x+c)/sin(d*x+c))^(7/2)*2^(1/2)

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{7/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

3.254 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=357

$$\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}+\frac{4\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}}-\frac{\text{ArcTan}}{\sqrt{2}\,a^2d(e}$$

```
[Out] 2/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^3/a^2/d
/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)
/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+c)
)^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/
(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/4*ln(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x
+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c)
)^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

Rubi [A]
time = 0.28, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3985, 3973, 3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 30}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{4\cos(c+dx)\cot^4(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{a^2d\sqrt{\sin(2c+2dx)}\,(e\cot(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(9/2)) - (4*Cos[c + d*x]*Cot[c
 + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(9/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^4*
EllipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(9/2)*Sqrt[Sin[2*c +
2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
 d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + Log[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*
Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2
*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= \frac{\int (-a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

$$= \frac{\int \left(a^2\sqrt{\tan(c+dx)} - 2a^2\sec(c+dx)\sqrt{\tan(c+dx)} + a^2 \sec\right.}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx}$$

$$= \frac{\int \sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\int \sec^2(c+dx)\sqrt{\tan(c+}}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(}$$

$$= -\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\int \cos(c+dx)\sqrt{\tan(c+dx)}}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+d}$$

$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{\left(4\cos^{\frac{9}{2}}\right.}{}$$

$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{(4\cos(c}{}$$

$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c}{a^2d(}$$

$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c}{a^2d(}$$

$$= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c}{a^2d(}$$

Mathematica [F]
time = 57.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 1504, normalized size = 4.21

method	result	size
default	Expression too large to display	1504

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2/d*(-1+cos(d*x+c))^2*(3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2)*cos(d*x+c)^2-3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^2+3*I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))-3*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+12*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2)*cos(d*x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2*2^(1/2))-24*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))-3*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-3*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+1
2*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-24*cos(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*
```

```
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+10*cos(d*x+c)^2*2^(1/2)-12*
2^(1/2)*cos(d*x+c)+2*2^(1/2))*cos(d*x+c)^3*(1+cos(d*x+c))^2/(e*cos(d*x+c)/s
in(d*x+c))^(9/2)/sin(d*x+c)^9*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(-9/2)*integrate(1/((a*sec(d*x + c) + a)^2*cot(d*x + c)^(9/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{9/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)^2), x)
```

3.255 $\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=389

$$\frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^5(c+dx)\csc(c+dx)F(c-}{3a^2d(e\cot(c}$$

```
[Out] 2/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(11/2)+2*cot(d*x+c)^5/a^2/d/(e*cot(d*
x+c))^(11/2)-4/3*cot(d*x+c)^4*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(11/2)-2/3*co
t(d*x+c)^5*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipti
cF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(11
/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(
1/2)/tan(d*x+c)^(11/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(
d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)-1/4*ln(1+
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/ta
n(d*x+c)^(11/2)
```

Rubi [A]

time = 0.29, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3985, 3973, 3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 30}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}+\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}+\frac{2\sqrt{\sin(2c+2dx)}\cot^5(c+dx)\csc(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{3a^2d(e\cot(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(5*a^2*d*(e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5)/(
a^2*d*(e*Cot[c + d*x])^(11/2)) - (4*Cot[c + d*x]^4*Csc[c + d*x])/(3*a^2*d*(
e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5*Csc[c + d*x]*EllipticF[c - Pi/4
 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d*x])^(11/2)) + ArcT
an[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*T
an[c + d*x]^(11/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d
*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2)) + Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*
x]^(11/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*
a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int (-a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int \left(a^2\tan^{\frac{3}{2}}(c+dx) - 2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx) + a^2\sec^2(\right.}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

$$= \frac{\int \tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} + \frac{\int \sec^2(c+dx)\tan^{\frac{3}{2}}(c}{a^2(e\cot(c+dx))^{11/2}\,\mathrm{ta}}$$

$$= \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} + \frac{}{3a^2(e\,\mathrm{c}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+}{3a^2d(e\,\mathrm{c}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+}{3a^2d(e\,\mathrm{c}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+}{3a^2d(e\,\mathrm{c}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+}{3a^2d(e\,\mathrm{c}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+}{3a^2d(e\,\mathrm{c}}$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+}{3a^2d(e\,\mathrm{c}}$$

Mathematica [F]
time = 63.45, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

[Out] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2), x]

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 731, normalized size = 1.88

method	result
default	$-\frac{(-1+\cos(dx+c))\left(15i\sin(dx+c)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}}\,(\cos^2(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -1/30/a^2/d*(-1+cos(d*x+c))*(15*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*I*EllipticPi((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d
x+c))^(1/2)((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*sin(d*x+c)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-15*sin(d*
x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)+50*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)-24*cos(d*x+c)^3*2^(1/2)+44*cos(d*x+c)^2*2^(1/2)-26*2^(1/2)*cos(d*x+c)+6*2
^(1/2))*cos(d*x+c)^3*(1+cos(d*x+c))^2/(e*cos(d*x+c)/sin(d*x+c))^(11/2)/sin(
d*x+c)^9*2^(1/2)

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(11/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{11/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(11/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(11/2)*(cos(c + d*x) + 1)^2), x)
```

3.256 $\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{a\log(\cos(c+dx))}{d}-\frac{16b\sec(c+dx)}{35d}+\frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d}-\frac{(35a+24b\sec(c+dx))\tan^4(c}{140d}$$

```
[Out] a*ln(cos(d*x+c))/d-16/35*b*sec(d*x+c)/d+1/70*(35*a+16*b*sec(d*x+c))*tan(d*x
+c)^2/d-1/140*(35*a+24*b*sec(d*x+c))*tan(d*x+c)^4/d+1/42*(7*a+6*b*sec(d*x+c
))*tan(d*x+c)^6/d
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3966, 3969, 3556, 2686, 8}

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{140d}+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{70d}+\frac{a\log(\cos(c+dx))}{d}-\frac{16b\sec(c+dx)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (16*b*Sec[c + d*x])/(35*d) + ((35*a + 16*b*Sec[c
+ d*x])*Tan[c + d*x]^2)/(70*d) - ((35*a + 24*b*Sec[c + d*x])*Tan[c + d*x]^4
)/(140*d) + ((7*a + 6*b*Sec[c + d*x])*Tan[c + d*x]^6)/(42*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
```

```
1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx = \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} - \frac{1}{7}\int (7a+6b\sec(c+dx))\tan^5($$
$$= -\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} + \frac{(7a+6b\sec(c+dx))\tan^6(}{42d}$$
$$= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b\sec(c+dx))\tan^4}{140d}$$
$$= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b\sec(c+dx))\tan^4}{140d}$$
$$= \frac{a\log(\cos(c+dx))}{d} + \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+}{}$$
$$= \frac{a\log(\cos(c+dx))}{d} - \frac{16b\sec(c+dx)}{35d} + \frac{(35a+16b\sec(c+dx))\tan^2}{70d}$$

Mathematica [A]
time = 0.50, size = 106, normalized size = 0.95

$$-\frac{b\sec(c+dx)}{d} + \frac{b\sec^3(c+dx)}{d} - \frac{3b\sec^5(c+dx)}{5d} + \frac{b\sec^7(c+dx)}{7d} + \frac{a(12\log(\cos(c+dx)) + 6\tan^2(c+dx) - 3\tan^4(c+dx) + 2\tan^6(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/d - (3*b*Sec[c + d*x]^5)/(5*d) +
 (b*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A]
time = 0.13, size = 159, normalized size = 1.43

method	result

derivativedivides	$\frac{b\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}\right.}{d}$
default	$\frac{b\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2\left(105b\,e^{13i(dx+c)}-315a\,e^{12i(dx+c)}+210b\,e^{11i(dx+c)}-945a\,e^{10i(dx+c)}+903b\,e^{9i(dx+c)}-1820a\,e^{8i(dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/7*sin(d*x+c)^8/cos(d*x+c)^7-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35*s
in(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)^6
+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a*(1/6*tan(d*x+c)^6-1/4*tan
(d*x+c)^4+1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.31, size = 94, normalized size = 0.85

$$\frac{420\,a\log\left(\cos\left(dx+c\right)\right)-\frac{420\,b\cos(dx+c)^6-630\,a\cos(dx+c)^5-420\,b\cos(dx+c)^4+315\,a\cos(dx+c)^3+252\,b\cos(dx+c)^2-70\,a\cos(dx+c)-60\,b}{\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*b*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*b*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*b*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*b)/cos(d*x + c)^7)/d
```

Fricas [A]
time = 3.61, size = 101, normalized size = 0.91

$$\frac{420\,a\cos\left(dx+c\right)^7\log\left(-\cos\left(dx+c\right)\right)-420\,b\cos\left(dx+c\right)^6+630\,a\cos\left(dx+c\right)^5+420\,b\cos\left(dx+c\right)^4-315\,a\cos\left(dx+c\right)^3-252\,b\cos\left(dx+c\right)^2+70\,a\cos\left(dx+c\right)+60\,b}{420\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*b*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*b*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*b*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*b)/(d*cos(d*x + c)^7)
```

Sympy [A]
time = 1.23, size = 148, normalized size = 1.33

$$\begin{cases} -\frac{a \log \left(\tan^{2}\left(c+dx\right)+1\right)}{2d} + \frac{a \tan^{6}\left(c+dx\right)}{6d} - \frac{a \tan^{4}\left(c+dx\right)}{4d} + \frac{a \tan^{2}\left(c+dx\right)}{2d} + \frac{b \tan^{6}\left(c+dx\right) \sec\left(c+dx\right)}{7d} - \frac{6b \tan^{4}\left(c+dx\right) \sec\left(c+dx\right)}{35d} + \frac{8b \tan^{2}\left(c+dx\right) \sec\left(c+dx\right)}{35d} - \frac{16b \sec\left(c+dx\right)}{35d} & \text{for}\: d \neq 0 \\ x\left(a + b \sec\left(c\right)\right) \tan^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6/(6*d) - a*
tan(c + d*x)**4/(4*d) + a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**6*sec(c +
 d*x)/(7*d) - 6*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) + 8*b*tan(c + d*x)**2
*sec(c + d*x)/(35*d) - 16*b*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a + b*sec(c
))*tan(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(103) = 206.
time = 3.70, size = 317, normalized size = 2.86

$$-\frac{420\, a \log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right) - 420\, a \log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right) + \frac{1089\, a + 384\, b + \frac{8463\, a\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1} + \frac{2688\, b\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1} + \frac{28749\, a\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}} + \frac{8064\, b\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}} + \frac{56035\, a\left(\cos\left(dx+c\right)-1\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}} + \frac{13440\, b\left(\cos\left(dx+c\right)-1\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}} + \frac{56035\, a\left(\cos\left(dx+c\right)-1\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}} + \frac{28749\, a\left(\cos\left(dx+c\right)-1\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}} + \frac{8463\, a\left(\cos\left(dx+c\right)-1\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}} + \frac{1089\, a\left(\cos\left(dx+c\right)-1\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}}{\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right)^{7}}}{420\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1089*a + 384*b + 84
63*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 28749*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 8064*b
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 56035*a*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 13440*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5
6035*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) -
1)^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 1)^7)/d
```

Mupad [B]
time = 5.24, size = 221, normalized size = 1.99

$$\frac{2\, a \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{12} - 14\, a \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{10} + \frac{128\, a \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{8}}{3} + \left(-\frac{128\, a}{3} - 32\, b\right) \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{6} + \left(14\, a + \frac{96\, b}{5}\right) \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{4} + \left(-2\, a - \frac{32\, b}{5}\right) \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{2} + \frac{32\, b}{35}}{d \left(\tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{14} - 7 \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{12} + 21 \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{10} - 35 \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{8} + 35 \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{6} - 21 \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{4} + 7 \tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{2} - 1\right)} - \frac{2\, a\, \mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\, x}{2}\right)^{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] ((32*b)/35 - tan(c/2 + (d*x)/2)^2*(2*a + (32*b)/5) + tan(c/2 + (d*x)/2)^4*(
14*a + (96*b)/5) - tan(c/2 + (d*x)/2)^6*((128*a)/3 + 32*b) + (128*a*tan(c/2
```

```
 + (d*x)/2)^8)/3 - 14*a*tan(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/
(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2
)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x
)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.257 $\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=84

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}-\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d}+\frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d}$$

```
[Out] -a*ln(cos(d*x+c))/d+8/15*b*sec(d*x+c)/d-1/30*(15*a+8*b*sec(d*x+c))*tan(d*x+
c)^2/d+1/20*(5*a+4*b*sec(d*x+c))*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3966, 3969, 3556, 2686, 8}

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{30d}-\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (8*b*Sec[c + d*x])/(15*d) - ((15*a + 8*b*Sec[c
 + d*x])*Tan[c + d*x]^2)/(30*d) + ((5*a + 4*b*Sec[c + d*x])*Tan[c + d*x]^4)
/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
```

```
1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d}-\frac{1}{5}\int (5a+4b\sec(c+dx))\tan$$
$$= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d}+\frac{(5a+4b\sec(c+dx))\tan^4}{20d}$$
$$= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d}+\frac{(5a+4b\sec(c+dx))\tan^4}{20d}$$
$$= -\frac{a\log(\cos(c+dx))}{d}-\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d}+\frac{(5a-}{}$$
$$= -\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}-\frac{(15a+8b\sec(c+dx))\tan}{30d}$$

Mathematica [A]
time = 0.22, size = 82, normalized size = 0.98

$$\frac{b\sec(c+dx)}{d}-\frac{2b\sec^3(c+dx)}{3d}+\frac{b\sec^5(c+dx)}{5d}-\frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (b*Sec[c + d*x])/d - (2*b*Sec[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.12, size = 123, normalized size = 1.46

method	result
derivativedivides	$\frac{b\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+\right.}{2}\right.}{d}$

default	$\frac{b\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c\right.}{2}\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2b\,e^{9i(dx+c)}-4a\,e^{8i(dx+c)}+\frac{8b\,e^{7i(dx+c)}}{3}-8a\,e^{6i(dx+c)}+\frac{116b\,e^{5i(dx+c)}}{15}-8a\,e^{4i(dx+c)}+\frac{8b\,e^{3i(dx+c)}}{3}-4a\,e^{2}}{d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/5*sin(d*x+c)^6/cos(d*x+c)^5-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*si
n(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a
*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.86

$$-\frac{60\,a\log\left(\cos\left(dx+c\right)\right)-\frac{60\,b\cos(dx+c)^4-60\,a\cos(dx+c)^3-40\,b\cos(dx+c)^2+15\,a\cos(dx+c)+12\,b}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*b*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*b)/cos(d*x + c)^5)/d
```

Fricas [A]
time = 3.33, size = 79, normalized size = 0.94

$$-\frac{60\,a\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-60\,b\cos\left(dx+c\right)^4+60\,a\cos\left(dx+c\right)^3+40\,b\cos\left(dx+c\right)^2-15\,a\cos\left(dx+c\right)-12\,b}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*b*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*b)/(d*cos(d*x
 + c)^5)
```

Sympy [A]
time = 0.55, size = 112, normalized size = 1.33

$$\begin{cases}\frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^4\left(c+dx\right)}{4d}-\frac{a\tan^2\left(c+dx\right)}{2d}+\frac{b\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{5d}-\frac{4b\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{15d}+\frac{8b\sec\left(c+dx\right)}{15d} & \text{for}\ d\neq 0\\ x\left(a+b\sec\left(c\right)\right)\tan^5\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4/(4*d) - a*t
an(c + d*x)**2/(2*d) + b*tan(c + d*x)**4*sec(c + d*x)/(5*d) - 4*b*tan(c + d
*x)**2*sec(c + d*x)/(15*d) + 8*b*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a + b*
sec(c))*tan(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(78) = 156.
time = 1.81, size = 248, normalized size = 2.95

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{137\,a+64\,b+\frac{805\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{320\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1970\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{640\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1970\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{805\,a(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{137\,a(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (137*a + 64*b + 805*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1970*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 640*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^5)/d
```

Mupad [B]
time = 5.85, size = 162, normalized size = 1.93

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(10\,a+\frac{32\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-2\,a-\frac{16\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{16\,b}{15}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*b)/15 - tan(c/2 + (d*x)/2)^2*(2*
a + (16*b)/3) + tan(c/2 + (d*x)/2)^4*(10*a + (32*b)/3) - 10*a*tan(c/2 + (d*
x)/2)^6 + 2*a*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2
 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
+ (d*x)/2)^10 - 1))
```

3.258 $\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=55

$$\frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}+\frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}$$

```
[Out] a*ln(cos(d*x+c))/d-2/3*b*sec(d*x+c)/d+1/6*(3*a+2*b*sec(d*x+c))*tan(d*x+c)^2
/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3966, 3969, 3556, 2686, 8}

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}+\frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (2*b*Sec[c + d*x])/(3*d) + ((3*a + 2*b*Sec[c + d*
x])*Tan[c + d*x]^2)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx &= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-\frac{1}{3}\int (3a+2b\sec(c+dx))\tan\\ &= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-a\int \tan(c+dx)\,dx-\frac{1}{3}(2b)\int\\ &= \frac{a\log(\cos(c+dx))}{d}+\frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-\frac{(2b)\text{Sub}}{}\\ &= \frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}+\frac{(3a+2b\sec(c+dx))\tan^2(c}{6d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 55, normalized size = 1.00

$$-\frac{b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.09, size = 83, normalized size = 1.51

method	result	siz
derivativedivides	$\frac{b\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	83
default	$\frac{b\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	83
risch	$-iax-\frac{2iac}{d}-\frac{2\left(3b\,e^{5i(dx+c)}-3a\,e^{4i(dx+c)}+2b\,e^{3i(dx+c)}-3a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+si
n(d*x+c)^2)*cos(d*x+c))+a*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 50, normalized size = 0.91

$$\frac{6\,a\log\left(\cos\left(dx+c\right)\right)-\frac{6\,b\cos\left(dx+c\right)^2-3\,a\cos\left(dx+c\right)-2\,b}{\cos\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*b)/
cos(d*x + c)^3)/d
```

Fricas [A]
time = 3.41, size = 57, normalized size = 1.04

$$\frac{6\,a\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-6\,b\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+2\,b}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*b*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

Sympy [A]
time = 0.22, size = 76, normalized size = 1.38

$$\begin{cases} -\frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^2\left(c+dx\right)}{2d}+\frac{b\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}-\frac{2b\sec\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x(a+b\sec\left(c\right))\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2/(2*d) + b*
tan(c + d*x)**2*sec(c + d*x)/(3*d) - 2*b*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a + b*sec(c))*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(51) = 102.
time = 0.81, size = 179, normalized size = 3.25

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a+8\,b+\frac{45\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a + 8*b + 45*a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 24*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

Mupad [B]
time = 2.23, size = 102, normalized size = 1.85

$$\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-2\,a-4\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] ((4*b)/3 - tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + 2*a*tan(c/2 + (d*x)/2)^4)/(d*
(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1
)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.259 $\int (a+b\sec(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=25

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {3969, 3556, 2686, 8}

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx = a\int \tan(c+dx)\,dx + b\int \sec(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{b\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}$$
$$= -\frac{a\log(\cos(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{a\log(\cos(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Maple [A]
time = 0.03, size = 23, normalized size = 0.92

method	result	size
derivativedivides	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
default	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
risch	$iax+\frac{2iac}{d}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{d(\mathrm{e}^{2i(dx+c)}+1)}-\frac{a\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*sec(d*x+c)+a*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.04

$$-\frac{a\log(\cos(dx+c)) - \frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
[Out] -(a*log(cos(d*x + c)) - b/cos(d*x + c))/d
```

Fricas [A]
time = 3.66, size = 34, normalized size = 1.36

$$-\frac{a\cos(dx+c)\log(-\cos(dx+c))-b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

Sympy [A]
time = 0.10, size = 37, normalized size = 1.48

$$\begin{cases} \frac{a\log(\tan^2(c+dx)+1)}{2d}+\frac{b\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\sec(c))\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + b*sec(c + d*x)/d, Ne(d, 0)),
(x*(a + b*sec(c))*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(25) = 50.
time = 0.48, size = 107, normalized size = 4.28

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B]
time = 1.30, size = 40, normalized size = 1.60

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.260 $\int \cot(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(1+\cos(c+dx))}{2d}$$

```
[Out] 1/2*(a+b)*ln(1-cos(d*x+c))/d+1/2*(a-b)*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {3968, 2747, 647, 31}

$$\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(\cos(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a + b)*Log[1 - Cos[c + d*x]])/(2*d) + ((a - b)*Log[1 + Cos[c + d*x]])/(2*
d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \int (b+a\cos(c+dx))\csc(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \frac{b+x}{a^2-x^2}\,dx,x,a\cos(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{-a-x}\,dx,x,a\cos(c+dx)\right)}{2d} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a-x}\,dx\right.}{2d}$$
$$= \frac{(a+b)\log(1-\cos(c+dx))}{2d} + \frac{(a-b)\log(1+\cos(c+dx))}{2d}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.40

$$-\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Log[Cos[c/2 + (d*x)/2]])/d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*(Log[
Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.08, size = 33, normalized size = 0.77

method	result	size
derivativedivides	$\frac{b\ln(\csc(dx+c)-\cot(dx+c))+a\ln(\sin(dx+c))}{d}$	33
default	$\frac{b\ln(\csc(dx+c)-\cot(dx+c))+a\ln(\sin(dx+c))}{d}$	33
risch	$-iax-\frac{2iac}{d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}+\frac{\ln(e^{i(dx+c)}-1)b}{d}+\frac{a\ln(e^{i(dx+c)}+1)}{d}-\frac{\ln(e^{i(dx+c)}+1)b}{d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*ln(csc(d*x+c)-cot(d*x+c))+a*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.79

$$\frac{(a-b)\log(\cos(dx+c)+1)+(a+b)\log(\cos(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(cos(d*x + c) + 1) + (a + b)*log(cos(d*x + c) - 1))/d
```

Fricas [A]
time = 3.30, size = 38, normalized size = 0.88

$$\frac{(a-b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+(a+b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*log(1/2*cos(d*x + c) + 1/2) + (a + b)*log(-1/2*cos(d*x + c) +
1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec\left(c+dx\right))\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x), x)
```

Giac [A]
time = 0.44, size = 61, normalized size = 1.42

$$\frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 1.31, size = 51, normalized size = 1.19

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*lo
g(tan(c/2 + (d*x)/2)))/d
```

3.261 $\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=72

$$-\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(1+\cos(c+dx))}{4d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] -1/4*(2*a+b)*ln(1-cos(d*x+c))/d-1/4*(2*a-b)*ln(1+cos(d*x+c))/d-1/2*cot(d*x+
c)^2*(a+b*sec(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3967, 3968, 2747, 647, 31}

$$-\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(\cos(c+dx)+1)}{4d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/4*((2*a + b)*Log[1 - Cos[c + d*x]])/d - ((2*a - b)*Log[1 + Cos[c + d*x]]
)/(4*d) - (Cot[c + d*x]^2*(a + b*Sec[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
```

```
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int \cot(c+dx)(-2a-b\sec(c\\ &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int (-b-2a\cos(c+dx))\csc(\\ &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{a\text{Subst}\left(\int \frac{-b+x}{4a^2-x^2}\,dx,x,-2a\cos\right.}{d}\\ &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{(2a-b)\text{Subst}\left(\int \frac{1}{2a-x}\,dx,x,-2\right.}{4d}\\ &= -\frac{(2a+b)\log(1-\cos(c+dx))}{4d}-\frac{(2a-b)\log(1+\cos(c+dx))}{4d}-\end{aligned}$$

Mathematica [A]

time = 1.40, size = 114, normalized size = 1.58

$$-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/8*(b*Csc[(c + d*x)/2]^2)/d + (b*Log[Cos[(c + d*x)/2]])/(2*d) - (b*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (b*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]

time = 0.12, size = 75, normalized size = 1.04

method	result
derivativedivides	$\frac{b\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$

default	$\frac{b\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{b\,e^{3i(dx+c)}+2a\,e^{2i(dx+c)}+b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-1\right)b}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))

Maxima [A]
time = 0.28, size = 62, normalized size = 0.86

$$-\frac{(2\,a-b)\log\left(\cos\left(dx+c\right)+1\right)+(2\,a+b)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,(b\cos(dx+c)+a)}{\cos(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/4*((2*a - b)*log(cos(d*x + c) + 1) + (2*a + b)*log(cos(d*x + c) - 1) - 2*(b*cos(d*x + c) + a)/(cos(d*x + c)^2 - 1))/d

Fricas [A]
time = 2.80, size = 99, normalized size = 1.38

$$\frac{2\,b\cos\left(dx+c\right)-\left((2\,a-b)\cos\left(dx+c\right)^2-2\,a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left((2\,a+b)\cos\left(dx+c\right)^2-2\,a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,a}{4\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/4*(2*b*cos(d*x + c) - ((2*a - b)*cos(d*x + c)^2 - 2*a + b)*log(1/2*cos(d*x + c) + 1/2) - ((2*a + b)*cos(d*x + c)^2 - 2*a - b)*log(-1/2*cos(d*x + c) + 1/2) + 2*a)/(d*cos(d*x + c)^2 - d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\cot^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c)),x)

[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(66) = 132$.

time = 0.50, size = 170, normalized size = 2.36

$$-\frac{2\,(2\,a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 4*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

Mupad [B]

time = 1.36, size = 86, normalized size = 1.19

$$\frac{a\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a}{8}-\frac{b}{8}\right)}{d}-\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a}{8}+\frac{b}{8}\right)}{d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+\frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a/8 - b/8))/d
- (cot(c/2 + (d*x)/2)^2*(a/8 + b/8))/d - (log(tan(c/2 + (d*x)/2))*(a + b/2)
)/d
```

3.262 $\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(1+\cos(c+dx))}{16d}-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+d$$

[Out] 1/16*(8*a+3*b)*ln(1-cos(d*x+c))/d+1/16*(8*a-3*b)*ln(1+cos(d*x+c))/d-1/4*cot (d*x+c)^4*(a+b*sec(d*x+c))/d+1/8*cot(d*x+c)^2*(4*a+3*b*sec(d*x+c))/d

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3967, 3968, 2747, 647, 31}

$$\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(\cos(c+dx)+1)}{16d}-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]

[Out] ((8*a + 3*b)*Log[1 - Cos[c + d*x]])/(16*d) + ((8*a - 3*b)*Log[1 + Cos[c + d*x]])/(16*d) - (Cot[c + d*x]^4*(a + b*Sec[c + d*x]))/(4*d) + (Cot[c + d*x]^2*(4*a + 3*b*Sec[c + d*x]))/(8*d)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
```

```
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{1}{4}\int \cot^3(c+dx)(-4a-3b\sec\\ &= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b\sec(c+d}{8d}\\ &= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b\sec(c+d}{8d}\\ &= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b\sec(c+d}{8d}\\ &= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b\sec(c+d}{8d}\\ &= \frac{(8a+3b)\log(1-\cos(c+dx))}{16d}+\frac{(8a-3b)\log(1+\cos(c+dx))}{16d}-\end{aligned}$$

Mathematica [A]
time = 0.36, size = 166, normalized size = 1.63

$$\frac{5b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}-\frac{5b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (5*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^4)/(64*d) - (3*b*Log[
Cos[(c + d*x)/2]])/(8*d) + (3*b*Log[Sin[(c + d*x)/2]])/(8*d) + (a*(2*Cot[c
+ d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*
d) - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.14, size = 111, normalized size = 1.09

method	result

derivativedivides	$\frac{b\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}\right.}{d}$
default	$\frac{b\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{5b\,e^{7i(dx+c)}+16a\,e^{6i(dx+c)}+3b\,e^{5i(dx+c)}-16a\,e^{4i(dx+c)}+3b\,e^{3i(dx+c)}+16a\,e^{2i(dx+c)}+5b\,e^{i(dx+c)}}{4d\left(e^{2i(dx+c)}-1\right)^4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*co
s(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/4*cot(d*x+c)
^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 99, normalized size = 0.97

$$\frac{(8\,a-3\,b)\log\left(\cos\left(dx+c\right)+1\right)+(8\,a+3\,b)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(5\,b\cos(dx+c)^3+8\,a\cos(dx+c)^2-3\,b\cos(dx+c)-6\,a\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((8*a - 3*b)*log(cos(d*x + c) + 1) + (8*a + 3*b)*log(cos(d*x + c) - 1)
 - 2*(5*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 3*b*cos(d*x + c) - 6*a)/(co
s(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.66, size = 168, normalized size = 1.65

$$-\frac{10\,b\cos\left(dx+c\right)^3+16\,a\cos\left(dx+c\right)^2-6\,b\cos\left(dx+c\right)-\left((8\,a-3\,b)\cos\left(dx+c\right)^4-2\,(8\,a-3\,b)\cos\left(dx+c\right)^2+8\,a-3\,b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left((8\,a+3\,b)\cos\left(dx+c\right)^4-2\,(8\,a+3\,b)\cos\left(dx+c\right)^2+8\,a+3\,b\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,a}{16\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*b*cos(d*x + c)^3 + 16*a*cos(d*x + c)^2 - 6*b*cos(d*x + c) - ((8*a
 - 3*b)*cos(d*x + c)^4 - 2*(8*a - 3*b)*cos(d*x + c)^2 + 8*a - 3*b)*log(1/2*
cos(d*x + c) + 1/2) - ((8*a + 3*b)*cos(d*x + c)^4 - 2*(8*a + 3*b)*cos(d*x +
 c)^2 + 8*a + 3*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*a)/(d*cos(d*x + c)^4 -
 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\cot^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(94) = 188.
time = 0.53, size = 266, normalized size = 2.61

$$\frac{4\,(8\,a+3\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}-\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(8*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 48*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 12*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

Mupad [B]
time = 1.34, size = 128, normalized size = 1.25

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3\,a}{16}-\frac{b}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a}{64}-\frac{b}{64}\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\left(-3\,a-2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{4}+\frac{b}{4}\right)}{16\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+\frac{3\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((3*a)/16 - b/8))/d - (tan(c/2 + (d*x)/2)^4*(a/64 - b
/64))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^4*(a/4
+ b/4 - tan(c/2 + (d*x)/2)^2*(3*a + 2*b)))/(16*d) + (log(tan(c/2 + (d*x)/2)
)*(a + (3*b)/8))/d
```

3.263 $\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(1+\cos(c+dx))}{32d}-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c}{}$$

```
[Out] -1/32*(16*a+5*b)*ln(1-cos(d*x+c))/d-1/32*(16*a-5*b)*ln(1+cos(d*x+c))/d-1/6*
cot(d*x+c)^6*(a+b*sec(d*x+c))/d+1/24*cot(d*x+c)^4*(6*a+5*b*sec(d*x+c))/d-1/
16*cot(d*x+c)^2*(8*a+5*b*sec(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3967, 3968, 2747, 647, 31}

$$-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(\cos(c+dx)+1)}{32d}-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/32*((16*a + 5*b)*Log[1 - Cos[c + d*x]])/d - ((16*a - 5*b)*Log[1 + Cos[c
+ d*x]])/(32*d) - (Cot[c + d*x]^6*(a + b*Sec[c + d*x]))/(6*d) + (Cot[c + d*
x]^4*(6*a + 5*b*Sec[c + d*x]))/(24*d) - (Cot[c + d*x]^2*(8*a + 5*b*Sec[c +
d*x]))/(16*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{1}{6}\int \cot^5(c+dx)(-6a-5b\sec\\ &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+d}{24d}\\ &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+d}{24d}\\ &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+d}{24d}\\ &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+d}{24d}\\ &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+d}{24d}\\ &= -\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(1+\cos(c+dx)}{32d}\end{aligned}$$

Mathematica [A]

time = 0.64, size = 216, normalized size = 1.66

$$-\frac{11b\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{b\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{5b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{5b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{a(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))}{12d}+\frac{11b\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-11*b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*x)/2]^4)/(32*d) - (b*Csc[
(c + d*x)/2]^6)/(384*d) + (5*b*Log[Cos[(c + d*x)/2]])/(16*d) - (5*b*Log[Sin
[(c + d*x)/2]])/(16*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) + (11*b*Sec
[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/(32*d) + (b*Sec[(c + d*x)/
2]^6)/(384*d)
```

Maple [A]

time = 0.13, size = 151, normalized size = 1.16

method	result
derivativedivides	$\frac{b\left(-\frac{\cos^{7}(dx+c)}{6\sin(dx+c)^{6}}+\frac{\cos^{7}(dx+c)}{24\sin(dx+c)^{4}}-\frac{\cos^{7}(dx+c)}{16\sin(dx+c)^{2}}-\frac{\left(\cos^{5}(dx+c)\right)}{16}-\frac{5\left(\cos^{3}(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+}{d}$
default	$\frac{b\left(-\frac{\cos^{7}(dx+c)}{6\sin(dx+c)^{6}}+\frac{\cos^{7}(dx+c)}{24\sin(dx+c)^{4}}-\frac{\cos^{7}(dx+c)}{16\sin(dx+c)^{2}}-\frac{\left(\cos^{5}(dx+c)\right)}{16}-\frac{5\left(\cos^{3}(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+}{d}$
risch	$iax+\frac{2iac}{d}+\frac{33b\,e^{11i(dx+c)}+144a\,e^{10i(dx+c)}+5b\,e^{9i(dx+c)}-288a\,e^{8i(dx+c)}+90b\,e^{7i(dx+c)}+544a\,e^{6i(dx+c)}+90b\,e^{5i(dx}}{24d\left(e^{2i(dx+c)}-1\right)^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+
c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/
2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]

time = 0.29, size = 133, normalized size = 1.02

$$-\frac{3\,(16\,a-5\,b)\log\left(\cos\left(dx+c\right)+1\right)+3\,(16\,a+5\,b)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,b\cos(dx+c)^{5}+72\,a\cos(dx+c)^{4}-40\,b\cos(dx+c)^{3}-108\,a\cos(dx+c)^{2}+15\,b\cos(dx+c)+44\,a\right)}{\cos(dx+c)^{6}-3\,\cos(dx+c)^{4}+3\,\cos(dx+c)^{2}-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(16*a - 5*b)*log(cos(d*x + c) + 1) + 3*(16*a + 5*b)*log(cos(d*x +
c) - 1) - 2*(33*b*cos(d*x + c)^5 + 72*a*cos(d*x + c)^4 - 40*b*cos(d*x + c)^
3 - 108*a*cos(d*x + c)^2 + 15*b*cos(d*x + c) + 44*a)/(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

Fricas [A]

time = 3.85, size = 237, normalized size = 1.82

$$\frac{66\,b\cos\left(dx+c\right)^{5}+144\,a\cos\left(dx+c\right)^{4}-80\,b\cos\left(dx+c\right)^{3}-216\,a\cos\left(dx+c\right)^{2}+30\,b\cos\left(dx+c\right)-3\left((16\,a-5\,b)\cos\left(dx+c\right)^{6}-3\,(16\,a-5\,b)\cos\left(dx+c\right)^{4}+3\,(16\,a-5\,b)\cos\left(dx+c\right)^{2}-16\,a+5\,b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left((16\,a+5\,b)\cos\left(dx+c\right)^{6}-3\,(16\,a+5\,b)\cos\left(dx+c\right)^{4}+3\,(16\,a+5\,b)\cos\left(dx+c\right)^{2}-16\,a-5\,b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+88\,a}{96\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*b*cos(d*x + c)^5 + 144*a*cos(d*x + c)^4 - 80*b*cos(d*x + c)^3 - 21
6*a*cos(d*x + c)^2 + 30*b*cos(d*x + c) - 3*((16*a - 5*b)*cos(d*x + c)^6 - 3
*(16*a - 5*b)*cos(d*x + c)^4 + 3*(16*a - 5*b)*cos(d*x + c)^2 - 16*a + 5*b)*
```

```
log(1/2*cos(d*x + c) + 1/2) - 3*((16*a + 5*b)*cos(d*x + c)^6 - 3*(16*a + 5*
b)*cos(d*x + c)^4 + 3*(16*a + 5*b)*cos(d*x + c)^2 - 16*a - 5*b)*log(-1/2*co
s(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(
d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx)) \cot^7 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(120) = 240.
time = 0.55, size = 358, normalized size = 2.75

$$-\frac{12\,(16\,a+5\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{45\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{352\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{110\,b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}-\frac{87\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{9\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(12*(16*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
 384*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 87*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 45*b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 + 352*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 110*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)
^3/(cos(d*x + c) - 1)^3 - 87*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 9*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)
^3)/d
```

Mupad [B]
time = 1.50, size = 170, normalized size = 1.31

$$\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{a}{32}-\frac{3\,b}{128}\right)}{d}-\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\left(\frac{29\,a}{2}+\frac{15\,b}{2}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(-2\,a-\frac{3\,b}{2}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{a}{6}+\frac{b}{6}\right)}{64\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{29\,a}{128}-\frac{15\,b}{128}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{a}{384}-\frac{b}{384}\right)}{d}+\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+\frac{5\,b}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(a/32 - (3*b)/128))/d - (cot(c/2 + (d*x)/2)^6*(a/6 +
b/6 - tan(c/2 + (d*x)/2)^2*(2*a + (3*b)/2) + tan(c/2 + (d*x)/2)^4*((29*a)/2
 + (15*b)/2)))/(64*d) - (tan(c/2 + (d*x)/2)^2*((29*a)/128 - (15*b)/128))/d
- (tan(c/2 + (d*x)/2)^6*(a/384 - b/384))/d + (a*log(tan(c/2 + (d*x)/2)^2 +
1))/d - (log(tan(c/2 + (d*x)/2))*(a + (5*b)/16))/d
```

3.264 $\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$

Optimal. Leaf size=102

$$-ax-\frac{5b\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a}{}$$

```
[Out] -a*x-5/16*b*arctanh(sin(d*x+c))/d+1/16*(16*a+5*b*sec(d*x+c))*tan(d*x+c)/d-1
/24*(8*a+5*b*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*b*sec(d*x+c))*tan(d*x+c
)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3966, 3855}

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}-\frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{24d}+\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{16d}-ax-\frac{5b\tanh^{-1}(\sin(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*b*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} - \frac{1}{6}\int (6a+5b\sec(c+dx))\tan$$
$$= -\frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} + \frac{(6a+5b\sec(c+dx))\tan^5(c}{30d}$$
$$= \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5b\sec(c+dx))\tan^3(c-}{24d}$$
$$= -ax + \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5b\sec(c+dx))}{24d}$$
$$= -ax - \frac{5b\tanh^{-1}(\sin(c+dx))}{16d} + \frac{(16a+5b\sec(c+dx))\tan(c+dx}{16d}$$

Mathematica [A]
time = 1.14, size = 103, normalized size = 1.01

$$\frac{-240a\text{ArcTan}(\tan(c+dx)) - 75b\tanh^{-1}(\sin(c+dx)) + \frac{1}{8}(295b + 1168a\cos(c+dx) + 140b\cos(2(c+dx)) + 568a\cos(3(c+dx)) + 165b\cos(4(c+dx)) + 184a\cos(5(c+dx)))\sec^5(c+dx)\tan(c+dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] (-240*a*ArcTan[Tan[c + d*x]] - 75*b*ArcTanh[Sin[c + d*x]] + ((295*b + 1168*
a*Cos[c + d*x] + 140*b*Cos[2*(c + d*x)] + 568*a*Cos[3*(c + d*x)] + 165*b*Co
s[4*(c + d*x)] + 184*a*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Tan[c + d*x])/8)/(2
40*d)
```

Maple [A]
time = 0.10, size = 143, normalized size = 1.40

method	result
derivativedivides	$\frac{b\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6} - \frac{\sin^7(dx+c)}{24\cos(dx+c)^4} + \frac{\sin^7(dx+c)}{16\cos(dx+c)^2} + \frac{\left(\sin^5(dx+c)\right)}{16} + \frac{5\left(\sin^3(dx+c)\right)}{48} + \frac{5\sin(dx+c)}{16} - \frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right) +}{d}$
default	$\frac{b\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6} - \frac{\sin^7(dx+c)}{24\cos(dx+c)^4} + \frac{\sin^7(dx+c)}{16\cos(dx+c)^2} + \frac{\left(\sin^5(dx+c)\right)}{16} + \frac{5\left(\sin^3(dx+c)\right)}{48} + \frac{5\sin(dx+c)}{16} - \frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right) +}{d}$
risch	$-ax - \frac{i\left(165b\,e^{11i(dx+c)} - 720a\,e^{10i(dx+c)} - 25b\,e^{9i(dx+c)} - 2160a\,e^{8i(dx+c)} + 450b\,e^{7i(dx+c)} - 3680a\,e^{6i(dx+c)} - 450b\,e^{5}\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*s
in(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c
```

```
)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(
d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 134, normalized size = 1.31

$$\frac{32\left(3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a-5\,b\left(\frac{2\left(33\,\sin\left(dx+c\right)^{5}-40\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{6}-3\,\sin\left(dx+c\right)^{4}+3\,\sin\left(dx+c\right)^{2}-1}+15\,\log\left(\sin\left(dx+c\right)+1\right)-15\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*b*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.10, size = 134, normalized size = 1.31

$$-\frac{480\,adx\cos\left(dx+c\right)^{6}+75\,b\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-75\,b\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-2\left(368\,a\cos\left(dx+c\right)^{5}+165\,b\cos\left(dx+c\right)^{4}-176\,a\cos\left(dx+c\right)^{3}-130\,b\cos\left(dx+c\right)^{2}+48\,a\cos\left(dx+c\right)+40\,b\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*b*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*b*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*b*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*b*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*b)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)\tan^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(94) = 188.
time = 2.35, size = 228, normalized size = 2.24

$$-\frac{240\left(dx+c\right)a+75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1520\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+425\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4128\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-990\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-4128\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-990\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1520\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+425\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(240*a*tan(1/2*d*x + 1/2*c)^11 - 75*b*
tan(1/2*d*x + 1/2*c)^11 - 1520*a*tan(1/2*d*x + 1/2*c)^9 + 425*b*tan(1/2*d*x
 + 1/2*c)^9 + 4128*a*tan(1/2*d*x + 1/2*c)^7 - 990*b*tan(1/2*d*x + 1/2*c)^7
- 4128*a*tan(1/2*d*x + 1/2*c)^5 - 990*b*tan(1/2*d*x + 1/2*c)^5 + 1520*a*tan
(1/2*d*x + 1/2*c)^3 + 425*b*tan(1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/
2*c) - 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 2.51, size = 331, normalized size = 3.25

$$\frac{\left(\frac{5\,b}{8}-2\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{38\,a}{3}-\frac{85\,b}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{33\,b}{4}-\frac{172\,a}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{172\,a}{5}+\frac{33\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\left(-\frac{38\,a}{3}-\frac{85\,b}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\left(2\,a+\frac{5\,b}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-\frac{5\,b\,\mathrm{atanh}\left(\frac{125\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,\left(20\,a^2\,b+\frac{125\,b^3}{64}\right)}+\frac{20\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20\,a^2\,b+\frac{125\,b^3}{64}}\right)}{8\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+\frac{25\,a\,b^2}{4}}+\frac{25\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(64\,a^3+\frac{25\,a\,b^2}{4}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*a + (5*b)/8) - tan(c/2 + (d*x)/2)^11*(2*a - (5*b)/8)
 - tan(c/2 + (d*x)/2)^3*((38*a)/3 + (85*b)/24) + tan(c/2 + (d*x)/2)^9*((38*
a)/3 - (85*b)/24) + tan(c/2 + (d*x)/2)^5*((172*a)/5 + (33*b)/4) - tan(c/2 +
 (d*x)/2)^7*((172*a)/5 - (33*b)/4))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2
 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c
/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (5*b*atanh((125*b^3*tan(c/
2 + (d*x)/2))/(64*(20*a^2*b + (125*b^3)/64)) + (20*a^2*b*tan(c/2 + (d*x)/2)
)/(20*a^2*b + (125*b^3)/64)))/(8*d) - (2*a*atan((64*a^3*tan(c/2 + (d*x)/2))
/((25*a*b^2)/4 + 64*a^3) + (25*a*b^2*tan(c/2 + (d*x)/2))/(4*((25*a*b^2)/4 +
 64*a^3))))/d
```

3.265 $\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=73

$$ax+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d}$$

```
[Out] a*x+3/8*b*arctanh(sin(d*x+c))/d-1/8*(8*a+3*b*sec(d*x+c))*tan(d*x+c)/d+1/12*
(4*a+3*b*sec(d*x+c))*tan(d*x+c)^3/d
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3966, 3855}

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}-\frac{\tan(c+dx)(8a+3b\sec(c+dx))}{8d}+ax+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*b*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*b*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx &= \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d}-\frac{1}{4}\int (4a+3b\sec(c+dx))\tan^2(\\ &= -\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3b\sec(c+dx))\tan^3(c+\\ &= ax-\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\frac{(4a+3b\sec(c+dx))\tan^3\\ &= ax+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d}+\end{aligned}$$

Mathematica [A]
time = 0.64, size = 79, normalized size = 1.08

$$\frac{48a\text{ArcTan}(\tan(c+dx))+18b\tanh^{-1}(\sin(c+dx))-(3b+32a\cos(c+dx)+15b\cos(2(c+dx))+16a\cos(3(c+dx)))\sec^3(c+dx)\tan(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (48*a*ArcTan[Tan[c + d*x]] + 18*b*ArcTanh[Sin[c + d*x]] - (3*b + 32*a*Cos[c
 + d*x] + 15*b*Cos[2*(c + d*x)] + 16*a*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan
[c + d*x])/(48*d)
```

Maple [A]
time = 0.10, size = 104, normalized size = 1.42

method	result
derivativedivides	$\frac{b\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+a\right.}{d}$
default	$\frac{b\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+a\right.}{d}$
risch	$ax+\frac{i(15b\,e^{7i(dx+c)}-48a\,e^{6i(dx+c)}-9b\,e^{5i(dx+c)}-96a\,e^{4i(dx+c)}+9b\,e^{3i(dx+c)}-80a\,e^{2i(dx+c)}-15b\,e^{i(dx+c)}-32a)}{12d\left(e^{2i(dx+c)}+1\right)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin
(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/3*tan(d*x+c)^3
-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 102, normalized size = 1.40

$$\frac{16\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a+3\,b\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log(\sin(dx+c)+1)-3\,\log(\sin(dx+c)-1)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*b*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.34, size = 112, normalized size = 1.53

$$\frac{48\,adx\cos(dx+c)^4+9\,b\cos(dx+c)^4\log(\sin(dx+c)+1)-9\,b\cos(dx+c)^4\log(-\sin(dx+c)+1)-2\left(32\,a\cos(dx+c)^3+15\,b\cos(dx+c)^2-8\,a\cos(dx+c)-6\,b\right)\sin(dx+c)}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*b*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec{(c+dx)})\tan^4{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(67) = 134.
time = 1.07, size = 172, normalized size = 2.36

$$\frac{24\,(dx+c)a+9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-104\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+33\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+104\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a + 9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 9*b*tan(1/2*
d*x + 1/2*c)^7 - 104*a*tan(1/2*d*x + 1/2*c)^5 + 33*b*tan(1/2*d*x + 1/2*c)^5
 + 104*a*tan(1/2*d*x + 1/2*c)^3 + 33*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/
2*d*x + 1/2*c) - 9*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/
d
```

Mupad [B]
time = 2.23, size = 267, normalized size = 3.66

$$\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+9\,a\,b^2}+\frac{9\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+9\,a\,b^2}\right)}{d}+\frac{3\,b\,\mathrm{atanh}\left(\frac{27\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(24\,a^2\,b+\frac{27\,b^3}{8}\right)}+\frac{24\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{24\,a^2\,b+\frac{27\,b^3}{8}}\right)}{4\,d}-\frac{\left(\frac{3\,b}{4}-2\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{26\,a}{3}-\frac{11\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{26\,a}{3}-\frac{11\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a+\frac{3\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atan((64*a^3*tan(c/2 + (d*x)/2))/(9*a*b^2 + 64*a^3) + (9*a*b^2*tan(c/2
 + (d*x)/2))/(9*a*b^2 + 64*a^3)))/d + (3*b*atanh((27*b^3*tan(c/2 + (d*x)/2)
)/(8*(24*a^2*b + (27*b^3)/8)) + (24*a^2*b*tan(c/2 + (d*x)/2))/(24*a^2*b + (
27*b^3)/8)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (3*b)/4) - tan(c/2 + (d*x)/
2)^7*(2*a - (3*b)/4) - tan(c/2 + (d*x)/2)^3*((26*a)/3 + (11*b)/4) + tan(c/2
 + (d*x)/2)^5*((26*a)/3 - (11*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2
 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.266 $\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$-ax-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] -a*x-1/2*b*arctanh(sin(d*x+c))/d+1/2*(2*a+b*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3966, 3855}

$$\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d}-ax-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + b*Sec[c + d*x])*Tan[c +
d*x])/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx &= \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}\int (2a+b\sec(c+dx))\,dx\\ &= -ax+\frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}b\int \sec(c+dx)\,dx\\ &= -ax-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 1.33

$$-\frac{a\text{ArcTan}(\tan(c+dx))}{d}-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.07, size = 67, normalized size = 1.49

method	result	size
derivativedivides	$\frac{b\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
default	$\frac{b\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
risch	$-ax-\frac{i\left(b\,e^{3i(dx+c)}-2a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-2a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{b\ln\left(e^{i(dx+c)}-i\right)}{2d}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(
d*x+c)))+a*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 65, normalized size = 1.44

$$-\frac{4\,(dx+c-\tan\,(dx+c))a+b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\,(\sin\,(dx+c)+1)-\log\,(\sin\,(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.
time = 3.97, size = 87, normalized size = 1.93

$$-\frac{4\,adx\cos\,(dx+c)^2+b\cos\,(dx+c)^2\log\,(\sin\,(dx+c)+1)-b\cos\,(dx+c)^2\log\,(-\sin\,(dx+c)+1)-2\,(2\,a\cos\,(dx+c)+b)\sin\,(dx+c)}{4\,d\cos\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + b)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx)) \tan^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(41) = 82.
time = 0.62, size = 115, normalized size = 2.56

$$-\frac{2\,(dx+c)a + b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/
2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.39, size = 96, normalized size = 2.13

$$\frac{a\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} - \frac{b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x))/(d*cos(c + d*x)) - (b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/d - (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b*sin(
c + d*x))/(2*d*cos(c + d*x)^2)
```

3.267 $\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$-ax-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}$$

```
[Out] -a*x-cot(d*x+c)*(a+b*sec(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] -(a*x) - (Cot[c + d*x]*(a + b*Sec[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}-\int a\,dx\\ &= -ax-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 43, normalized size = 1.65

$$-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] -((b*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

Maple [A]
time = 0.07, size = 35, normalized size = 1.35

method	result	size
derivativedivides	$\frac{-\frac{b}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35
default	$\frac{-\frac{b}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35
risch	$-ax-\frac{2i(b\,\mathrm{e}^{i(dx+c)}+a)}{d(\mathrm{e}^{2i(dx+c)}-1)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-b/sin(d*x+c)+a*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 31, normalized size = 1.19

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{b}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a + b/sin(d*x + c))/d
```

Fricas [A]
time = 2.92, size = 33, normalized size = 1.27

$$-\frac{adx\sin(dx+c)+a\cos(dx+c)+b}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + b)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

Giac [A]
time = 0.45, size = 52, normalized size = 2.00

$$-\frac{2\,(dx+c)a - a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a - a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c) + (a
+ b)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 1.30, size = 48, normalized size = 1.85

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{d}-\frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a/2 - b/2))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) - a
*x
```

3.268 $\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=55

$$ax-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}$$

```
[Out] a*x-1/3*cot(d*x+c)^3*(a+b*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*b*sec(d*x+c))
/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3967, 8}

$$-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + b*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
b*Sec[c + d*x]))/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{1}{3}\int \cot^2(c+dx)(-3a-2b\sec\\ &= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx}{3d}\\ &= ax-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 62, normalized size = 1.13

$$\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

Maple [A]
time = 0.09, size = 86, normalized size = 1.56

method	result	size
derivativedivides	$\frac{b\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	86
default	$\frac{b\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$	86
risch	$ax+\frac{2i\left(3b\,e^{5i(dx+c)}+6a\,e^{4i(dx+c)}-2b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}+4a\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/3/sin(d*x+c)^3*cos(d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+c
os(d*x+c)^2)*sin(d*x+c))+a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.48, size = 59, normalized size = 1.07

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+\frac{\left(3\,\sin(dx+c)^2-1\right)b}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*b/sin(d*x + c)^3)/d
```

Fricas [A]
time = 2.17, size = 87, normalized size = 1.58

$$\frac{4\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2-3\,a\cos\left(dx+c\right)+3\,\left(adx\cos\left(dx+c\right)^2-adx\right)\sin\left(dx+c\right)-2\,b}{3\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) + 3*(a*d*x*
cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 2*b)/((d*cos(d*x + c)^2 - d)*sin(d*x
 + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\cot^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.
time = 0.48, size = 112, normalized size = 2.04

$$\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 24\,(dx + c)a - 15\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 9\,b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a - b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a
- 15*a*tan(1/2*d*x + 1/2*c) + 9*b*tan(1/2*d*x + 1/2*c) + (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 9*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 1.56, size = 90, normalized size = 1.64

$$a\,x + \frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3\,\left(\frac{a}{24} - \frac{b}{24}\right)}{d} - \frac{\cot\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3\,\left(\left(-5\,a - 3\,b\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + \frac{a}{3} + \frac{b}{3}\right)}{8\,d} - \frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,\left(\frac{5\,a}{8} - \frac{3\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 +
 b/3 - tan(c/2 + (d*x)/2)^2*(5*a + 3*b)))/(8*d) - (tan(c/2 + (d*x)/2)*((5*a
)/8 - (3*b)/8))/d
```

3.269 $\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$-ax-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx)}{15d}$$

```
[Out] -a*x-1/5*cot(d*x+c)^5*(a+b*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*b*sec(d*x
+c))/d-1/15*cot(d*x+c)*(15*a+8*b*sec(d*x+c))/d
```

Rubi [A]

time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{3967, 8\}$

$$-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + b*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*b*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*b*Sec[c + d*x]))/(15
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{1}{5}\int \cot^4(c+dx)(-5a-4b\sec(
\\
&= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx)}{15d}
\\
&= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx)}{15d}
\\
&= -ax-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c}{15d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 79, normalized size = 0.94

$$-\frac{b\csc(c+dx)}{d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc^5(c+dx)}{5d}-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) + (2*b*Csc[c + d*x]^3)/(3*d) - (b*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

Maple [A]
time = 0.09, size = 129, normalized size = 1.54

method	result
derivativedivides	$\frac{b\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(\right.}{3}\right.}{d}$
default	$\frac{b\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(\right.}{3}\right.}{d}$
risch	$-ax-\frac{2i\left(15b\,e^{9i(dx+c)}+45a\,e^{8i(dx+c)}-20b\,e^{7i(dx+c)}-90a\,e^{6i(dx+c)}+58b\,e^{5i(dx+c)}+140a\,e^{4i(dx+c)}-20b\,e^{3i(dx+c)}-\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/s
in(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+
a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 79, normalized size = 0.94

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+\frac{\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)b}{\sin(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*b/sin(d*x + c)^5)/d
```

Fricas [A]
time = 3.86, size = 130, normalized size = 1.55

$$-\frac{23\,a\cos\left(dx+c\right)^{5}+15\,b\cos\left(dx+c\right)^{4}-35\,a\cos\left(dx+c\right)^{3}-20\,b\cos\left(dx+c\right)^{2}+15\,a\cos\left(dx+c\right)+15\,\left(adx\cos\left(dx+c\right)^{4}-2\,adx\cos\left(dx+c\right)^{2}+adx\right)\sin\left(dx+c\right)+8\,b}{15\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(23*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 35*a*cos(d*x + c)^3 - 20
*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^4 - 2*a*d*x*
cos(d*x + c)^2 + a*d*x)*sin(d*x + c) + 8*b)/((d*cos(d*x + c)^4 - 2*d*cos(d*
x + c)^2 + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\cot^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.
time = 0.49, size = 170, normalized size = 2.02

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+25\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-480\,(dx+c)a+330\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-150\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{330\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+150\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-25\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a+3\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 35*a*tan(1
/2*d*x + 1/2*c)^3 + 25*b*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a + 330*a*t
an(1/2*d*x + 1/2*c) - 150*b*tan(1/2*d*x + 1/2*c) - (330*a*tan(1/2*d*x + 1/2
*c)^4 + 150*b*tan(1/2*d*x + 1/2*c)^4 - 35*a*tan(1/2*d*x + 1/2*c)^2 - 25*b*t
an(1/2*d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 1.39, size = 132, normalized size = 1.57

$$\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{a}{160}-\frac{b}{160}\right)}{d}-\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\left(22\,a+10\,b\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(-\frac{7\,a}{3}-\frac{5\,b}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{a}{5}+\frac{b}{5}\right)}{32\,d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{7\,a}{96}-\frac{5\,b}{96}\right)}{d}-a\,x+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{11\,a}{16}-\frac{5\,b}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + b/5
 - tan(c/2 + (d*x)/2)^2*((7*a)/3 + (5*b)/3) + tan(c/2 + (d*x)/2)^4*(22*a +
10*b)))/(32*d) - (tan(c/2 + (d*x)/2)^3*((7*a)/96 - (5*b)/96))/d - a*x + (ta
n(c/2 + (d*x)/2)*((11*a)/16 - (5*b)/16))/d
```

3.270 $\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$ax-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}+\frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d}$$

```
[Out] a*x-1/7*cot(d*x+c)^7*(a+b*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*b*sec(d*x+
c))/d+1/35*cot(d*x+c)*(35*a+16*b*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+24*
b*sec(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}+\frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + b*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*b*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*b*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*b*Sec[c + d*x]))/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{1}{7}\int \cot^6(c+dx)(-7a-6b\sec$$
$$= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+d}{35d}$$
$$= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+d}{35d}$$
$$= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+d}{35d}$$
$$= ax - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c}{35d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.05, size = 92, normalized size = 0.83

$$\frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{d} + \frac{3b\csc^5(c+dx)}{5d} - \frac{b\csc^7(c+dx)}{7d} - \frac{a\cot^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\tan^2(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/d + (3*b*Csc[c + d*x]^5)/(5*d) - (b
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

Maple [A]

time = 0.13, size = 162, normalized size = 1.46

method	result
derivativedivides	$\frac{b\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c}{7}\right.}{d}$
default	$\frac{b\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c}{7}\right.}{d}$
risch	$ax+\frac{2i\left(105b\,e^{13i(dx+c)}+420a\,e^{12i(dx+c)}-210b\,e^{11i(dx+c)}-1260a\,e^{10i(dx+c)}+903b\,e^{9i(dx+c)}+3080a\,e^{8i(dx+c)}-636b\right.}{105d\left(e^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/
sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/7*cot(d*x+c)^7+1/5*c
ot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.48, size = 100, normalized size = 0.90

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a+\frac{3\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)b}{\sin(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*b/sin(d*x + c)^7)/d
```

Fricas [A]
time = 4.03, size = 179, normalized size = 1.61

$$\frac{176\,a\cos\left(dx+c\right)^7+105\,b\cos\left(dx+c\right)^6-406\,a\cos\left(dx+c\right)^5-210\,b\cos\left(dx+c\right)^4+350\,a\cos\left(dx+c\right)^3+168\,b\cos\left(dx+c\right)^2-105\,a\cos\left(dx+c\right)+105\left(adx\cos\left(dx+c\right)^6-3\,adx\cos\left(dx+c\right)^4+3\,adx\cos\left(dx+c\right)^2-adx\right)\sin\left(dx+c\right)-48\,b}{105\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^7 + 105*b*cos(d*x + c)^6 - 406*a*cos(d*x + c)^5 -
 210*b*cos(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 168*b*cos(d*x + c)^2 - 105*a
*cos(d*x + c) + 105*(a*d*x*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^4 + 3*a*d*
x*cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 48*b)/((d*cos(d*x + c)^6 - 3*d*cos
(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)\cot^8\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**8, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(103) = 206.
time = 0.52, size = 225, normalized size = 2.03

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+147\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1295\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-735\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+13440\,(dx+c)a-9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3675\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3675\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1295\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-735\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+189\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+147\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a-15\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 - 189*a*
tan(1/2*d*x + 1/2*c)^5 + 147*b*tan(1/2*d*x + 1/2*c)^5 + 1295*a*tan(1/2*d*x
+ 1/2*c)^3 - 735*b*tan(1/2*d*x + 1/2*c)^3 + 13440*(d*x + c)*a - 9765*a*tan(
1/2*d*x + 1/2*c) + 3675*b*tan(1/2*d*x + 1/2*c) + (9765*a*tan(1/2*d*x + 1/2*
c)^6 + 3675*b*tan(1/2*d*x + 1/2*c)^6 - 1295*a*tan(1/2*d*x + 1/2*c)^4 - 735*
b*tan(1/2*d*x + 1/2*c)^4 + 189*a*tan(1/2*d*x + 1/2*c)^2 + 147*b*tan(1/2*d*x
 + 1/2*c)^2 - 15*a - 15*b)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 1.62, size = 174, normalized size = 1.57

$$a\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{37\,a}{384}-\frac{7\,b}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{9\,a}{640}-\frac{7\,b}{640}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{a}{896}-\frac{b}{896}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\left(-93\,a-35\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{37\,a}{3}+7\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-\frac{9\,a}{5}-\frac{7\,b}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{7}+\frac{b}{7}\right)}{128\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{93\,a}{128}-\frac{35\,b}{128}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + b/cos(c + d*x)),x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)^3*((37*a)/384 - (7*b)/128))/d - (tan(c/2 + (d*x)/
2)^5*((9*a)/640 - (7*b)/640))/d + (tan(c/2 + (d*x)/2)^7*(a/896 - b/896))/d
- (cot(c/2 + (d*x)/2)^7*(a/7 + b/7 - tan(c/2 + (d*x)/2)^2*((9*a)/5 + (7*b)/
5) + tan(c/2 + (d*x)/2)^4*((37*a)/3 + 7*b) - tan(c/2 + (d*x)/2)^6*(93*a + 3
5*b)))/(128*d) - (tan(c/2 + (d*x)/2)*((93*a)/128 - (35*b)/128))/d
```

3.271 $\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$

Optimal. Leaf size=185

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{2a^2\sec^2(c+dx)}{d}-\frac{8ab\sec^3(c+dx)}{3d}+\frac{3a^2\sec^4(c+dx)}{2d}+\frac{12ab\sec^5(c+$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-2*a^2*sec(d*x+c)^2/d-8/3*a*b*sec(d
*x+c)^3/d+3/2*a^2*sec(d*x+c)^4/d+12/5*a*b*sec(d*x+c)^5/d-2/3*a^2*sec(d*x+c)
^6/d-8/7*a*b*sec(d*x+c)^7/d+1/8*a^2*sec(d*x+c)^8/d+2/9*a*b*sec(d*x+c)^9/d+1
/10*b^2*tan(d*x+c)^10/d
```

Rubi [A]
time = 0.09, antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 962}

$$\frac{(a^2-4b^2)\sec^8(c+dx)}{8d}-\frac{(2a^2-3b^2)\sec^6(c+dx)}{3d}+\frac{(3a^2-2b^2)\sec^4(c+dx)}{2d}-\frac{(4a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^9(c+dx)}{9d}-\frac{8ab\sec^7(c+dx)}{7d}+\frac{12ab\sec^5(c+dx)}{5d}-\frac{8ab\sec^3(c+dx)}{3d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^{10}(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((4*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (8*a*b*Sec[c + d*x]^3)/(3*d) + ((3*a^2 - 2*b^2)*Sec[c +
 d*x]^4)/(2*d) + (12*a*b*Sec[c + d*x]^5)/(5*d) - ((2*a^2 - 3*b^2)*Sec[c + d
*x]^6)/(3*d) - (8*a*b*Sec[c + d*x]^7)/(7*d) + ((a^2 - 4*b^2)*Sec[c + d*x]^8
)/(8*d) + (2*a*b*Sec[c + d*x]^9)/(9*d) + (b^2*Sec[c + d*x]^10)/(10*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^4}{x}\,dx, x, b\sec(c+dx)\right)}{b^8d}$$

$$= \frac{\text{Subst}\left(\int \left(2ab^8+\frac{a^2b^8}{x}-b^6(4a^2-b^2)\,x-8ab^6x^2+2b^4(3a^2-2b^2)\right.\right.}{}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{(4a^2-b^2)\sec^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.46, size = 173, normalized size = 0.94

$$\frac{-2520a^2\log(\cos(c+dx))+5040ab\sec(c+dx)-1260(4a^2-b^2)\sec^2(c+dx)-6720ab\sec^3(c+dx)+1260(3a^2-2b^2)\sec^4(c+dx)+6048ab\sec^5(c+dx)-840(2a^2-3b^2)\sec^6(c+dx)-2880ab\sec^7(c+dx)+315(a^2-4b^2)\sec^8(c+dx)+560ab\sec^9(c+dx)+252b^2\sec^{10}(c+dx)}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] (-2520*a^2*Log[Cos[c + d*x]] + 5040*a*b*Sec[c + d*x] - 1260*(4*a^2 - b^2)*S
ec[c + d*x]^2 - 6720*a*b*Sec[c + d*x]^3 + 1260*(3*a^2 - 2*b^2)*Sec[c + d*x]
^4 + 6048*a*b*Sec[c + d*x]^5 - 840*(2*a^2 - 3*b^2)*Sec[c + d*x]^6 - 2880*a*
b*Sec[c + d*x]^7 + 315*(a^2 - 4*b^2)*Sec[c + d*x]^8 + 560*a*b*Sec[c + d*x]^
9 + 252*b^2*Sec[c + d*x]^10)/(2520*d)
```

Maple [A]
time = 0.19, size = 224, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^{10}(dx+c)\right)}{10\cos(dx+c)^{10}}+2ba\left(\frac{\sin^{10}(dx+c)}{9\cos(dx+c)^9}-\frac{\sin^{10}(dx+c)}{63\cos(dx+c)^7}+\frac{\sin^{10}(dx+c)}{105\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{63\cos(dx+c)^3}+\frac{\sin^{10}(dx+c)}{9\cos(dx+c)}+\frac{\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8(s}{}\right.}{}\right.}{}$
default	$\frac{\frac{b^2\left(\sin^{10}(dx+c)\right)}{10\cos(dx+c)^{10}}+2ba\left(\frac{\sin^{10}(dx+c)}{9\cos(dx+c)^9}-\frac{\sin^{10}(dx+c)}{63\cos(dx+c)^7}+\frac{\sin^{10}(dx+c)}{105\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{63\cos(dx+c)^3}+\frac{\sin^{10}(dx+c)}{9\cos(dx+c)}+\frac{\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8(s}{}\right.}{}\right.}{}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{4ba\,e^{19i(dx+c)}-8a^2e^{18i(dx+c)}+2b^2e^{18i(dx+c)}+\frac{44ba\,e^{17i(dx+c)}}{3}-40a^2e^{16i(dx+c)}+\frac{1072ba\,e^{15i(dx+c)}}{15}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/10*b^2*sin(d*x+c)^10/cos(d*x+c)^10+2*b*a*(1/9*sin(d*x+c)^10/cos(d*x+
c)^9-1/63*sin(d*x+c)^10/cos(d*x+c)^7+1/105*sin(d*x+c)^10/cos(d*x+c)^5-1/63*
```

```
sin(d*x+c)^10/cos(d*x+c)^3+1/9*sin(d*x+c)^10/cos(d*x+c)+1/9*(128/35+sin(d*x
+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+a
^2*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(
cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 174, normalized size = 0.94

$$-\frac{2520\,a^{2}\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,ab\cos(dx+c)^{9}-6720\,ab\cos(dx+c)^{7}-1260\,(4\,a^{2}-b^{2})\cos(dx+c)^{8}+6048\,ab\cos(dx+c)^{5}+1260\,(3\,a^{2}-2\,b^{2})\cos(dx+c)^{6}-2880\,ab\cos(dx+c)^{3}-840\,(2\,a^{2}-3\,b^{2})\cos(dx+c)^{4}+560\,ab\cos(dx+c)+315\,(a^{2}-4\,b^{2})\cos(dx+c)^{2}+252\,b^{2}}{\cos(dx+c)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a*b*cos(d*x + c)^9 - 6720*a*b*c
os(d*x + c)^7 - 1260*(4*a^2 - b^2)*cos(d*x + c)^8 + 6048*a*b*cos(d*x + c)^5
 + 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 - 2880*a*b*cos(d*x + c)^3 - 840*(2*a
^2 - 3*b^2)*cos(d*x + c)^4 + 560*a*b*cos(d*x + c) + 315*(a^2 - 4*b^2)*cos(d
*x + c)^2 + 252*b^2)/cos(d*x + c)^10)/d
```

Fricas [A]
time = 3.47, size = 181, normalized size = 0.98

$$-\frac{2520\,a^{2}\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,ab\cos\left(dx+c\right)^{9}+6720\,ab\cos\left(dx+c\right)^{7}+1260\,(4\,a^{2}-b^{2})\cos\left(dx+c\right)^{8}-6048\,ab\cos\left(dx+c\right)^{5}-1260\,(3\,a^{2}-2\,b^{2})\cos\left(dx+c\right)^{6}+2880\,ab\cos\left(dx+c\right)^{3}+840\,(2\,a^{2}-3\,b^{2})\cos\left(dx+c\right)^{4}-560\,ab\cos\left(dx+c\right)-315\,(a^{2}-4\,b^{2})\cos\left(dx+c\right)^{2}-252\,b^{2}}{2520\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a*b*cos(d*x + c
)^9 + 6720*a*b*cos(d*x + c)^7 + 1260*(4*a^2 - b^2)*cos(d*x + c)^8 - 6048*a*
b*cos(d*x + c)^5 - 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 + 2880*a*b*cos(d*x +
 c)^3 + 840*(2*a^2 - 3*b^2)*cos(d*x + c)^4 - 560*a*b*cos(d*x + c) - 315*(a^
2 - 4*b^2)*cos(d*x + c)^2 - 252*b^2)/(d*cos(d*x + c)^10)
```

Sympy [A]
time = 3.28, size = 314, normalized size = 1.70

$$\begin{cases} \frac{a^{2}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a^{2}\tan^{8}\left(c+dx\right)}{8d}-\frac{a^{2}\tan^{6}\left(c+dx\right)}{6d}+\frac{a^{2}\tan^{4}\left(c+dx\right)}{4d}-\frac{a^{2}\tan^{2}\left(c+dx\right)}{2d}+\frac{2ab\tan^{8}\left(c+dx\right)\sec\left(c+dx\right)}{9d}-\frac{16ab\tan^{6}\left(c+dx\right)\sec\left(c+dx\right)}{63d}+\frac{32ab\tan^{4}\left(c+dx\right)\sec\left(c+dx\right)}{105d}-\frac{128ab\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{315d}+\frac{256ab\sec\left(c+dx\right)}{315d}+\frac{b^{2}\tan^{8}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}-\frac{b^{2}\tan^{6}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}+\frac{b^{2}\tan^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}-\frac{b^{2}\tan^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{10d}+\frac{b^{2}\sec^{2}\left(c+dx\right)}{10d} & \text{for}\: d\neq 0 \\ x\left(a+b\sec\left(c\right)\right)^{2}\tan^{9}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8/(8*d)
 - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d
*x)**2/(2*d) + 2*a*b*tan(c + d*x)**8*sec(c + d*x)/(9*d) - 16*a*b*tan(c + d*
x)**6*sec(c + d*x)/(63*d) + 32*a*b*tan(c + d*x)**4*sec(c + d*x)/(105*d) - 1
28*a*b*tan(c + d*x)**2*sec(c + d*x)/(315*d) + 256*a*b*sec(c + d*x)/(315*d)
```

```
+ b**2*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) - b**2*tan(c + d*x)**6*sec(c
+ d*x)**2/(10*d) + b**2*tan(c + d*x)**4*sec(c + d*x)**2/(10*d) - b**2*tan(c
 + d*x)**2*sec(c + d*x)**2/(10*d) + b**2*sec(c + d*x)**2/(10*d), Ne(d, 0)),
 (x*(a + b*sec(c))**2*tan(c)**9, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(169) = 338.
time = 6.35, size = 489, normalized size = 2.64

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{7381\,a^2+4096\,ab+\frac{78850\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{40960\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{382545\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{184320\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1114200\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{491520\,ab(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2171610\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{860160\,ab(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{2736972\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{516096\,ab(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-\frac{258048\,b^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{2171610\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{1114200\,a^2(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{382545\,a^2(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}+\frac{78850\,a^2(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}+\frac{7381\,a^2(\cos(dx+c)-1)^{10}}{(\cos(dx+c)+1)^{10}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{10}}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (7381*a^2 + 40
96*a*b + 78850*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40960*a*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 382545*a^2*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 184320*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 111420
0*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 491520*a*b*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 2171610*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c
) + 1)^4 + 860160*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2736972*a
^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 516096*a*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 - 258048*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 2171610*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8
/(cos(d*x + c) + 1)^8 + 78850*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9
 + 7381*a^2*(cos(d*x + c) - 1)^10/(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 1)^10)/d
```

Mupad [B]
time = 5.07, size = 344, normalized size = 1.86

$$\frac{\frac{512\,a\,b}{315}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(20\,a^2+\frac{512\,b\,a}{7}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+\frac{1024\,b\,a}{63}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{740\,a^2}{3}+\frac{1024\,b\,a}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{272\,a^2}{3}+\frac{4096\,b\,a}{21}\right)+\frac{740\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-\frac{272\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}+20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(348\,a^2+\frac{1024\,a\,b}{5}-\frac{512\,b^2}{5}\right)}{d\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{20}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}+45\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}-120\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+210\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-252\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+210\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-120\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+45\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((512*a*b)/315 + tan(c/2 + (d*x)/2)^4*((512*a*b)/7 + 20*a^2) - tan(c/2 + (d
*x)/2)^2*((1024*a*b)/63 + 2*a^2) + tan(c/2 + (d*x)/2)^8*((1024*a*b)/3 + (74
0*a^2)/3) - tan(c/2 + (d*x)/2)^6*((4096*a*b)/21 + (272*a^2)/3) + (740*a^2*t
an(c/2 + (d*x)/2)^12)/3 - (272*a^2*tan(c/2 + (d*x)/2)^14)/3 + 20*a^2*tan(c/
2 + (d*x)/2)^16 - 2*a^2*tan(c/2 + (d*x)/2)^18 - tan(c/2 + (d*x)/2)^10*((102
4*a*b)/5 + 348*a^2 - (512*b^2)/5))/(d*(45*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2
 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8 - 252*t
an(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2 + (d*x)/2)^1
4 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2
)^20 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.272 $\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal. Leaf size=149

$$\frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{3a^2\sec^2(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{d}-\frac{3a^2\sec^4(c+dx)}{4d}-\frac{6ab\sec^5(c+dx)}{5d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+3/2*a^2*sec(d*x+c)^2/d+2*a*b*sec(d*
x+c)^3/d-3/4*a^2*sec(d*x+c)^4/d-6/5*a*b*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c)^6
/d+2/7*a*b*sec(d*x+c)^7/d+1/8*b^2*tan(d*x+c)^8/d
```

Rubi [A]

time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 962}

$$\frac{(a^2-3b^2)\sec^6(c+dx)}{6d}-\frac{3(a^2-b^2)\sec^4(c+dx)}{4d}+\frac{(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^7(c+dx)}{7d}-\frac{6ab\sec^5(c+dx)}{5d}+\frac{2ab\sec^3(c+dx)}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((3*a^2 - b^2)*Sec[c +
 d*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/d - (3*(a^2 - b^2)*Sec[c + d*x]^4)/
(4*d) - (6*a*b*Sec[c + d*x]^5)/(5*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^6)/(6*d)
 + (2*a*b*Sec[c + d*x]^7)/(7*d) + (b^2*Sec[c + d*x]^8)/(8*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^3}{x}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(2ab^6+\frac{a^2b^6}{x}-b^4(3a^2-b^2)\,x-6ab^4x^2+3b^2(a^2-b^2)\,x\right.\right.}{b^6 d}$$

$$= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{(3a^2-b^2)\sec^2(c+dx)}{2d}+$$

Mathematica [A]
time = 0.36, size = 138, normalized size = 0.93

$$\frac{840a^2\log(\cos(c+dx))-1680ab\sec(c+dx)+420(3a^2-b^2)\sec^2(c+dx)+1680ab\sec^3(c+dx)-630(a^2-b^2)\sec^4(c+dx)-1008ab\sec^5(c+dx)+140(a^2-3b^2)\sec^6(c+dx)+240ab\sec^7(c+dx)+105b^2\sec^8(c+dx)}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (840*a^2*Log[Cos[c + d*x]] - 1680*a*b*Sec[c + d*x] + 420*(3*a^2 - b^2)*Sec[
c + d*x]^2 + 1680*a*b*Sec[c + d*x]^3 - 630*(a^2 - b^2)*Sec[c + d*x]^4 - 100
8*a*b*Sec[c + d*x]^5 + 140*(a^2 - 3*b^2)*Sec[c + d*x]^6 + 240*a*b*Sec[c + d
*x]^7 + 105*b^2*Sec[c + d*x]^8)/(840*d)
```

Maple [A]
time = 0.15, size = 184, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^8(dx+c)\right)}{8\cos(dx+c)^8}+2ba\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2\right.}{}\right.}{7}\right.}{d}$
default	$\frac{\frac{b^2\left(\sin^8(dx+c)\right)}{8\cos(dx+c)^8}+2ba\left(\frac{\sin^8(dx+c)}{7\cos(dx+c)^7}-\frac{\sin^8(dx+c)}{35\cos(dx+c)^5}+\frac{\sin^8(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^8(dx+c)}{7\cos(dx+c)}-\frac{\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2\right.}{}\right.}{7}\right.}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2\left(210ba\,e^{15i(dx+c)}-315a^2e^{14i(dx+c)}+105b^2e^{14i(dx+c)}+630ba\,e^{13i(dx+c)}-1260a^2e^{12i(dx+c)}+22\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*b^2*sin(d*x+c)^8/cos(d*x+c)^8+2*b*a*(1/7*sin(d*x+c)^8/cos(d*x+c)^7
-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35*sin(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+
```

```
c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*c
os(d*x+c))+a^2*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2+ln(cos(d
*x+c))))
```

Maxima [A]
time = 0.27, size = 139, normalized size = 0.93

$$\frac{840\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{1680\,ab\cos\left(dx+c\right)^7-1680\,ab\cos\left(dx+c\right)^5-420\,\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^6+1008\,ab\cos\left(dx+c\right)^3+630\,\left(a^2-b^2\right)\cos\left(dx+c\right)^4-240\,ab\cos\left(dx+c\right)-140\,\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^2-105\,b^2}{\cos\left(dx+c\right)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a*b*cos(d*x + c)^7 - 1680*a*b*cos(
d*x + c)^5 - 420*(3*a^2 - b^2)*cos(d*x + c)^6 + 1008*a*b*cos(d*x + c)^3 + 6
30*(a^2 - b^2)*cos(d*x + c)^4 - 240*a*b*cos(d*x + c) - 140*(a^2 - 3*b^2)*co
s(d*x + c)^2 - 105*b^2)/cos(d*x + c)^8)/d
```

Fricas [A]
time = 4.03, size = 146, normalized size = 0.98

$$\frac{840\,a^2\cos\left(dx+c\right)^8\log\left(-\cos\left(dx+c\right)\right)-1680\,ab\cos\left(dx+c\right)^7+1680\,ab\cos\left(dx+c\right)^5+420\,\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^6-1008\,ab\cos\left(dx+c\right)^3-630\,\left(a^2-b^2\right)\cos\left(dx+c\right)^4+240\,ab\cos\left(dx+c\right)+140\,\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^2+105\,b^2}{840\,d\cos\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a*b*cos(d*x + c)^7
+ 1680*a*b*cos(d*x + c)^5 + 420*(3*a^2 - b^2)*cos(d*x + c)^6 - 1008*a*b*cos
(d*x + c)^3 - 630*(a^2 - b^2)*cos(d*x + c)^4 + 240*a*b*cos(d*x + c) + 140*(
a^2 - 3*b^2)*cos(d*x + c)^2 + 105*b^2)/(d*cos(d*x + c)^8)
```

Sympy [A]
time = 1.64, size = 252, normalized size = 1.69

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^6\left(c+dx\right)}{6d}-\frac{a^2\tan^4\left(c+dx\right)}{4d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{7d}-\frac{12ab\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{35d}+\frac{16ab\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{35d}-\frac{32ab\sec\left(c+dx\right)}{35d}+\frac{b^2\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}-\frac{b^2\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}+\frac{b^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{8d}-\frac{b^2\sec^2\left(c+dx\right)}{8d} & \text{for}\ d\neq 0 \\ x\left(a+b\sec\left(c\right)\right)^2\tan^7\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6/(6*d
) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c +
 d*x)**6*sec(c + d*x)/(7*d) - 12*a*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) +
16*a*b*tan(c + d*x)**2*sec(c + d*x)/(35*d) - 32*a*b*sec(c + d*x)/(35*d) + b
**2*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) - b**2*tan(c + d*x)**4*sec(c + d*
x)**2/(8*d) + b**2*tan(c + d*x)**2*sec(c + d*x)**2/(8*d) - b**2*sec(c + d*x
)**2/(8*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(137) = 274.
time = 3.95, size = 415, normalized size = 2.79

$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2283\,a^2+1536\,ab+\frac{19944\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12288\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{77364\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{43008\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{175448\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{86016\,ab(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{231490\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{53760\,ab(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{26880\,b^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{175448\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{77364\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{19944\,a^2(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{2283\,a^2(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2283*a^2 + 1536
*a*b + 19944*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12288*a*b*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 77364*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 43008*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 175448*a^2
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 86016*a*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 231490*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^
4 + 53760*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 26880*b^2*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1994
4*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2283*a^2*(cos(d*x + c) -
1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^8)/d
```

Mupad [B]
time = 5.17, size = 280, normalized size = 1.88

$$-\frac{\frac{64\,a\,b}{35}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(16\,a^2+\frac{256\,b\,a}{5}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2+\frac{512\,b\,a}{35}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{170\,a^2}{3}+\frac{512\,b\,a}{5}\right)-\frac{170\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{3}+16\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-2\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{256\,a^2}{3}+64\,a\,b-32\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}-8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-56\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+70\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-56\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,a^2\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((64*a*b)/35 + tan(c/2 + (d*x)/2)^4*((256*a*b)/5 + 16*a^2) - tan(c/2 + (d
*x)/2)^2*((512*a*b)/35 + 2*a^2) - tan(c/2 + (d*x)/2)^6*((512*a*b)/5 + (170*
a^2)/3) - (170*a^2*tan(c/2 + (d*x)/2)^10)/3 + 16*a^2*tan(c/2 + (d*x)/2)^12
- 2*a^2*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^8*(64*a*b + (256*a^2)/3
- 32*b^2))/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/
2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*ta
n(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
 - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.273 $\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=115

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}+\frac{2ab\sec^5(c+dx}{5d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-a^2*sec(d*x+c)^2/d-4/3*a*b*sec(d*x
+c)^3/d+1/4*a^2*sec(d*x+c)^4/d+2/5*a*b*sec(d*x+c)^5/d+1/6*b^2*tan(d*x+c)^6/
d
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 962}

$$\frac{(a^2-2b^2)\sec^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((2*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (4*a*b*Sec[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sec[c + d
*x]^4)/(4*d) + (2*a*b*Sec[c + d*x]^5)/(5*d) + (b^2*Sec[c + d*x]^6)/(6*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4d}$$

$$= \frac{\text{Subst}\left(\int \left(2ab^4+\frac{a^2b^4}{x}-b^2(2a^2-b^2)\,x-4ab^2x^2+(a^2-2b^2)\,x^3+\right.\right.}{b^4d}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{(2a^2-b^2)\sec^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.28, size = 105, normalized size = 0.91

$$\frac{-60a^2\log(\cos(c+dx))+120ab\sec(c+dx)+30(-2a^2+b^2)\sec^2(c+dx)-80ab\sec^3(c+dx)+15(a^2-2b^2)\sec^4(c+dx)+24ab\sec^5(c+dx)+10b^2\sec^6(c+dx)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-60*a^2*Log[Cos[c + d*x]] + 120*a*b*Sec[c + d*x] + 30*(-2*a^2 + b^2)*Sec[c
 + d*x]^2 - 80*a*b*Sec[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sec[c + d*x]^4 + 24*a*
b*Sec[c + d*x]^5 + 10*b^2*Sec[c + d*x]^6)/(60*d)
```

Maple [A]
time = 0.14, size = 148, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+2ba\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a^2\left(\left(\tan\right.\right.}{d}$
default	$\frac{\frac{b^2\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+2ba\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+a^2\left(\left(\tan\right.\right.}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{4ba\,e^{11i(dx+c)}-4a^2e^{10i(dx+c)}+2b^2e^{10i(dx+c)}+\frac{28ba\,e^{9i(dx+c)}}{3}-12a^2e^{8i(dx+c)}+\frac{104ba\,e^{7i(dx+c)}}{5}-16a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*b^2*sin(d*x+c)^6/cos(d*x+c)^6+2*b*a*(1/5*sin(d*x+c)^6/cos(d*x+c)^5
-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2
-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 0.94

$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{120\,ab\cos\left(dx+c\right)^5-80\,ab\cos\left(dx+c\right)^3-30\,\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^4+24\,ab\cos\left(dx+c\right)+15\,\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2+10\,b^2}{\cos\left(dx+c\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a*b*cos(d*x + c)^5 - 80*a*b*cos(d*x
+ c)^3 - 30*(2*a^2 - b^2)*cos(d*x + c)^4 + 24*a*b*cos(d*x + c) + 15*(a^2 -
2*b^2)*cos(d*x + c)^2 + 10*b^2)/cos(d*x + c)^6)/d
```

Fricas [A]
time = 5.05, size = 115, normalized size = 1.00

$$-\frac{60\,a^2\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-120\,ab\cos\left(dx+c\right)^5+80\,ab\cos\left(dx+c\right)^3+30\,\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^4-24\,ab\cos\left(dx+c\right)-15\,\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-10\,b^2}{60\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a*b*cos(d*x + c)^5 +
80*a*b*cos(d*x + c)^3 + 30*(2*a^2 - b^2)*cos(d*x + c)^4 - 24*a*b*cos(d*x +
c) - 15*(a^2 - 2*b^2)*cos(d*x + c)^2 - 10*b^2)/(d*cos(d*x + c)^6)
```

Sympy [A]
time = 0.81, size = 189, normalized size = 1.64

$$\begin{cases} \frac{a^{2}\log{\left(\tan^{2}{\left(c+dx\right)}+1\right)}}{2d}+\frac{a^{2}\tan^{4}{\left(c+dx\right)}}{4d}-\frac{a^{2}\tan^{2}{\left(c+dx\right)}}{2d}+\frac{2ab\tan^{4}{\left(c+dx\right)}\sec{\left(c+dx\right)}}{5d}-\frac{8ab\tan^{2}{\left(c+dx\right)}\sec{\left(c+dx\right)}}{15d}+\frac{16ab\sec{\left(c+dx\right)}}{15d}+\frac{b^{2}\tan^{4}{\left(c+dx\right)}\sec^{2}{\left(c+dx\right)}}{6d}-\frac{b^{2}\tan^{2}{\left(c+dx\right)}\sec^{2}{\left(c+dx\right)}}{6d}+\frac{b^{2}\sec^{2}{\left(c+dx\right)}}{6d} & \text{for}\: d\neq 0 \\ x\left(a+b\sec{\left(c\right)}\right)^{2}\tan^{5}{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4/(4*d)
 - a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c + d*x)**4*sec(c + d*x)/(5*d) -
8*a*b*tan(c + d*x)**2*sec(c + d*x)/(15*d) + 16*a*b*sec(c + d*x)/(15*d) + b*
*2*tan(c + d*x)**4*sec(c + d*x)**2/(6*d) - b**2*tan(c + d*x)**2*sec(c + d*x
)**2/(6*d) + b**2*sec(c + d*x)**2/(6*d), Ne(d, 0)), (x*(a + b*sec(c))**2*ta
n(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(107) = 214.
time = 1.92, size = 341, normalized size = 2.97

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{147\,a^2+128\,ab+\frac{1002\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{768\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2925\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1920\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4140\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{1280\,ab(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{640\,b^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1002\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{147\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (147*a^2 + 128*a*b +
 1002*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 768*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2925*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 1920*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4140*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 + 1280*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 - 640*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^2*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

Mupad [B]
time = 4.97, size = 215, normalized size = 1.87

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}+\frac{\frac{32\,a\,b}{15}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\,a^2+32\,b\,a\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+\frac{64\,b\,a}{5}\right)+12\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(20\,a^2+\frac{64\,a\,b}{3}-\frac{32\,b^2}{3}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d + ((32*a*b)/15 + tan(c/2 + (d*x)/2)^4
*(32*a*b + 12*a^2) - tan(c/2 + (d*x)/2)^2*((64*a*b)/5 + 2*a^2) + 12*a^2*tan
(c/2 + (d*x)/2)^8 - 2*a^2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*((64
*a*b)/3 + 20*a^2 - (32*b^2)/3))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (
d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 +
 (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.274 $\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{b^2\sec^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+1/2*(a^2-b^2)*sec(d*x+c)^2/d+2/3*a*
b*sec(d*x+c)^3/d+1/4*b^2*sec(d*x+c)^4/d
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((a^2 - b^2)*Sec[c + d
*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x]^4)/(4*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$

$$= -\frac{\text{Subst}\left(\int \left(2ab^2+\frac{a^2b^2}{x}-(a^2-b^2)\,x-2ax^2-x^3\right)\,dx, x, b\sec(c\right.}{b^2d}$$

$$= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}+\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+$$

Mathematica [A]
time = 0.46, size = 74, normalized size = 0.85

$$\frac{12a^2\log(\cos(c+dx))-24ab\sec(c+dx)+6(a^2-b^2)\sec^2(c+dx)+8ab\sec^3(c+dx)+3b^2\sec^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (12*a^2*Log[Cos[c + d*x]] - 24*a*b*Sec[c + d*x] + 6*(a^2 - b^2)*Sec[c + d*x
]^2 + 8*a*b*Sec[c + d*x]^3 + 3*b^2*Sec[c + d*x]^4)/(12*d)
```

Maple [A]
time = 0.10, size = 108, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+2ba\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
default	$\frac{\frac{b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+2ba\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2\left(6ba\,e^{7i(dx+c)}-3a^2e^{6i(dx+c)}+3b^2e^{6i(dx+c)}+10ba\,e^{5i(dx+c)}-6a^2e^{4i(dx+c)}+10ba\,e^{3i(dx+c)}-3a^2\right.}{3d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^2*sin(d*x+c)^4/cos(d*x+c)^4+2*b*a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/2*tan(
d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 0.86

$$\frac{12\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{24\,ab\cos(dx+c)^3-8\,ab\cos(dx+c)-6\,(a^2-b^2)\cos(dx+c)^2-3\,b^2}{\cos(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c
) - 6*(a^2 - b^2)*cos(d*x + c)^2 - 3*b^2)/cos(d*x + c)^4)/d
```

Fricas [A]
time = 3.56, size = 82, normalized size = 0.94

$$\frac{12\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-24\,ab\cos\left(dx+c\right)^3+8\,ab\cos\left(dx+c\right)+6\left(a^2-b^2\right)\cos\left(dx+c\right)^2+3\,b^2}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a*b*cos(d*x + c)^3 + 8*
a*b*cos(d*x + c) + 6*(a^2 - b^2)*cos(d*x + c)^2 + 3*b^2)/(d*cos(d*x + c)^4)
```

Sympy [A]
time = 0.33, size = 126, normalized size = 1.45

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}-\frac{4ab\sec\left(c+dx\right)}{3d}+\frac{b^2\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}-\frac{b^2\sec^2\left(c+dx\right)}{4d} & \text{for } d\neq 0\\ x\left(a+b\sec\left(c\right)\right)^2\tan^3\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2/(2*d
) + 2*a*b*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 4*a*b*sec(c + d*x)/(3*d) + b
**2*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) - b**2*sec(c + d*x)**2/(4*d), Ne(
d, 0)), (x*(a + b*sec(c))**2*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(81) = 162$.
time = 0.92, size = 267, normalized size = 3.07

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{25\,a^2+32\,ab+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{128\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{198\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{96\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{48\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{124\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{25\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (25*a^2 + 32*a*b +
124*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 128*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 198*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 96
```

```
*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 48*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
+ 25*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1)^4)/d
```

Mupad [B]
time = 3.73, size = 151, normalized size = 1.74

$$-\frac{\frac{8\,a\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+\frac{32\,b\,a}{3}\right)-2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(4\,a^2+8\,a\,b-4\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,a^2\,\mathrm{atanh}\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((8*a*b)/3 - tan(c/2 + (d*x)/2)^2*((32*a*b)/3 + 2*a^2) - 2*a^2*tan(c/2 +
(d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a*b + 4*a^2 - 4*b^2))/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.275 $\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d+1/2*b^2*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3970, 45}

$$-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (b^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)\,dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 0.89

$$\frac{-2a^2\log(\cos(c+dx))+4ab\sec(c+dx)+b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
[Out] (-2*a^2*Log[Cos[c + d*x]] + 4*a*b*Sec[c + d*x] + b^2*Sec[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.05, size = 40, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sec^2(dx+c)\right)}{2}+2ab\sec(dx+c)+a^2\ln(\sec(dx+c))}{d}$	40
default	$\frac{\frac{b^2\left(\sec^2(dx+c)\right)}{2}+2ab\sec(dx+c)+a^2\ln(\sec(dx+c))}{d}$	40
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2b\left(2a\,e^{3i(dx+c)}+b\,e^{2i(dx+c)}+2\,e^{i(dx+c)}a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*b^2*sec(d*x+c)^2+2*a*b*sec(d*x+c)+a^2*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.89

$$-\frac{2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{4\,ab\cos(dx+c)+b^2}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a*b*cos(d*x + c) + b^2)/cos(d*x + c)^2)/
d
```

Fricas [A]
time = 3.18, size = 51, normalized size = 1.09

$$-\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,ab\cos\left(dx+c\right)-b^2}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a*b*cos(d*x + c) - b^2)/(
d*cos(d*x + c)^2)
```

Sympy [A]
time = 0.13, size = 60, normalized size = 1.28

$$\begin{cases} \frac{a^2 \log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{2ab\sec\left(c+dx\right)}{d} + \frac{b^2\sec^2\left(c+dx\right)}{2d} & \text{for}\ d \neq 0 \\ x\left(a+b\sec\left(c\right)\right)^2\tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + 2*a*b*sec(c + d*x)/d + b**
2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(45) = 90.
time = 0.56, size = 191, normalized size = 4.06

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3\,a^2+8\,ab+\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a^2 + 8*a*b + 6*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
2)/d
```

Mupad [B]
time = 1.53, size = 81, normalized size = 1.72

$$\frac{4\,a\,b-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(4\,a\,b-2\,b^2\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2\,a^2\,\mathrm{atanh}\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (4*a*b - tan(c/2 + (d*x)/2)^2*(4*a*b - 2*b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2
*tan(c/2 + (d*x)/2)^2 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.276 $\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(1+\sec(c+dx))}{2d}$$

```
[Out] a^2*ln(cos(d*x+c))/d+1/2*(a+b)^2*ln(1-sec(d*x+c))/d+1/2*(a-b)^2*ln(1+sec(d*
x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3970, 1816}

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(\sec(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + ((a + b)^2*Log[1 - Sec[c + d*x]])/(2*d) + ((a -
 b)^2*Log[1 + Sec[c + d*x]])/(2*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{b^2\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{b^2\text{Subst}\left(\int \left(\frac{(a+b)^2}{2b^2(b-x)}+\frac{a^2}{b^2x}-\frac{(a-b)^2}{2b^2(b+x)}\right)dx,x,b\sec(c+dx)\right)}{d}\\ &= \frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 53, normalized size = 0.87

$$\frac{(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b^2\log(\cos(c+dx))+(a+b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
[Out] ((a - b)^2*Log[Cos[(c + d*x)/2]] - b^2*Log[Cos[c + d*x]] + (a + b)^2*Log[Si
n[(c + d*x)/2]])/d
```

Maple [A]
time = 0.11, size = 48, normalized size = 0.79

method	result
derivativedivides	$\frac{b^2\ln(\tan(dx+c))+2ba\ln(\csc(dx+c)-\cot(dx+c))+a^2\ln(\sin(dx+c))}{d}$
default	$\frac{b^2\ln(\tan(dx+c))+2ba\ln(\csc(dx+c)-\cot(dx+c))+a^2\ln(\sin(dx+c))}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}+\frac{2\ln(e^{i(dx+c)}-1)ba}{d}+\frac{\ln(e^{i(dx+c)}-1)b^2}{d}+\frac{a^2\ln(e^{i(dx+c)}+1)}{d}-\frac{2\ln(e^{i}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*ln(tan(d*x+c))+2*b*a*ln(csc(d*x+c)-cot(d*x+c))+a^2*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 62, normalized size = 1.02

$$-\frac{2\,b^2\log\left(\cos\left(dx+c\right)\right)-\left(a^2-2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*(2*b^2*log(cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1))/d
```

Fricas [A]
time = 2.42, size = 68, normalized size = 1.11

$$-\frac{2\,b^2\log\left(-\cos\left(dx+c\right)\right)-\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2+2\,ab+b^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(-cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(1/2*cos(d*x + c) +
 1/2) - (a^2 + 2*a*b + b^2)*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \cot(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x), x)
```

Giac [A]
time = 0.46, size = 101, normalized size = 1.66

$$-\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+2\,b^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-(a^2+2\,ab+b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 2*b^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2*a*b + b^2)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 1.43, size = 96, normalized size = 1.57

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{b^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{b^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{d}+\frac{2\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (b^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*lo
g(tan(c/2 + (d*x)/2)^2 + 1))/d - (b^2*log(tan(c/2 + (d*x)/2)^2 - 1))/d + (2
*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.277 $\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=92

$$-\frac{a^2\log(\cos(c+dx))}{d}-\frac{a(a+b)\log(1-\sec(c+dx))}{2d}-\frac{a(a-b)\log(1+\sec(c+dx))}{2d}-\frac{\cot^2(c+dx)\,(a^2+b^2+$$

```
[Out] -a^2*ln(cos(d*x+c))/d-1/2*a*(a+b)*ln(1-sec(d*x+c))/d-1/2*a*(a-b)*ln(1+sec(d
*x+c))/d-1/2*cot(d*x+c)^2*(a^2+b^2+2*a*b*sec(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3970, 1819, 815}

$$-\frac{\cot^2(c+dx)\,(a^2+2ab\sec(c+dx)+b^2)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}-\frac{a(a+b)\log(1-\sec(c+dx))}{2d}-\frac{a(a-b)\log(\sec(c+dx)+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) - (a*(a + b)*Log[1 - Sec[c + d*x]])/(2*d) - (a
*(a - b)*Log[1 + Sec[c + d*x]])/(2*d) - (Cot[c + d*x]^2*(a^2 + b^2 + 2*a*b*
Sec[c + d*x]))/(2*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{b^4\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\text{Subst}\left(\int \frac{-2a^2-2ax}{x(b^2-x^2)}\,d\right.}{2d}$$
$$= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\text{Subst}\left(\int \left(-\frac{a(a+b)}{b^2(b-x)}\right.\right.}{}$$
$$= -\frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d} - \frac{a(a-b)\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.49, size = 82, normalized size = 0.89

$$-\frac{(a+b)^2\csc^2\left(\frac{1}{2}(c+dx)\right)+8a\left((a-b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+(a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a-b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/8*((a + b)^2*Csc[(c + d*x)/2]^2 + 8*a*((a - b)*Log[Cos[(c + d*x)/2]] + (
a + b)*Log[Sin[(c + d*x)/2]]) + (a - b)^2*Sec[(c + d*x)/2]^2)/d
```

Maple [A]
time = 0.12, size = 92, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{b^2}{2\sin(dx+c)^2}+2ba\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$
default	$\frac{-\frac{b^2}{2\sin(dx+c)^2}+2ba\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2ba\,e^{3i(dx+c)}+2a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}+2ba\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b^2/sin(d*x+c)^2+2*b*a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*
x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.78

$$-\frac{\left(a^2-ab\right)\log\left(\cos\left(dx+c\right)+1\right)+\left(a^2+ab\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,ab\cos(dx+c)+a^2+b^2}{\cos(dx+c)^2-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a^2 - a*b)*log(cos(d*x + c) + 1) + (a^2 + a*b)*log(cos(d*x + c) - 1)
 - (2*a*b*cos(d*x + c) + a^2 + b^2)/(cos(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 4.31, size = 113, normalized size = 1.23

$$\frac{2\,ab\cos\left(dx+c\right)+a^2+b^2-\left(\left(a^2-ab\right)\cos\left(dx+c\right)^2-a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\left(a^2+ab\right)\cos\left(dx+c\right)^2-a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c) + a^2 + b^2 - ((a^2 - a*b)*cos(d*x + c)^2 - a^2 + a
*b)*log(1/2*cos(d*x + c) + 1/2) - ((a^2 + a*b)*cos(d*x + c)^2 - a^2 - a*b)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(86) = 172.
time = 0.52, size = 209, normalized size = 2.27

$$\frac{8\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-4\left(a^2+ab\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a^2+2\,ab+b^2+\frac{4\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
```

```
 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*(a^2 + a*b)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 4*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B]
time = 1.36, size = 98, normalized size = 1.07

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,{\left(a-b\right)}^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+b\,a\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{8}+\frac{a\,b}{4}+\frac{b^2}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a - b)^2)/(8
*d) - (log(tan(c/2 + (d*x)/2))*(a*b + a^2))/d - (cot(c/2 + (d*x)/2)^2*((a*b
)/4 + a^2/8 + b^2/8))/d
```

3.278 $\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\log(1+\sec(c+dx))}{8d}+\frac{a\cot^2(c+dx)(2a+}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d+1/8*a*(4*a+3*b)*ln(1-sec(d*x+c))/d+1/8*a*(4*a-3*b)*ln(
1+sec(d*x+c))/d+1/4*a*cot(d*x+c)^2*(2*a+3*b*sec(d*x+c))/d-1/4*cot(d*x+c)^4*
(a^2+b^2+2*a*b*sec(d*x+c))/d
```

Rubi [A]

time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 1819, 837, 815}

$$-\frac{\cot^4(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\cos(c+dx))}{d}+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}+\frac{a(4a-3b)\log(\sec(c+dx)+1)}{8d}+\frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + (a*(4*a + 3*b)*Log[1 - Sec[c + d*x]])/(8*d) + (
a*(4*a - 3*b)*Log[1 + Sec[c + d*x]])/(8*d) + (a*Cot[c + d*x]^2*(2*a + 3*b*S
ec[c + d*x]))/(4*d) - (Cot[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sec[c + d*x]))/(4*
d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b^6\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d} + \frac{b^4\text{Subst}\left(\int \frac{-4a^2-6ax}{x(b^2-x^2)^2}\,d\right.}{4d}$$

$$= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\right.}{4d}$$

$$= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\right.}{4d}$$

$$= \frac{a^2\log(\cos(c+dx))}{d} + \frac{a(4a+3b)\log(1-\sec(c+dx))}{8d} + \frac{a(4a-3\ldots}{}$$

Mathematica [A]
time = 3.04, size = 148, normalized size = 1.17

$$\frac{2(7a^2+10ab+3b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)-(a+b)^2\csc^4\left(\frac{1}{2}(c+dx)\right)+16a\left((4a-3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+(4a+3b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+2(7a^2-10ab+3b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)-(a-b)^2\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(7*a^2 + 10*a*b + 3*b^2)*Csc[(c + d*x)/2]^2 - (a + b)^2*Csc[(c + d*x)/2]
^4 + 16*a*((4*a - 3*b)*Log[Cos[(c + d*x)/2]] + (4*a + 3*b)*Log[Sin[(c + d*x
)/2]]) + 2*(7*a^2 - 10*a*b + 3*b^2)*Sec[(c + d*x)/2]^2 - (a - b)^2*Sec[(c +
 d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.12, size = 136, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{b^2\left(\cos^4(dx+c)\right)}{4\sin(dx+c)^4}+2ba\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a^2\left(-\frac{\left(\cot\right.}{}\right.}{d}$
default	$\frac{-\frac{b^2\left(\cos^4(dx+c)\right)}{4\sin(dx+c)^4}+2ba\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a^2\left(-\frac{\left(\cot\right.}{}\right.}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{5ba\,\mathrm{e}^{7i(dx+c)}+8a^2\mathrm{e}^{6i(dx+c)}+4b^2\mathrm{e}^{6i(dx+c)}+3ba\,\mathrm{e}^{5i(dx+c)}-8a^2\mathrm{e}^{4i(dx+c)}+3ba\,\mathrm{e}^{3i(dx+c)}+8a^2\mathrm{e}^{2i(dx}}{2d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*b^2/sin(d*x+c)^4*cos(d*x+c)^4+2*b*a*(-1/4/sin(d*x+c)^4*cos(d*x+c)
^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc
(d*x+c)-cot(d*x+c)))+a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))
))
```

Maxima [A]
time = 0.29, size = 122, normalized size = 0.97

$$\frac{(4\,a^2-3\,ab)\log\left(\cos\left(dx+c\right)+1\right)+(4\,a^2+3\,ab)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(5\,ab\cos(dx+c)^3-3\,ab\cos(dx+c)+2\,(2\,a^2+b^2)\cos(dx+c)^2-3\,a^2-b^2\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((4*a^2 - 3*a*b)*log(cos(d*x + c) + 1) + (4*a^2 + 3*a*b)*log(cos(d*x +
c) - 1) - 2*(5*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c) + 2*(2*a^2 + b^2)*co
s(d*x + c)^2 - 3*a^2 - b^2)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.23, size = 203, normalized size = 1.61

$$-\frac{10\,ab\cos\left(dx+c\right)^3-6\,ab\cos\left(dx+c\right)+4\,(2\,a^2+b^2)\cos\left(dx+c\right)^2-6\,a^2-2\,b^2-\left((4\,a^2-3\,ab)\cos\left(dx+c\right)^4-2\,(4\,a^2-3\,ab)\cos\left(dx+c\right)^2+4\,a^2-3\,ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left((4\,a^2+3\,ab)\cos\left(dx+c\right)^4-2\,(4\,a^2+3\,ab)\cos\left(dx+c\right)^2+4\,a^2+3\,ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{8\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a*b*cos(d*x + c)^3 - 6*a*b*cos(d*x + c) + 4*(2*a^2 + b^2)*cos(d*x
+ c)^2 - 6*a^2 - 2*b^2 - ((4*a^2 - 3*a*b)*cos(d*x + c)^4 - 2*(4*a^2 - 3*a*b
)*cos(d*x + c)^2 + 4*a^2 - 3*a*b)*log(1/2*cos(d*x + c) + 1/2) - ((4*a^2 + 3
*a*b)*cos(d*x + c)^4 - 2*(4*a^2 + 3*a*b)*cos(d*x + c)^2 + 4*a^2 + 3*a*b)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\cot^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(118) = 236.
time = 0.56, size = 360, normalized size = 2.86

$$-\frac{64\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{2\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-8\,(4\,a^2+3\,ab)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a^2+2\,ab+b^2+\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{16\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{36\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 12*a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*a*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 8*(4*a^2 + 3*a*b)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 12*a
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 36*a*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 1.45, size = 164, normalized size = 1.30

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{5\,a^2}{32}-\frac{3\,a\,b}{16}+\frac{b^2}{32}+\frac{{(a-b)}^2}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,{(a-b)}^2}{64\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+\frac{3\,b\,a}{4}\right)}{d}-\frac{a^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(3\,a^2+4\,a\,b+b^2)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/32 - (3*a*b)/16 + b^2/32 + (a - b)^2/32))/d
- (tan(c/2 + (d*x)/2)^4*(a - b)^2)/(64*d) + (log(tan(c/2 + (d*x)/2))*((3*a*
b)/4 + a^2))/d - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2
)^4*((a*b)/2 + a^2/4 + b^2/4 - tan(c/2 + (d*x)/2)^2*(4*a*b + 3*a^2 + b^2)))
/(16*d)
```

3.279 $\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=157

$$-a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\frac{5ab\sec(c+$$

```
[Out] -a^2*x-5/8*a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a*b*sec(d*x+c)*ta
n(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^
2*tan(d*x+c)^5/d+1/3*a*b*sec(d*x+c)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5ab\tan^3(c+dx)\sec(c+dx)}{12d}+\frac{5ab\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a*b*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a*b*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = \int \left(a^2\tan^6(c+dx)+2ab\sec(c+dx)\tan^6(c+dx)+b^2\sec^2(c+d\right.$$

$$= a^2\int \tan^6(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^6(c+dx)\,dx+b^2$$

$$= \frac{a^2\tan^5(c+dx)}{5d}+\frac{ab\sec(c+dx)\tan^5(c+dx)}{3d}-a^2\int \tan^4(c+dx$$

$$= -\frac{a^2\tan^3(c+dx)}{3d}-\frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx}{5d}$$

$$= \frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-$$

$$= -a^2x-\frac{5ab\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5ab\sec(c+d}{}$$

Mathematica [A]
time = 1.42, size = 293, normalized size = 1.87

$$\frac{16800ab\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec^7(c+dx)\left(8820a^2(c+dx)\cos(3(c+dx))+2940a^2(c+dx)\cos(5(c+dx))+420a^2c\cos(7(c+dx))+420a^2dx\cos(7(c+dx))-3444a^2\sin(3(c+dx))+1260b^2\sin(3(c+dx))-980ab\sin(4(c+dx))-1988a^2\sin(5(c+dx))-420b^2\sin(5(c+dx))-1155ab\sin(6(c+dx))-644a^2\sin(7(c+dx))+60b^2\sin(7(c+dx))\right)+5950ab\sec^5(c+dx)\tan(c+dx)-2100\sec^6(c+dx)\left(7a^2(c+dx)-(a^2+b^2)\tan(c+dx)\right)}{26880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (16800*a*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) - Sec[c + d*x]^7*(8820*a^2*(c + d*x)*Cos[3*(c + d*x)]
 + 2940*a^2*(c + d*x)*Cos[5*(c + d*x)] + 420*a^2*c*Cos[7*(c + d*x)] + 420*a
^2*d*x*Cos[7*(c + d*x)] - 3444*a^2*Sin[3*(c + d*x)] + 1260*b^2*Sin[3*(c + d
*x)] - 980*a*b*Sin[4*(c + d*x)] - 1988*a^2*Sin[5*(c + d*x)] - 420*b^2*Sin[5
*(c + d*x)] - 1155*a*b*Sin[6*(c + d*x)] - 644*a^2*Sin[7*(c + d*x)] + 60*b^2
*Sin[7*(c + d*x)]) + 5950*a*b*Sec[c + d*x]^5*Tan[c + d*x] - 2100*Sec[c + d*
x]^6*(7*a^2*(c + d*x) - (a^2 + b^2)*Tan[c + d*x]))/(26880*d)
```

Maple [A]

time = 0.12, size = 168, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^7(dx+c)\right)}{7\cos(dx+c)^7}+2ba\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^7(dx+c)}{24\cos(dx+c)^4}+\frac{\sin^7(dx+c)}{16\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{16}+\frac{5\left(\sin^3(dx+c)\right)}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+\right.}{d}$
default	$\frac{\frac{b^2\left(\sin^7(dx+c)\right)}{7\cos(dx+c)^7}+2ba\left(\frac{\sin^7(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^7(dx+c)}{24\cos(dx+c)^4}+\frac{\sin^7(dx+c)}{16\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{16}+\frac{5\left(\sin^3(dx+c)\right)}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+\right.}{d}$
risch	$-a^2x-\frac{i(1155ba\,e^{13i(dx+c)}-2520a^2e^{12i(dx+c)}+840b^2e^{12i(dx+c)}+980ba\,e^{11i(dx+c)}-10080a^2e^{10i(dx+c)}+2975ba\,e^{9i(d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*b^2*sin(d*x+c)^7/cos(d*x+c)^7+2*b*a*(1/6*sin(d*x+c)^7/cos(d*x+c)^6
-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x
+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a^2
*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A]

time = 0.52, size = 150, normalized size = 0.96

$$\frac{240\,b^2\tan(dx+c)^7+112\left(3\,\tan(dx+c)^5-5\,\tan(dx+c)^3-15\,dx-15\,c+15\,\tan(dx+c)\right)a^2-35\,ab\left(\frac{2\left(33\,\sin(dx+c)^5-40\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}+15\,\log\left(\sin(dx+c)+1\right)-15\,\log\left(\sin(dx+c)-1\right)\right)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/1680*(240*b^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a*b*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Fricas [A]

time = 3.06, size = 184, normalized size = 1.17

$$-\frac{1680\,a^2dx\cos(dx+c)^7+525\,ab\cos(dx+c)^7\log\left(\sin(dx+c)+1\right)-525\,ab\cos(dx+c)^7\log\left(-\sin(dx+c)+1\right)-2\left(1155\,ab\cos(dx+c)^5+8\left(161\,a^2-15\,b^2\right)\cos(dx+c)^6-910\,ab\cos(dx+c)^3-8\left(77\,a^2-45\,b^2\right)\cos(dx+c)^4+280\,ab\cos(dx+c)+24\left(7\,a^2-15\,b^2\right)\cos(dx+c)^2+120\,b^2\right)\sin(dx+c)}{1680\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a*b*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a*b*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1155*a*b*cos(
d*x + c)^5 + 8*(161*a^2 - 15*b^2)*cos(d*x + c)^6 - 910*a*b*cos(d*x + c)^3 -
 8*(77*a^2 - 45*b^2)*cos(d*x + c)^4 + 280*a*b*cos(d*x + c) + 24*(7*a^2 - 15
*b^2)*cos(d*x + c)^2 + 120*b^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**6, x)
```

Giac [A]
time = 2.52, size = 282, normalized size = 1.80

$$-\frac{840\,(dx+c)a^2+525\,ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-525\,ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(840\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-525\,ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-6160\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+3500\,ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+19768\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-9905\,ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-28896\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+7680\,b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+19768\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+9905\,ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-6160\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3500\,ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+840\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+525\,ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(840*a^2*tan(1/2*d*x + 1/2*c)^
13 - 525*a*b*tan(1/2*d*x + 1/2*c)^13 - 6160*a^2*tan(1/2*d*x + 1/2*c)^11 + 3
500*a*b*tan(1/2*d*x + 1/2*c)^11 + 19768*a^2*tan(1/2*d*x + 1/2*c)^9 - 9905*a
*b*tan(1/2*d*x + 1/2*c)^9 - 28896*a^2*tan(1/2*d*x + 1/2*c)^7 + 7680*b^2*tan
(1/2*d*x + 1/2*c)^7 + 19768*a^2*tan(1/2*d*x + 1/2*c)^5 + 9905*a*b*tan(1/2*d
*x + 1/2*c)^5 - 6160*a^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b*tan(1/2*d*x + 1/
2*c)^3 + 840*a^2*tan(1/2*d*x + 1/2*c) + 525*a*b*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B]
time = 2.64, size = 403, normalized size = 2.57

$$\frac{\left(\frac{5ab}{4}-2a^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+\left(\frac{44a^2}{3}-\frac{25ab}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{283ab}{12}-\frac{706a^2}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\left(\frac{344a^2}{5}-\frac{128b^2}{7}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\left(-\frac{706a^2}{15}-\frac{283ba}{12}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\left(\frac{44a^2}{3}+\frac{25ba}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\left(-2a^2-\frac{5ba}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-1\right)}-\frac{2a^2\,\mathrm{atan}\left(\frac{64a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+25a^4b^2}+\frac{25a^4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+25a^4b^2}\right)}{d}-\frac{5ab\,\mathrm{atanh}\left(\frac{40a^5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^5b+\frac{125a^3b^3}{8}}+\frac{125a^3b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\left(40a^5b+\frac{125a^3b^3}{8}\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*((344*a^2)/5 - (128*b^2)/7) + tan(c/2 + (d*x)/2)^13*(
(5*a*b)/4 - 2*a^2) + tan(c/2 + (d*x)/2)^3*((25*a*b)/3 + (44*a^2)/3) - tan(c
/2 + (d*x)/2)^11*((25*a*b)/3 - (44*a^2)/3) - tan(c/2 + (d*x)/2)^5*((283*a*b
)/12 + (706*a^2)/15) + tan(c/2 + (d*x)/2)^9*((283*a*b)/12 - (706*a^2)/15) -
 tan(c/2 + (d*x)/2)*((5*a*b)/4 + 2*a^2))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*ta
n(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21
*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 -
1)) - (2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2) + (25*a
^4*b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2)))/d - (5*a*b*atanh((40*a^5
*b*tan(c/2 + (d*x)/2))/(40*a^5*b + (125*a^3*b^3)/8) + (125*a^3*b^3*tan(c/2
+ (d*x)/2))/(8*(40*a^5*b + (125*a^3*b^3)/8))))/(4*d)
```

3.280 $\int (a+b\sec(c+dx))^2 \tan^4(c+dx)\,dx$

Optimal. Leaf size=116

$$a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+d}{}$$

```
[Out] a^2*x+3/4*a*b*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a*b*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*b^2*
tan(d*x+c)^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\sec(c+dx)}{4d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a*b
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a*b*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx &= \int \left(a^2\tan^4(c+dx)+2ab\sec(c+dx)\tan^4(c+dx)+b^2\sec^2(c+dx\right.\\ &= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^4(c+dx)\,dx+b^2\int\\ &= \frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\\ &= -\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}\\ &= a^2x+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)}{4d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 355 vs. 2(116) = 232.
time = 0.97, size = 355, normalized size = 3.06

$$\frac{\sec^5(c+dx)\left(60a^2c\cos(5(c+dx))+60a^2dx\cos(5(c+dx))-45ab\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+45ab\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+150a\cos(c+dx)\left(4a(c+dx)-3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+75a\cos(3(c+dx))\left(4a(c+dx)-3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-80a^2\sin(c+dx)+120b^2\sin(c+dx)-60ab\sin(2(c+dx))-160a^2\sin(3(c+dx))-60b^2\sin(3(c+dx))-150ab\sin(4(c+dx))-80a^2\sin(5(c+dx))+12b^2\sin(5(c+dx))\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^5*(60*a^2*c*Cos[5*(c + d*x)] + 60*a^2*d*x*Cos[5*(c + d*x)] -
45*a*b*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b*C
```

```
os[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 150*a*Cos[c + d*
x]*(4*a*(c + d*x) - 3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*b*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 75*a*Cos[3*(c + d*x)]*(4*a*(c + d*x
) - 3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*b*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) - 80*a^2*Sin[c + d*x] + 120*b^2*Sin[c + d*x] - 60*a*b
*Sin[2*(c + d*x)] - 160*a^2*Sin[3*(c + d*x)] - 60*b^2*Sin[3*(c + d*x)] - 15
0*a*b*Sin[4*(c + d*x)] - 80*a^2*Sin[5*(c + d*x)] + 12*b^2*Sin[5*(c + d*x)])
)/(960*d)
```

Maple [A]
time = 0.09, size = 129, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+2ba\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\left(\tan^3(dx\right.}{3}\right.}{d}$
default	$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+2ba\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\left(\tan^3(dx\right.}{3}\right.}{d}$
risch	$a^2x+\frac{i\left(75ba\,e^{9i(dx+c)}-120a^2e^{8i(dx+c)}+60b^2e^{8i(dx+c)}+30ba\,e^{7i(dx+c)}-360a^2e^{6i(dx+c)}-440a^2e^{4i(dx+c)}+120b^2e^{4i(\right.}{30d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^2*sin(d*x+c)^5/cos(d*x+c)^5+2*b*a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 118, normalized size = 1.02

$$\frac{24\,b^2\tan\left(dx+c\right)^5+40\,\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2+15\,ab\left(\frac{2\,\left(5\,\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(24*b^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a*b*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

Fricas [A]
time = 2.99, size = 151, normalized size = 1.30

$$\frac{120\,a^2dx\cos\left(dx+c\right)^5+45\,ab\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,ab\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(75\,ab\cos\left(dx+c\right)^3+4\,\left(20\,a^2-3\,b^2\right)\cos\left(dx+c\right)^4-30\,ab\cos\left(dx+c\right)-4\,\left(5\,a^2-6\,b^2\right)\cos\left(dx+c\right)^2-12\,b^2\right)\sin\left(dx+c\right)}{120\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a*b*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(75*a*b*cos(d*x + c
)^3 + 4*(20*a^2 - 3*b^2)*cos(d*x + c)^4 - 30*a*b*cos(d*x + c) - 4*(5*a^2 -
6*b^2)*cos(d*x + c)^2 - 12*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(106) = 212.
time = 1.22, size = 220, normalized size = 1.90

$$\frac{60\,(dx+c)a^2+45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-45\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-320\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-192\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-320\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+45\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 45
*a*b*tan(1/2*d*x + 1/2*c)^9 - 320*a^2*tan(1/2*d*x + 1/2*c)^7 + 210*a*b*tan(
1/2*d*x + 1/2*c)^7 + 520*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*b^2*tan(1/2*d*x +
 1/2*c)^5 - 320*a^2*tan(1/2*d*x + 1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 45*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 2.56, size = 332, normalized size = 2.86

$$\frac{\left(2\,a^2-\frac{3\,a\,b}{2}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(7\,a\,b-\frac{32\,a^2}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{52\,a^2}{3}-\frac{32\,b^2}{5}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{32\,a^2}{3}-7\,b\,a\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2+\frac{3\,b\,a}{2}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{64\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+36\,a^4\,b^2}+\frac{36\,a^4\,b^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+36\,a^4\,b^2}\right)}{d}+\frac{3\,a\,b\,\mathrm{atanh}\left(\frac{48\,a^5\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{48\,a^5\,b+27\,a^3\,b^3}+\frac{27\,a^3\,b^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{48\,a^5\,b+27\,a^3\,b^3}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*((52*a^2)/3 - (32*b^2)/5) - tan(c/2 + (d*x)/2)^9*((3*
a*b)/2 - 2*a^2) - tan(c/2 + (d*x)/2)^3*(7*a*b + (32*a^2)/3) + tan(c/2 + (d*
```

```
x)/2)^7*(7*a*b - (32*a^2)/3) + tan(c/2 + (d*x)/2)*((3*a*b)/2 + 2*a^2))/(d*(
5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
- 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) + (2*a^2*atan((64*a^
6*tan(c/2 + (d*x)/2))/(64*a^6 + 36*a^4*b^2) + (36*a^4*b^2*tan(c/2 + (d*x)/2
))/(64*a^6 + 36*a^4*b^2)))/d + (3*a*b*atanh((48*a^5*b*tan(c/2 + (d*x)/2))/(
48*a^5*b + 27*a^3*b^3) + (27*a^3*b^3*tan(c/2 + (d*x)/2))/(48*a^5*b + 27*a^3
*b^3)))/(2*d)
```

3.281 $\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=70

$$-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] -a^2*x-a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d*x+c)
/d+1/3*b^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[
c + d*x]*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx &= \int \left(a^2\tan^2(c+dx)+2ab\sec(c+dx)\tan^2(c+dx)+b^2\sec^2(c+d\right.\\ &= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^2(c+dx)\,dx+b^2\\ &= \frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}-a^2\int 1\,dx-(ab)\int\\ &= -a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(70) = 140.
time = 1.24, size = 201, normalized size = 2.87

$$\frac{\sec^3(c+dx)\left(-9a\cos(c+dx)\left(a(c+dx)-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3a\cos(3(c+dx))\left(a(c+dx)-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+2\left(3a^2+b^2+6ab\cos(c+dx)+\left(3a^2-b^2\right)\cos(2(c+dx))\right)\sin(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*a*Cos[c + d*x]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*a*Cos[3
*(c + d*x)]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(3*a^2 + b^2 + 6*a*b*Cos[c + d
*x] + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

Maple [A]
time = 0.10, size = 92, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+2ba\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+2ba\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$
risch	$-a^2x-\frac{2i\left(3ba\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-3ba\,e^{i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{ba\ln\left(e^{i(dx+c)}+i\right)}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3+2*b*a*(1/2*sin(d*x+c)^3/cos(d*x+c)^2
+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 1.17

$$\frac{2\,b^2\tan\left(dx+c\right)^3-6\,\left(dx+c-\tan\left(dx+c\right)\right)a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Fricas [A]
time = 4.07, size = 115, normalized size = 1.64

$$-\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(3\,ab\cos\left(dx+c\right)+\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\tan^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(68) = 136.
time = 0.73, size = 158, normalized size = 2.26

$$-\frac{3\,(dx+c)a^2+3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 1.67, size = 227, normalized size = 3.24

$$-\frac{\frac{b^2\sin(3\,c+3\,d\,x)}{12}-\frac{b^2\sin(c+d\,x)}{4}-\frac{a^2\sin(3\,c+3\,d\,x)}{4}-\frac{a^2\sin(c+d\,x)}{4}+\frac{3\,a^2\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}-\frac{a\,b\sin(2\,c+2\,d\,x)}{2}+\frac{3\,a\,b\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}}{d\left(\frac{3\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] -((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x))
/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*
x)/4))
```

3.282 $\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=48

$$-a^2x-\frac{a^2\cot(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-b^2*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3971, 3554, 8, 2686, 3852}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d - (b^2*Cot[c + d*x])/d - (2*a*b*Csc[c + d*x
])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)+2ab\cot(c+dx)\csc(c+dx)+b^2\csc^2(c+dx)\right.\\ &= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cot(c+dx)\csc(c+dx)\,dx+b^2\int\\ &= -\frac{a^2\cot(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\text{Subst}(\int 1\,dx,x,\csc(c+dx))}{d}\\ &= -a^2x-\frac{a^2\cot(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 39, normalized size = 0.81

$$-\frac{(a^2+b^2)\cot(c+dx)+a(a(c+dx)+2b\csc(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
[Out] -(((a^2 + b^2)*Cot[c + d*x] + a*(a*(c + d*x) + 2*b*Csc[c + d*x]))/d)
```

Maple [A]
time = 0.08, size = 49, normalized size = 1.02

method	result	size
risch	$-a^2x-\frac{2i\left(2ba\,e^{i(dx+c)}+a^2+b^2\right)}{d\left(e^{2i(dx+c)}-1\right)}$	47
derivativedivides	$\frac{-b^2\cot(dx+c)-\frac{2ba}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	49
default	$\frac{-b^2\cot(dx+c)-\frac{2ba}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-b^2*cot(d*x+c)-2*b*a/sin(d*x+c)+a^2*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 47, normalized size = 0.98

$$-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,ab}{\sin(dx+c)}+\frac{b^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a*b/sin(d*x + c) + b^2/tan(d*x + c))/d
```

Fricas [A]
time = 4.64, size = 44, normalized size = 0.92

$$-\frac{a^2dx\sin\left(dx+c\right)+2\,ab+\left(a^2+b^2\right)\cos\left(dx+c\right)}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*sin(d*x + c) + 2*a*b + (a^2 + b^2)*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cot^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 80, normalized size = 1.67

$$-\frac{2\left(dx+c\right)a^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a^2+2\,ab+b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a^2 - a^2*tan(1/2*d*x + 1/2*c) + 2*a*b*tan(1/2*d*x + 1/2*
c) - b^2*tan(1/2*d*x + 1/2*c) + (a^2 + 2*a*b + b^2)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 1.43, size = 58, normalized size = 1.21

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a-b\right)}^2}{2\,d}-\frac{\frac{a^2}{2}+a\,b+\frac{b^2}{2}}{d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
[Out] (tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (a*b + a^2/2 + b^2/2)/(d*tan(c/2 + (
d*x)/2)) - a^2*x
```

3.283 $\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=85

$$a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-1/3*b^2*cot(d*x+c)^3/d+2*a*b*
csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d
```

Rubi [A]

time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3971, 3554, 8, 2686, 2687, 30}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*
x]^3)/(3*d) + (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx &= \int \left(a^2\cot^4(c+dx)+2ab\cot^3(c+dx)\csc(c+dx)+b^2\cot^2(c+dx\right.\\ &= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cot^3(c+dx)\csc(c+dx)\,dx+b^2\int\\ &= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\mathrm{Subst}\left(\int(-1+x\right.}{d}\\ &= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c+}{d}\\ &= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\,\mathrm{cs}}{}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 122, normalized size = 1.44

$$-\frac{\csc^3(c+dx)\left(-4ab+3b^2\cos(c+dx)+12ab\cos(2(c+dx))+4a^2\cos(3(c+dx))+b^2\cos(3(c+dx))-9a^2c\sin(c+dx)-9a^2dx\sin(c+dx)+3a^2c\sin(3(c+dx))+3a^2dx\sin(3(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*(Csc[c + d*x]^3*(-4*a*b + 3*b^2*Cos[c + d*x] + 12*a*b*Cos[2*(c + d*x)
] + 4*a^2*Cos[3*(c + d*x)] + b^2*Cos[3*(c + d*x)] - 9*a^2*c*Sin[c + d*x] -
9*a^2*d*x*Sin[c + d*x] + 3*a^2*c*Sin[3*(c + d*x)] + 3*a^2*d*x*Sin[3*(c + d*
x)]))/d
```

Maple [A]
time = 0.09, size = 111, normalized size = 1.31

method	result

derivativedivides	$\frac{-\frac{b^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+2ba\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)+a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
default	$\frac{-\frac{b^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+2ba\left(-\frac{\cos^4(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^4(dx+c)}{3\sin(dx+c)}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}\right)+a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
risch	$a^2x+\frac{2i\left(6ba\,e^{5i(dx+c)}+6a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-4ba\,e^{3i(dx+c)}-6a^2e^{2i(dx+c)}+6ba\,e^{i(dx+c)}+4a^2+b^2\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b^2/sin(d*x+c)^3*cos(d*x+c)^3+2*b*a*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/3*c
ot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.48, size = 76, normalized size = 0.89

$$\frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2+\frac{2\left(3\,\sin(dx+c)^2-1\right)ab}{\sin(dx+c)^3}-\frac{b^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a*b/sin(d*x + c)^3 - b^2/tan(d*x + c)^3)/d
```

Fricas [A]
time = 3.86, size = 102, normalized size = 1.20

$$\frac{6\,ab\cos\left(dx+c\right)^2+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^3-3\,a^2\cos\left(dx+c\right)-4\,ab+3\left(a^2dx\cos\left(dx+c\right)^2-a^2dx\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a*b*cos(d*x + c)^2 + (4*a^2 + b^2)*cos(d*x + c)^3 - 3*a^2*cos(d*x +
c) - 4*a*b + 3*(a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x + c))/((d*cos(d*x
 + c)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cot^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(79) = 158.
time = 0.51, size = 176, normalized size = 2.07

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,(dx+c)a^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+18\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2-2\,ab-b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 18*a*b
*tan(1/2*d*x + 1/2*c) - 3*b^2*tan(1/2*d*x + 1/2*c) + (15*a^2*tan(1/2*d*x +
1/2*c)^2 + 18*a*b*tan(1/2*d*x + 1/2*c)^2 + 3*b^2*tan(1/2*d*x + 1/2*c)^2 - a
^2 - 2*a*b - b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 1.46, size = 118, normalized size = 1.39

$$a^2\,x+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,{\left(a-b\right)}^2}{24\,d}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a\,\left(a-b\right)}{2}+\frac{{\left(a-b\right)}^2}{8}\right)}{d}-\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{2\,a\,b}{3}+\frac{a^2}{3}+\frac{b^2}{3}-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(5\,a^2+6\,a\,b+b^2\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - (tan(c/2 + (d*x)/2)*((a*(
a - b))/2 + (a - b)^2/8))/d - (cot(c/2 + (d*x)/2)^3*((2*a*b)/3 + a^2/3 + b^
2/3 - tan(c/2 + (d*x)/2)^2*(6*a*b + 5*a^2 + b^2)))/(8*d)
```

3.284 $\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=122

$$-a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-\frac{2ab\csc(c+dx)}{d}+\frac{4ab\csc^3(c+dx}{3d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d-1/5*b
^2*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d+4/3*a*b*csc(d*x+c)^3/d-2/5*a*b*csc(d*x
+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d + (4*a
*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)+2ab\cot^5(c+dx)\csc(c+dx)+b^2\cot^4(c+dx\right.$$

$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cot^5(c+dx)\csc(c+dx)\,dx+b^2\int$$

$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int(-1+x\right.}{}$$

$$= \frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}+a^2\int \cot^2(c$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+}{5d}$$

$$= -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\text{c}}{}$$

Mathematica [A]
time = 0.62, size = 198, normalized size = 1.62

$$-\frac{\csc^5(c+dx)\left(116ab+10(5a^2+3b^2)\cos(c+dx)-80ab\cos(2(c+dx))-25a^2\cos(3(c+dx))+15b^2\cos(3(c+dx))+60ab\cos(4(c+dx))+23a^2\cos(5(c+dx))+3b^2\cos(5(c+dx))+150a^2c\sin(c+dx)+150a^2dx\sin(c+dx)-75a^2c\sin(3(c+dx))-75a^2dx\sin(3(c+dx))+15a^2c\sin(5(c+dx))+15a^2dx\sin(5(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/240*(Csc[c + d*x]^5*(116*a*b + 10*(5*a^2 + 3*b^2)*Cos[c + d*x] - 80*a*b*
Cos[2*(c + d*x)] - 25*a^2*Cos[3*(c + d*x)] + 15*b^2*Cos[3*(c + d*x)] + 60*a
```

```
*b*Cos[4*(c + d*x)] + 23*a^2*Cos[5*(c + d*x)] + 3*b^2*Cos[5*(c + d*x)] + 15
0*a^2*c*Sin[c + d*x] + 150*a^2*d*x*Sin[c + d*x] - 75*a^2*c*Sin[3*(c + d*x)]
 - 75*a^2*d*x*Sin[3*(c + d*x)] + 15*a^2*c*Sin[5*(c + d*x)] + 15*a^2*d*x*Sin
[5*(c + d*x)]))/d
```

Maple [A]
time = 0.10, size = 154, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{b^2\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+2ba\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\cdots\right.}{d}$
default	$\frac{-\frac{b^2\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+2ba\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\cdots\right.}{d}$
risch	$-a^2x-\frac{2i\left(30ba\,e^{9i(dx+c)}+45a^2e^{8i(dx+c)}+15b^2e^{8i(dx+c)}-40ba\,e^{7i(dx+c)}-90a^2e^{6i(dx+c)}+116ba\,e^{5i(dx+c)}+140a^2e^{4i(\cdots}\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b^2/sin(d*x+c)^5*cos(d*x+c)^5+2*b*a*(-1/5/sin(d*x+c)^5*cos(d*x+c)
^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 96, normalized size = 0.79

$$-\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)ab}{\sin(dx+c)^5}+\frac{3\,b^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a*b/sin(d*x + c
)^5 + 3*b^2/tan(d*x + c)^5)/d
```

Fricas [A]
time = 2.74, size = 152, normalized size = 1.25

$$-\frac{30\,ab\cos(dx+c)^4+(23\,a^2+3\,b^2)\cos(dx+c)^5-35\,a^2\cos(dx+c)^3-40\,ab\cos(dx+c)^2+15\,a^2\cos(dx+c)+16\,ab+15\left(a^2dx\cos(dx+c)^4-2\,a^2dx\cos(dx+c)^2+a^2dx\right)\sin(dx+c)}{15\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^4 + (23*a^2 + 3*b^2)*cos(d*x + c)^5 - 35*a^2*cos
(d*x + c)^3 - 40*a*b*cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 16*a*b + 15*(a^
2*d*x*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^2 + a^2*d*x)*sin(d*x + c))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \cot^6(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(112) = 224.
time = 0.57, size = 273, normalized size = 2.24

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+50\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-480\,(dx+c)a^2+330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-300\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+300\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-50\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2+6\,ab+3\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 - 35*a^2*tan(1/2*d*x + 1/2*c)^3 + 50*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 15*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a^2 + 330*a^2*t
an(1/2*d*x + 1/2*c) - 300*a*b*tan(1/2*d*x + 1/2*c) + 30*b^2*tan(1/2*d*x + 1
/2*c) - (330*a^2*tan(1/2*d*x + 1/2*c)^4 + 300*a*b*tan(1/2*d*x + 1/2*c)^4 +
30*b^2*tan(1/2*d*x + 1/2*c)^4 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 - 50*a*b*tan(
1/2*d*x + 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)
/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 1.50, size = 191, normalized size = 1.57

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-b)^2}{160\,d}-a^2\,x-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^2}{16}-\frac{a\,b}{12}+\frac{b^2}{48}+\frac{(a-b)^2}{96}\right)}{d}-\frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(22\,a^2+20\,a\,b+2\,b^2)+\frac{a^2}{5}+\frac{b^2}{5}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{7\,a^2}{3}+\frac{10\,a\,b}{3}+b^2\right)}{32\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{21\,a^2}{32}-\frac{9\,a\,b}{16}+\frac{b^2}{32}+\frac{(a-b)^2}{32}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) - a^2*x - (tan(c/2 + (d*x)/2)^3*(a
^2/16 - (a*b)/12 + b^2/48 + (a - b)^2/96))/d - ((2*a*b)/5 + tan(c/2 + (d*x)
/2)^4*(20*a*b + 22*a^2 + 2*b^2) + a^2/5 + b^2/5 - tan(c/2 + (d*x)/2)^2*((10
*a*b)/3 + (7*a^2)/3 + b^2))/(32*d*tan(c/2 + (d*x)/2)^5) + (tan(c/2 + (d*x)/
2)*((21*a^2)/32 - (9*a*b)/16 + b^2/32 + (a - b)^2/32))/d
```

3.285 $\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=153

$$a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+\frac{2ab\csc(c+dx)}{d}-$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-1/7*a^
2*cot(d*x+c)^7/d-1/7*b^2*cot(d*x+c)^7/d+2*a*b*csc(d*x+c)/d-2*a*b*csc(d*x+c)
^3/d+6/5*a*b*csc(d*x+c)^5/d-2/7*a*b*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)/(7*d) - (b^2*Cot[c + d*x]^7)/(7*d) + (2*
a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/d + (6*a*b*Csc[c + d*x]^5)/(5*
d) - (2*a*b*Csc[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a^2\cot^8(c+dx)+2ab\cot^7(c+dx)\csc(c+dx)+b^2\cot^6(c+dx\right.$$

$$= a^2\int \cot^8(c+dx)\,dx+(2ab)\int \cot^7(c+dx)\csc(c+dx)\,dx+b^2\int$$

$$= -\frac{a^2\cot^7(c+dx)}{7d}-a^2\int \cot^6(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int(-1+x\right.}{}$$

$$= \frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+a^2\int \cot^4(c$$

$$= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c}{7d}$$

$$= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+}{7d}$$

$$= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot}{}$$

Mathematica [A]
time = 0.87, size = 257, normalized size = 1.68

$$-\frac{\csc^7(c+dx)\left(-1272ab+525b^2\cos(c+dx)+3612ab\cos(2(c+dx))+1176a^2\cos(3(c+dx))+315b^2\cos(3(c+dx))-840ab\cos(4(c+dx))-392a^2\cos(5(c+dx))+105b^2\cos(5(c+dx))+420ab\cos(6(c+dx))+176a^2\cos(7(c+dx))+15b^2\cos(7(c+dx))-3675a^2c\sin(c+dx)-3675a^2dx\sin(c+dx)+2205a^2c\sin(3(c+dx))+2205a^2dx\sin(3(c+dx))-735a^2c\sin(5(c+dx))-735a^2dx\sin(5(c+dx))+105a^2c\sin(7(c+dx))+105a^2dx\sin(7(c+dx))\right)}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/6720*(Csc[c + d*x]^7*(-1272*a*b + 525*b^2*Cos[c + d*x] + 3612*a*b*Cos[2*
(c + d*x)] + 1176*a^2*Cos[3*(c + d*x)] + 315*b^2*Cos[3*(c + d*x)] - 840*a*b
*Cos[4*(c + d*x)] - 392*a^2*Cos[5*(c + d*x)] + 105*b^2*Cos[5*(c + d*x)] + 4
20*a*b*Cos[6*(c + d*x)] + 176*a^2*Cos[7*(c + d*x)] + 15*b^2*Cos[7*(c + d*x)
] - 3675*a^2*c*Sin[c + d*x] - 3675*a^2*d*x*Sin[c + d*x] + 2205*a^2*c*Sin[3*
(c + d*x)] + 2205*a^2*d*x*Sin[3*(c + d*x)] - 735*a^2*c*Sin[5*(c + d*x)] - 7
35*a^2*d*x*Sin[5*(c + d*x)] + 105*a^2*c*Sin[7*(c + d*x)] + 105*a^2*d*x*Sin[
7*(c + d*x)]))/d
```

Maple [A]
time = 0.14, size = 187, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{b^2(\cos^7(dx+c))}{7\sin(dx+c)^7}+2ba\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos^4(dx+c))}{5}+\frac{8(\cos}{}\right.}{7}\right.}{d}$
default	$\frac{-\frac{b^2(\cos^7(dx+c))}{7\sin(dx+c)^7}+2ba\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos^4(dx+c))}{5}+\frac{8(\cos}{}\right.}{7}\right.}{d}$
risch	$a^2x+\frac{2i(210ba\,e^{13i(dx+c)}+420a^2e^{12i(dx+c)}+105b^2e^{12i(dx+c)}-420ba\,e^{11i(dx+c)}-1260a^2e^{10i(dx+c)}+1806ba\,e^{9i(dx+c)}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*b^2/sin(d*x+c)^7*cos(d*x+c)^7+2*b*a*(-1/7/sin(d*x+c)^7*cos(d*x+c)
^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*
x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))+a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d
*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 116, normalized size = 0.76

$$\frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^2+\frac{6\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)ab}{\sin(dx+c)^7}-\frac{15\,b^2}{\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a*b/sin(d*x + c)^7 - 15*b^2/tan(d*x + c)^7)/d
```

Fricas [A]
time = 3.18, size = 206, normalized size = 1.35

$$\frac{210\,ab\cos\left(dx+c\right)^{6}+\left(176\,a^{2}+15\,b^{2}\right)\cos\left(dx+c\right)^{7}-406\,a^{2}\cos\left(dx+c\right)^{5}-420\,ab\cos\left(dx+c\right)^{4}+350\,a^{2}\cos\left(dx+c\right)^{3}+336\,ab\cos\left(dx+c\right)^{2}-105\,a^{2}\cos\left(dx+c\right)-96\,ab+105\,\left(a^{2}dx\cos\left(dx+c\right)^{6}-3\,a^{2}dx\cos\left(dx+c\right)^{4}+3\,a^{2}dx\cos\left(dx+c\right)^{2}-a^{2}dx\right)\sin\left(dx+c\right)}{105\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(210*a*b*cos(d*x + c)^6 + (176*a^2 + 15*b^2)*cos(d*x + c)^7 - 406*a^2
*cos(d*x + c)^5 - 420*a*b*cos(d*x + c)^4 + 350*a^2*cos(d*x + c)^3 + 336*a*b
*cos(d*x + c)^2 - 105*a^2*cos(d*x + c) - 96*a*b + 105*(a^2*d*x*cos(d*x + c)
^6 - 3*a^2*d*x*cos(d*x + c)^4 + 3*a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x
 + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(141) = 282.
time = 0.64, size = 366, normalized size = 2.39

$$\frac{15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+15\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-189\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+294\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1295\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1470\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+315\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+13440\,\left(dx+c\right)a^{2}-9765\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7350\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-525\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{9765\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+7350\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+525\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1295\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1470\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-315\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+189\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+294\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+105\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a^{2}-30\,ab-15\,b^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 15
*b^2*tan(1/2*d*x + 1/2*c)^7 - 189*a^2*tan(1/2*d*x + 1/2*c)^5 + 294*a*b*tan(
1/2*d*x + 1/2*c)^5 - 105*b^2*tan(1/2*d*x + 1/2*c)^5 + 1295*a^2*tan(1/2*d*x
+ 1/2*c)^3 - 1470*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*b^2*tan(1/2*d*x + 1/2*c)
^3 + 13440*(d*x + c)*a^2 - 9765*a^2*tan(1/2*d*x + 1/2*c) + 7350*a*b*tan(1/2
*d*x + 1/2*c) - 525*b^2*tan(1/2*d*x + 1/2*c) + (9765*a^2*tan(1/2*d*x + 1/2*
c)^6 + 7350*a*b*tan(1/2*d*x + 1/2*c)^6 + 525*b^2*tan(1/2*d*x + 1/2*c)^6 - 1
295*a^2*tan(1/2*d*x + 1/2*c)^4 - 1470*a*b*tan(1/2*d*x + 1/2*c)^4 - 315*b^2*
tan(1/2*d*x + 1/2*c)^4 + 189*a^2*tan(1/2*d*x + 1/2*c)^2 + 294*a*b*tan(1/2*d
*x + 1/2*c)^2 + 105*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2 - 30*a*b - 15*b^2)/
tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 1.50, size = 258, normalized size = 1.69

$$a^2\,x+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,{\left(a-b\right)}^2}{896\,d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{3\,a^2}{32}-\frac{5\,a\,b}{48}+\frac{b^2}{48}+\frac{{\left(a-b\right)}^2}{384}\right)}{d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{a^2}{80}-\frac{3\,a\,b}{160}+\frac{b^2}{160}+\frac{{\left(a-b\right)}^2}{640}\right)}{d}-\frac{\frac{2\,a\,b}{7}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{37\,a^2}{3}+14\,a\,b+3\,b^2\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(93\,a^2+70\,a\,b+5\,b^2\right)+\frac{a^2}{7}+\frac{b^2}{7}-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{9\,a^2}{5}+\frac{14\,a\,b}{5}+b^2\right)}{128\,d\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{23\,a^2}{32}-\frac{17\,a\,b}{32}+\frac{b^2}{32}+\frac{{\left(a-b\right)}^2}{128}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8*(a + b/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)^7*(a - b)^2)/(896*d) + (tan(c/2 + (d*x)/2)^3*((
3*a^2)/32 - (5*a*b)/48 + b^2/48 + (a - b)^2/384))/d - (tan(c/2 + (d*x)/2)^5
*(a^2/80 - (3*a*b)/160 + b^2/160 + (a - b)^2/640))/d - ((2*a*b)/7 + tan(c/2
 + (d*x)/2)^4*(14*a*b + (37*a^2)/3 + 3*b^2) - tan(c/2 + (d*x)/2)^6*(70*a*b
+ 93*a^2 + 5*b^2) + a^2/7 + b^2/7 - tan(c/2 + (d*x)/2)^2*((14*a*b)/5 + (9*a
^2)/5 + b^2))/(128*d*tan(c/2 + (d*x)/2)^7) - (tan(c/2 + (d*x)/2)*((23*a^2)/
32 - (17*a*b)/32 + b^2/32 + (a - b)^2/128))/d
```

3.286 $\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{ab^8d}+\frac{(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx)}{b^7d}-\frac{a(a^4-4a^2b$$

```
[Out] -ln(cos(d*x+c))/a/d-(a^2-b^2)^4*ln(a+b*sec(d*x+c))/a/b^8/d+(a^6-4*a^4*b^2+6
*a^2*b^4-4*b^6)*sec(d*x+c)/b^7/d-1/2*a*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^2/b
^6/d+1/3*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^3/b^5/d-1/4*a*(a^2-4*b^2)*sec(d*x
+c)^4/b^4/d+1/5*(a^2-4*b^2)*sec(d*x+c)^5/b^3/d-1/6*a*sec(d*x+c)^6/b^2/d+1/7
*sec(d*x+c)^7/b/d
```

Rubi [A]
time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$-\frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{ab^8d}-\frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d}+\frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d}-\frac{a(a^4-4a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d}+\frac{(a^4-4a^2b^2+6b^4)\sec^3(c+dx)}{3b^5d}+\frac{(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx)}{b^7d}-\frac{a\sec^6(c+dx)}{6b^2d}-\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^7(c+dx)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/(a*b^8
*d) + ((a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x])/(b^7*d) - (a*(a^
4 - 4*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) + ((a^4 - 4*a^2*b^2 + 6*b^
4)*Sec[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4*d) +
 ((a^2 - 4*b^2)*Sec[c + d*x]^5)/(5*b^3*d) - (a*Sec[c + d*x]^6)/(6*b^2*d) +
Sec[c + d*x]^7/(7*b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^4}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-4a^4b^2+6a^2b^4-4b^6}{a^6}\right)+\frac{b^8}{ax}-a(a^4-4a^2b^2+6b^4)\,x+(a^4-4a^2b^2+6\right.\right.}{b}$$

$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{ab^8 d} + \frac{(a^6-4a^4b^2+6a^2b^4-4b^6}{b^7 d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 520 vs. $2(250) = 500$.
time = 6.24, size = 520, normalized size = 2.08

$$\frac{(a^7-4a^5b^2+6a^3b^4-4ab^6)(b+a\cos(c+dx))\log(\cos(c+dx))\sec(c+dx)}{b^8d(a+b\sec(c+dx))}+\frac{(-a^8+4a^6b^2-6a^4b^4+4a^2b^6-b^8)(b+a\cos(c+dx))\log(b+a\cos(c+dx))\sec(c+dx)}{ab^8d(a+b\sec(c+dx))}-\frac{(-a^2+2b^2)(a^4-2a^2b^2+2b^4)(b+a\cos(c+dx))\sec^2(c+dx)}{b^7d(a+b\sec(c+dx))}-\frac{a(a^4-4a^2b^2+6b^4)(b+a\cos(c+dx))\sec^3(c+dx)}{2b^6d(a+b\sec(c+dx))}+\frac{(a^4-4a^2b^2+6b^4)(b+a\cos(c+dx))\sec^4(c+dx)}{3b^5d(a+b\sec(c+dx))}+\frac{a(-a+2b)(a+2b)(b+a\cos(c+dx))\sec^5(c+dx)}{4b^4d(a+b\sec(c+dx))}-\frac{(-a+2b)(a+2b)(b+a\cos(c+dx))\sec^6(c+dx)}{5b^3d(a+b\sec(c+dx))}-\frac{a(b+a\cos(c+dx))\sec^7(c+dx)}{6b^2d(a+b\sec(c+dx))}+\frac{(b+a\cos(c+dx))\sec^8(c+dx)}{7bd(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*(b + a*Cos[c + d*x])*Log[Cos[c + d
*x]]*Sec[c + d*x])/(b^8*d*(a + b*Sec[c + d*x])) + ((-a^8 + 4*a^6*b^2 - 6*a^
4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])*Log[b + a*Cos[c + d*x]]*Sec[c
 + d*x])/(a*b^8*d*(a + b*Sec[c + d*x])) - ((-a^2 + 2*b^2)*(a^4 - 2*a^2*b^2
+ 2*b^4)*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(b^7*d*(a + b*Sec[c + d*x]))
- (a*(a^4 - 4*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])*Sec[c + d*x]^3)/(2*b^6*
d*(a + b*Sec[c + d*x])) + ((a^4 - 4*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])*S
ec[c + d*x]^4)/(3*b^5*d*(a + b*Sec[c + d*x])) + (a*(-a + 2*b)*(a + 2*b)*(b
+ a*Cos[c + d*x])*Sec[c + d*x]^5)/(4*b^4*d*(a + b*Sec[c + d*x])) - ((-a + 2
*b)*(a + 2*b)*(b + a*Cos[c + d*x])*Sec[c + d*x]^6)/(5*b^3*d*(a + b*Sec[c +
d*x])) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]^7)/(6*b^2*d*(a + b*Sec[c + d*
x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^8)/(7*b*d*(a + b*Sec[c + d*x]))
```

Maple [A]
time = 0.20, size = 273, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{(-a^8+4a^6b^2-6a^4b^4+4b^6a^2-b^8)\ln(b+a\cos(dx+c))}{b^8 a}-\frac{a}{6b^2\cos(dx+c)^6}-\frac{-a^2+4b^2}{5b^3\cos(dx+c)^5}-\frac{-a^4+4b^2a^2-6b^4}{3b^5\cos(dx+c)^3}-\frac{-a^6+4a^4b^2-6a^2b^4+4}{b^7\cos(dx+c)}}{d}$
default	$\frac{\frac{(-a^8+4a^6b^2-6a^4b^4+4b^6a^2-b^8)\ln(b+a\cos(dx+c))}{b^8 a}-\frac{a}{6b^2\cos(dx+c)^6}-\frac{-a^2+4b^2}{5b^3\cos(dx+c)^5}-\frac{-a^4+4b^2a^2-6b^4}{3b^5\cos(dx+c)^3}-\frac{-a^6+4a^4b^2-6a^2b^4+4}{b^7\cos(dx+c)}}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-a^8+4*a^6*b^2-6*a^4*b^4+4*a^2*b^6-b^8)/b^8/a*ln(b+a*cos(d*x+c))-1/6/
b^2*a/cos(d*x+c)^6-1/5*(-a^2+4*b^2)/b^3/cos(d*x+c)^5-1/3*(-a^4+4*a^2*b^2-6*
b^4)/b^5/cos(d*x+c)^3-(-a^6+4*a^4*b^2-6*a^2*b^4+4*b^6)/b^7/cos(d*x+c)-1/4*(
a^2-4*b^2)/b^4*a/cos(d*x+c)^4-1/2*(a^4-4*a^2*b^2+6*b^4)/b^6*a/cos(d*x+c)^2+
(a^6-4*a^4*b^2+6*a^2*b^4-4*b^6)/b^8*a*ln(cos(d*x+c))+1/7/b/cos(d*x+c)^7)
```

Maxima [A]
time = 0.31, size = 268, normalized size = 1.07

$$\frac{\frac{420\,(a^7-4\,a^5b^2+6\,a^3b^4-4\,ab^6)\,\log(\cos(dx+c))}{b^8}-\frac{420\,(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\,\log(a\cos(dx+c)+b)}{ab^8}-\frac{70\,ab^5\cos(dx+c)-420\,(a^6-4\,a^4b^2+6\,a^2b^4-4\,b^6)\cos(dx+c)^6-60\,b^6+210\,(a^5b-4\,a^3b^3+6\,ab^5)\cos(dx+c)^5-140\,(a^4b^2-4\,a^2b^4+6\,b^6)\cos(dx+c)^4+105\,(a^3b^3-4\,ab^5)\cos(dx+c)^3-84\,(a^2b^4-4\,b^6)\cos(dx+c)^2}{b^7\cos(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(420*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(cos(d*x + c))/b^8 -
420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a*cos(d*x + c) + b)
/(a*b^8) - (70*a*b^5*cos(d*x + c) - 420*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^
6)*cos(d*x + c)^6 - 60*b^6 + 210*(a^5*b - 4*a^3*b^3 + 6*a*b^5)*cos(d*x + c)
^5 - 140*(a^4*b^2 - 4*a^2*b^4 + 6*b^6)*cos(d*x + c)^4 + 105*(a^3*b^3 - 4*a*
b^5)*cos(d*x + c)^3 - 84*(a^2*b^4 - 4*b^6)*cos(d*x + c)^2)/(b^7*cos(d*x + c
)^7))/d
```

Fricas [A]
time = 4.10, size = 293, normalized size = 1.17

$$-\frac{70\,a^2b^6\cos(dx+c)+420\,(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\cos(dx+c)^7\log(a\cos(dx+c)+b)-420\,(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6)\cos(dx+c)^7\log(-\cos(dx+c))-60\,ab^7-420\,(a^7b-4\,a^5b^3+6\,a^3b^5-4\,ab^7)\cos(dx+c)^6+210\,(a^6b^2-4\,a^4b^4+6\,a^2b^6)\cos(dx+c)^5-140\,(a^5b^3-4\,a^3b^5+6\,ab^7)\cos(dx+c)^4+105\,(a^4b^4-4\,a^2b^6)\cos(dx+c)^3-84\,(a^3b^5-4\,ab^7)\cos(dx+c)^2}{420\,ab^8d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/420*(70*a^2*b^6*cos(d*x + c) + 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*cos(d*x + c)^7*log(a*cos(d*x + c) + b) - 420*(a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^7*log(-cos(d*x + c)) - 60*a*b^7 - 420*(
a^7*b - 4*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^6 + 210*(a^6*b^2 - 4*
a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^5 - 140*(a^5*b^3 - 4*a^3*b^5 + 6*a*b^7)*c
os(d*x + c)^4 + 105*(a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^3 - 84*(a^3*b^5 - 4*
a*b^7)*cos(d*x + c)^2)/(a*b^8*d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**9/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. $2(238) = 476$.

time = 5.27, size = 1768, normalized size = 7.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(210*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(a + b - 2*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^8 - 420*(a^7 - 4*a^
5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 1))/b^8 - 210*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + 2*b^8)*log(abs(
2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^8*abs(a))
 + (1089*a^7 - 840*a^6*b - 4356*a^5*b^2 + 3080*a^4*b^3 + 6534*a^3*b^4 - 408
8*a^2*b^5 - 4356*a*b^6 + 2232*b^7 + 7623*a^7*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 5040*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 31332*a^5*b^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 19040*a^4*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 48258*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 26
096*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 33012*a*b^6*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 14784*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 22869*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a^6*b*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 95676*a^5*b^2*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 47880*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 151494*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 67368*a^2*b
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 107436*a*b^6*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 40152*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 38115*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 16800*a^6*b*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 160860*a^5*b^2*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 + 62720*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 258930*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 86240*
a^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 192220*a*b^6*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3 + 53760*b^7*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 38115*a^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 12600*a^
6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 160860*a^5*b^2*(cos(d*x + c
) - 1)^4/(cos(d*x + c) + 1)^4 + 45080*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 + 258930*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 5
6840*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 192220*a*b^6*(cos(
d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 24360*b^7*(cos(d*x + c) - 1)^4/(cos(
```

```
d*x + c) + 1)^4 + 22869*a^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 504
0*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 95676*a^5*b^2*(cos(d*x
+ c) - 1)^5/(cos(d*x + c) + 1)^5 + 16800*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 151494*a^3*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 18480*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 107436*a*b^6*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 6720*b^7*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 7623*a^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 84
0*a^6*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 31332*a^5*b^2*(cos(d*x
+ c) - 1)^6/(cos(d*x + c) + 1)^6 + 2520*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d
*x + c) + 1)^6 + 48258*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 -
2520*a^2*b^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 33012*a*b^6*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 840*b^7*(cos(d*x + c) - 1)^6/(cos(d*x
 + c) + 1)^6 + 1089*a^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a^
5*b^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 6534*a^3*b^4*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a*b^6*(cos(d*x + c) - 1)^7/(cos(d*x +
c) + 1)^7)/(b^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^7))/d
```

Mupad [B]
time = 2.97, size = 631, normalized size = 2.52

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}-\frac{\frac{2\,\left(105\,a^6-385\,a^4\,b^2+511\,a^2\,b^4-279\,b^6\right)}{105\,b^7}+\frac{2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\,\left(a^6+a^5\,b-3\,a^4\,b^2-3\,a^3\,b^3+3\,a^2\,b^4+3\,a\,b^5-b^6\right)}{b^7}-\frac{2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\,\left(6\,a^6+5\,a^5\,b-20\,a^4\,b^2-17\,a^3\,b^3+22\,a^2\,b^4+19\,a\,b^5-8\,b^6\right)}{b^7}-\frac{4\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\,\left(30\,a^6+15\,a^5\,b-112\,a^4\,b^2-54\,a^3\,b^3+154\,a^2\,b^4+71\,a\,b^5-96\,b^6\right)}{3\,b^7}+\frac{2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\,\left(45\,a^6+30\,a^5\,b-161\,a^4\,b^2-108\,a^3\,b^3+203\,a^2\,b^4+142\,a\,b^5-87\,b^6\right)}{3\,b^7}+\frac{2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\,\left(75\,a^6+25\,a^5\,b-285\,a^4\,b^2-85\,a^3\,b^3+401\,a^2\,b^4+95\,a\,b^5-239\,b^6\right)}{5\,b^7}-\frac{2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\,\left(90\,a^6+15\,a^5\,b-340\,a^4\,b^2-45\,a^3\,b^3+466\,a^2\,b^4+45\,a\,b^5-264\,b^6\right)}{15\,b^7}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-35\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+35\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-21\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+7\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,\left(-a^7+4\,a^5\,b^2-6\,a^3\,b^4+4\,a\,b^6\right)}{b^8\,d}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,\left(a^2-b^2\right)^4}{a\,b^8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(105*a^6 - 279*b^6 + 511*a^2*b^4
- 385*a^4*b^2))/(105*b^7) + (2*tan(c/2 + (d*x)/2)^12*(3*a*b^5 + a^5*b + a^6
 - b^6 + 3*a^2*b^4 - 3*a^3*b^3 - 3*a^4*b^2))/b^7 - (2*tan(c/2 + (d*x)/2)^10
*(19*a*b^5 + 5*a^5*b + 6*a^6 - 8*b^6 + 22*a^2*b^4 - 17*a^3*b^3 - 20*a^4*b^2
))/b^7 - (4*tan(c/2 + (d*x)/2)^6*(71*a*b^5 + 15*a^5*b + 30*a^6 - 96*b^6 + 1
54*a^2*b^4 - 54*a^3*b^3 - 112*a^4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^8*(
142*a*b^5 + 30*a^5*b + 45*a^6 - 87*b^6 + 203*a^2*b^4 - 108*a^3*b^3 - 161*a^
4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^4*(95*a*b^5 + 25*a^5*b + 75*a^6 - 2
39*b^6 + 401*a^2*b^4 - 85*a^3*b^3 - 285*a^4*b^2))/(5*b^7) - (2*tan(c/2 + (d
*x)/2)^2*(45*a*b^5 + 15*a^5*b + 90*a^6 - 264*b^6 + 466*a^2*b^4 - 45*a^3*b^3
 - 340*a^4*b^2))/(15*b^7))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/
2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*
x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (log(tan
(c/2 + (d*x)/2)^2 - 1)*(4*a*b^6 - a^7 - 6*a^3*b^4 + 4*a^5*b^2))/(b^8*d) - (
log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^4)
/(a*b^8*d)
```

3.287 $\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=170

$$\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6d}+\frac{(a^4-3a^2b^2+3b^4)\sec(c+dx)}{b^5d}-\frac{a(a^2-3b^2)\sec^2(c+dx}{2b^4d}$$

```
[Out] ln(cos(d*x+c))/a/d-(a^2-b^2)^3*ln(a+b*sec(d*x+c))/a/b^6/d+(a^4-3*a^2*b^2+3*
b^4)*sec(d*x+c)/b^5/d-1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/b^4/d+1/3*(a^2-3*b^2)*
sec(d*x+c)^3/b^3/d-1/4*a*sec(d*x+c)^4/b^2/d+1/5*sec(d*x+c)^5/b/d
```

Rubi [A]

time = 0.10, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$-\frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6d}-\frac{a(a^2-3b^2)\sec^2(c+dx)}{2b^4d}+\frac{(a^2-3b^2)\sec^3(c+dx)}{3b^3d}+\frac{(a^4-3a^2b^2+3b^4)\sec(c+dx)}{b^5d}-\frac{a\sec^4(c+dx)}{4b^2d}+\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/(a*b^6*d)
 + ((a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x])/(b^5*d) - (a*(a^2 - 3*b^2)*Sec[
c + d*x]^2)/(2*b^4*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^3)/(3*b^3*d) - (a*Sec[c
 + d*x]^4)/(4*b^2*d) + Sec[c + d*x]^5/(5*b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2(-a^2+b^2)}{a^4}\right)+\frac{b^6}{ax}+a(a^2-3b^2)\,x-(a^2-3b^2)\,x^2+ax^3-\right.\right.}{b^6 d}$$
$$= \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6 d} + \frac{(a^4-3a^2b^2+3b^4)\sec(c+}{b^5 d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 371 vs. 2(170) = 340.
time = 6.19, size = 371, normalized size = 2.18

$$\frac{(a^5-3a^3b^2+3ab^4)(b+a\cos(c+dx))\log(\cos(c+dx))\sec(c+dx)}{b^6d(a+b\sec(c+dx))}+\frac{(-a^6+3a^4b^2-3a^2b^4+b^6)(b+a\cos(c+dx))\log(b+a\cos(c+dx))\sec(c+dx)}{ab^6d(a+b\sec(c+dx))}+\frac{(a^4-3a^2b^2+3b^4)(b+a\cos(c+dx))\sec^2(c+dx)}{b^5d(a+b\sec(c+dx))}+\frac{a(-a^2+3b^2)(b+a\cos(c+dx))\sec^3(c+dx)}{2b^4d(a+b\sec(c+dx))}+\frac{(a^2-3b^2)(b+a\cos(c+dx))\sec^4(c+dx)}{3b^3d(a+b\sec(c+dx))}-\frac{a(b+a\cos(c+dx))\sec^5(c+dx)}{4b^2d(a+b\sec(c+dx))}+\frac{(b+a\cos(c+dx))\sec^6(c+dx)}{5bd(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^5 - 3*a^3*b^2 + 3*a*b^4)*(b + a*Cos[c + d*x])*Log[Cos[c + d*x]]*Sec[c +
 d*x])/(b^6*d*(a + b*Sec[c + d*x])) + ((-a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6)
*(b + a*Cos[c + d*x])*Log[b + a*Cos[c + d*x]]*Sec[c + d*x])/(a*b^6*d*(a + b
*Sec[c + d*x])) + ((a^4 - 3*a^2*b^2 + 3*b^4)*(b + a*Cos[c + d*x])*Sec[c + d
*x]^2)/(b^5*d*(a + b*Sec[c + d*x])) + (a*(-a^2 + 3*b^2)*(b + a*Cos[c + d*x]
)*Sec[c + d*x]^3)/(2*b^4*d*(a + b*Sec[c + d*x])) + ((a^2 - 3*b^2)*(b + a*Co
s[c + d*x])*Sec[c + d*x]^4)/(3*b^3*d*(a + b*Sec[c + d*x])) - (a*(b + a*Cos[
c + d*x])*Sec[c + d*x]^5)/(4*b^2*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]^6)/(5*b*d*(a + b*Sec[c + d*x]))
```

Maple [A]
time = 0.13, size = 184, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{\left(-a^6+3a^4b^2-3a^2b^4+b^6\right)\ln(b+a\cos(dx+c))}{b^6 a}-\frac{a}{4b^2\cos(dx+c)^4}-\frac{-a^2+3b^2}{3b^3\cos(dx+c)^3}-\frac{-a^4+3b^2a^2-3b^4}{b^5\cos(dx+c)}-\frac{\left(a^2-3b^2\right)a}{2b^4\cos(dx+c)^2}+\frac{\left(a^4-3b^2a\right.}{}}{d}$
default	$\frac{\frac{\left(-a^6+3a^4b^2-3a^2b^4+b^6\right)\ln(b+a\cos(dx+c))}{b^6 a}-\frac{a}{4b^2\cos(dx+c)^4}-\frac{-a^2+3b^2}{3b^3\cos(dx+c)^3}-\frac{-a^4+3b^2a^2-3b^4}{b^5\cos(dx+c)}-\frac{\left(a^2-3b^2\right)a}{2b^4\cos(dx+c)^2}+\frac{\left(a^4-3b^2a\right.}{}}{d}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2a^4e^{9i(dx+c)}-6b^2a^2e^{9i(dx+c)}+6b^4e^{9i(dx+c)}-2b\,a^3e^{8i(dx+c)}+6b^3a\,e^{8i(dx+c)}+8a^4e^{7i(dx+c)}-\frac{64b^2a^2e^{7i}}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^6/a*ln(b+a*cos(d*x+c))-1/4/b^2*a/cos(
d*x+c)^4-1/3*(-a^2+3*b^2)/b^3/cos(d*x+c)^3-(-a^4+3*a^2*b^2-3*b^4)/b^5/cos(d
*x+c)-1/2*(a^2-3*b^2)/b^4*a/cos(d*x+c)^2+(a^4-3*a^2*b^2+3*b^4)/b^6*a*ln(cos
(d*x+c))+1/5/b/cos(d*x+c)^5)
```

Maxima [A]
time = 0.27, size = 183, normalized size = 1.08

$$\frac{\frac{60\,(a^5-3\,a^3b^2+3\,ab^4)\log(\cos(dx+c))}{b^6}-\frac{60\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\log(a\cos(dx+c)+b)}{ab^6}-\frac{15\,ab^3\cos(dx+c)-60\,(a^4-3\,a^2b^2+3\,b^4)\cos(dx+c)^4-12\,b^4+30\,(a^3b-3\,ab^3)\cos(dx+c)^3-20\,(a^2b^2-3\,b^4)\cos(dx+c)^2}{b^5\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(cos(d*x + c))/b^6 - 60*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a*b^6) - (15*a*b^3*cos(d*
x + c) - 60*(a^4 - 3*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 - 12*b^4 + 30*(a^3*b -
 3*a*b^3)*cos(d*x + c)^3 - 20*(a^2*b^2 - 3*b^4)*cos(d*x + c)^2)/(b^5*cos(d*
x + c)^5))/d
```

Fricas [A]
time = 2.77, size = 205, normalized size = 1.21

$$-\frac{15\,a^2b^4\cos(dx+c)+60\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\cos(dx+c)^5\log(a\cos(dx+c)+b)-60\,(a^6-3\,a^4b^2+3\,a^2b^4)\cos(dx+c)^5\log(-\cos(dx+c))-12\,ab^5-60\,(a^5b-3\,a^3b^3+3\,ab^5)\cos(dx+c)^4+30\,(a^4b^2-3\,a^2b^4)\cos(dx+c)^3-20\,(a^3b^3-3\,ab^5)\cos(dx+c)^2}{60\,ab^6d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*b^4*cos(d*x + c) + 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
(d*x + c)^5*log(a*cos(d*x + c) + b) - 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cos(
d*x + c)^5*log(-cos(d*x + c)) - 12*a*b^5 - 60*(a^5*b - 3*a^3*b^3 + 3*a*b^5)
*cos(d*x + c)^4 + 30*(a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^3 - 20*(a^3*b^3 - 3
*a*b^5)*cos(d*x + c)^2)/(a*b^6*d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(162) = 324.
time = 3.32, size = 1052, normalized size = 6.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(30*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(abs(a + b - 2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^6 - 60*(a^5 - 3*a^3*b^2 + 3*a*b^4
)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 30*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - 2*b^6)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 2*abs(a)))/(b^6*abs(a)) + (137*a^5 - 120*a^4*b - 411*a^3*b^2 +
320*a^2*b^3 + 411*a*b^4 - 264*b^5 + 685*a^5*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 480*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2175*a^3*b^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1360*a^2*b^3*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 2295*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1200*b^5*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1370*a^5*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 720*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 447
0*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2000*a^2*b^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5070*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 1920*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1370*a
^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 480*a^4*b*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3 - 4470*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 1200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5070*a*b^4
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 720*b^5*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 685*a^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1
20*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2175*a^3*b^2*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 240*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 2295*a*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 120*
b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^5*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 - 411*a^3*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1
)^5 + 411*a*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/(b^6*((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

Mupad [B]
time = 2.40, size = 395, normalized size = 2.32

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(a^4-3\,a^2\,b^2+3\,b^4\right)}{b^6\,d}-\frac{\frac{2\,\left(15\,a^4-40\,a^2\,b^2+33\,b^4\right)}{15\,b^5}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a^4+a^3\,b-2\,a^2\,b^2-2\,a\,b^3+b^4\right)}{b^5}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(4\,a^4+3\,a^3\,b-10\,a^2\,b^2-8\,a\,b^3+6\,b^4\right)}{b^5}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^4+3\,a^3\,b-34\,a^2\,b^2-6\,a\,b^3+30\,b^4\right)}{3\,b^5}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(18\,a^4+9\,a^3\,b-50\,a^2\,b^2-24\,a\,b^3+48\,b^4\right)}{3\,b^5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(a^2-b^2\right)^3}{a\,b^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^4 + 3*b^4 - 3*a^2*b^2))/(b^6*d) - ((2*(
15*a^4 + 33*b^4 - 40*a^2*b^2))/(15*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a^3*b -
2*a*b^3 + a^4 + b^4 - 2*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^6*(3*a^3*b -
8*a*b^3 + 4*a^4 + 6*b^4 - 10*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^2*(3*a^3
```

```
*b - 6*a*b^3 + 12*a^4 + 30*b^4 - 34*a^2*b^2))/(3*b^5) + (2*tan(c/2 + (d*x)/
2)^4*(9*a^3*b - 24*a*b^3 + 18*a^4 + 48*b^4 - 50*a^2*b^2))/(3*b^5))/(d*(5*ta
n(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*
tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^
2)*(a^2 - b^2)^3)/(a*b^6*d)
```

3.288 $\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{a\sec^2(c+dx)}{2b^2d}+\frac{\sec^3(c+dx)}{3bd}$$

```
[Out] -ln(cos(d*x+c))/a/d-(a^2-b^2)^2*ln(a+b*sec(d*x+c))/a/b^4/d+(a^2-2*b^2)*sec(
d*x+c)/b^3/d-1/2*a*sec(d*x+c)^2/b^2/d+1/3*sec(d*x+c)^3/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-\frac{a\sec^2(c+dx)}{2b^2d}-\frac{\log(\cos(c+dx))}{ad}+\frac{\sec^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/(a*b^4
*d) + ((a^2 - 2*b^2)*Sec[c + d*x])/(b^3*d) - (a*Sec[c + d*x]^2)/(2*b^2*d) +
 Sec[c + d*x]^3/(3*b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^4d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^4d}$$

$$= -\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sec(c+dx)}{b^3d}-$$

Mathematica [A]
time = 0.39, size = 108, normalized size = 1.00

$$\frac{6a^2(a^2-2b^2)\log(\cos(c+dx))-6(a^2-b^2)^2\log(b+a\cos(c+dx))+6ab(a^2-2b^2)\sec(c+dx)-3a^2b^2\sec^2(c+dx)+2ab^3\sec^3(c+dx)}{6ab^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
[Out] (6*a^2*(a^2 - 2*b^2)*Log[Cos[c + d*x]] - 6*(a^2 - b^2)^2*Log[b + a*Cos[c +
d*x]] + 6*a*b*(a^2 - 2*b^2)*Sec[c + d*x] - 3*a^2*b^2*Sec[c + d*x]^2 + 2*a*b
^3*Sec[c + d*x]^3)/(6*a*b^4*d)
```

Maple [A]
time = 0.13, size = 115, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{\left(-a^4+2b^2a^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a}-\frac{a}{2b^2\cos(dx+c)^2}-\frac{-a^2+2b^2}{b^3\cos(dx+c)}+\frac{\left(a^2-2b^2\right)a\ln(\cos(dx+c))}{b^4}+\frac{1}{3b\cos(dx+c)^3}}{d}$
default	$\frac{\frac{\left(-a^4+2b^2a^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a}-\frac{a}{2b^2\cos(dx+c)^2}-\frac{-a^2+2b^2}{b^3\cos(dx+c)}+\frac{\left(a^2-2b^2\right)a\ln(\cos(dx+c))}{b^4}+\frac{1}{3b\cos(dx+c)^3}}{d}$
risch	$\frac{ix}{a}+\frac{2ic}{ad}+\frac{2a^2e^{5i(dx+c)}-4b^2e^{5i(dx+c)}-2ba\,e^{4i(dx+c)}+4a^2e^{3i(dx+c)}-\frac{16b^2e^{3i(dx+c)}}{3}-2ba\,e^{2i(dx+c)}+2a^2e^{i(dx+c)}-4b^2}{d\,b^3\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*((-a^4+2*a^2*b^2-b^4)/b^4/a*ln(b+a*cos(d*x+c))-1/2/b^2*a/cos(d*x+c)^2-(
-a^2+2*b^2)/b^3/cos(d*x+c)+(a^2-2*b^2)/b^4*a*ln(cos(d*x+c))+1/3/b/cos(d*x+c
)^3)
```

Maxima [A]
time = 0.28, size = 110, normalized size = 1.02

$$\frac{\frac{6\,(a^3-2\,ab^2)\log(\cos(dx+c))}{b^4}-\frac{6\,(a^4-2\,a^2b^2+b^4)\log(a\cos(dx+c)+b)}{ab^4}-\frac{3\,ab\cos(dx+c)-6\,(a^2-2\,b^2)\cos(dx+c)^2-2\,b^2}{b^3\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^3 - 2*a*b^2)*log(cos(d*x + c))/b^4 - 6*(a^4 - 2*a^2*b^2 + b^4)*lo
g(a*cos(d*x + c) + b)/(a*b^4) - (3*a*b*cos(d*x + c) - 6*(a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*b^2)/(b^3*cos(d*x + c)^3))/d
```

Fricas [A]
time = 2.89, size = 129, normalized size = 1.19

$$-\frac{3\,a^2b^2\cos\left(dx+c\right)+6\left(a^4-2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3\log\left(a\cos\left(dx+c\right)+b\right)-6\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-2\,ab^3-6\left(a^3b-2\,ab^3\right)\cos\left(dx+c\right)^2}{6\,ab^4d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*b^2*cos(d*x + c) + 6*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^3*log
(a*cos(d*x + c) + b) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)
) - 2*a*b^3 - 6*(a^3*b - 2*a*b^3)*cos(d*x + c)^2)/(a*b^4*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^5\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(104) = 208.
time = 1.95, size = 560, normalized size = 5.19

$$-\frac{\frac{3\left(a^3-2\,ab^2\right)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^4}-\frac{6\left(a^3-2\,ab^2\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^4}-\frac{3\left(a^4-2\,a^2b^2+2\,b^4\right)\log\left(\frac{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{b^4|a|}+\frac{11\,a^3-12\,a^2b-22\,ab^2+20\,b^3+\frac{33\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{24\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{78\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{12\,a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{78\,ab^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{12\,b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{22\,ab^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{b^4\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2))/b^4 - 6*(a^3 - 2*a*b^2)*log(abs(-(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 1))/b^4 - 3*(a^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b +
 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
```

```
 - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^4*abs(a)) + (1
1*a^3 - 12*a^2*b - 22*a*b^2 + 20*b^3 + 33*a^3*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 24*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 78*a*b^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 48*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 33*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12*a^2*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 78*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 12*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 22*a*b^2*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3)/(b^4*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

Mupad [B]
time = 1.88, size = 227, normalized size = 2.10

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\frac{2\,\left(3\,a^2-5\,b^2\right)}{3\,b^3}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+a\,b-4\,b^2\right)}{b^3}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2+a\,b-b^2\right)}{b^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(a^2-2\,b^2\right)}{b^4\,d}-\frac{\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(a^2-b^2\right)^2}{a\,b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(3*a^2 - 5*b^2))/(3*b^3) - (2*tan
(c/2 + (d*x)/2)^2*(a*b + 2*a^2 - 4*b^2))/b^3 + (2*tan(c/2 + (d*x)/2)^4*(a*b
 + a^2 - b^2))/b^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + t
an(c/2 + (d*x)/2)^6 - 1)) + (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^2 - 2*b^2))
/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a
^2 - b^2)^2)/(a*b^4*d)
```

3.289 $\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] ln(cos(d*x+c))/a/d-(a^2-b^2)*ln(a+b*sec(d*x+c))/a/b^2/d+sec(d*x+c)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$-\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d}+\frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a*b^2*d) +
 Sec[c + d*x]/(b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^2 d}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx, x, b\sec(c+dx)\right)}{b^2 d}$$
$$= \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2 d} + \frac{\sec(c+dx)}{bd}$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 0.88

$$\frac{a^2\log(\cos(c+dx)) + (-a^2+b^2)\log(b+a\cos(c+dx)) + ab\sec(c+dx)}{ab^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a^2*Log[Cos[c + d*x]] + (-a^2 + b^2)*Log[b + a*Cos[c + d*x]] + a*b*Sec[c +
 d*x])/(a*b^2*d)
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{\left(-a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2 a}+\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}}{d}$
default	$\frac{\frac{\left(-a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2 a}+\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}}{d}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{a}+1\right)}{b^2 d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{a}+1\right)}{ad}+\frac{a\ln(\mathrm{e}^{2i(dx}}{b^2 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-a^2+b^2)/b^2/a*ln(b+a*cos(d*x+c))+1/b^2*a*ln(cos(d*x+c))+1/b/cos(d*x
+c))
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.97

$$\frac{\frac{a\log(\cos(dx+c))}{b^2} - \frac{(a^2-b^2)\log(a\cos(dx+c)+b)}{ab^2} + \frac{1}{b\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(cos(d*x + c))/b^2 - (a^2 - b^2)*log(a*cos(d*x + c) + b)/(a*b^2) + 1/
(b*cos(d*x + c)))/d
```

Fricas [A]
time = 3.10, size = 69, normalized size = 1.17

$$\frac{a^2\cos(dx+c)\log(-\cos(dx+c)) - (a^2-b^2)\cos(dx+c)\log(a\cos(dx+c)+b) + ab}{ab^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(-cos(d*x + c)) - (a^2 - b^2)*cos(d*x + c)*log(a*cos(d
*x + c) + b) + a*b)/(a*b^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(59) = 118.
time = 0.83, size = 289, normalized size = 4.90

$$-\frac{\frac{a\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^2}-\frac{2a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}-\frac{(a^2-2b^2)\log\left(\frac{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2|a|\right|}{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2|a|\right|}\right)}{b^2|a|}+\frac{2\left(a-2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2))/b^2 - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^
2 - (a^2 - 2*b^2)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
```

```
 + 2*abs(a)))/(b^2*abs(a)) + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 1.54, size = 115, normalized size = 1.95

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{b^2\,d}-\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,\left(\frac{a}{b^2}-\frac{1}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 -
1)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/
2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d
```

3.290 $\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=35

$$-\frac{\log(\cos(c+dx))}{ad}-\frac{\log(a+b\sec(c+dx))}{ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-ln(a+b*sec(d*x+c))/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3970, 36, 29, 31}

$$-\frac{\log(a+b\sec(c+dx))}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Log[a + b*Sec[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sec(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sec(c+dx)\right)}{ad}$$
$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{\log(a+b\sec(c+dx))}{ad}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.54

$$-\frac{\log(b+a\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] -(Log[b + a*Cos[c + d*x]]/(a*d))
```

Maple [A]
time = 0.04, size = 33, normalized size = 0.94

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a}+\frac{\ln(\sec(dx+c))}{a}}{d}$	33
default	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a}+\frac{\ln(\sec(dx+c))}{a}}{d}$	33
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{ad}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a*ln(a+b*sec(d*x+c))+1/a*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.54

$$-\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

Fricas [A]
time = 3.34, size = 19, normalized size = 0.54

$$-\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(27) = 54.
time = 3.39, size = 82, normalized size = 2.34

$$\begin{cases} \frac{\tilde{\infty}x\tan\left(c\right)}{\sec\left(c\right)} & \text{for } a=0\wedge b=0\wedge d=0\\ \frac{x\tan\left(c\right)}{a+b\sec\left(c\right)} & \text{for } d=0\\ \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2ad} & \text{for } b=0\\ -\frac{1}{bd\sec\left(c+dx\right)} & \text{for } a=0\\ -\frac{\log\left(\frac{a}{b}+\sec\left(c+dx\right)\right)}{ad}+\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2ad} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*tan(c)/sec(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x*tan(c)/
(a + b*sec(c)), Eq(d, 0)), (log(tan(c + d*x)**2 + 1)/(2*a*d), Eq(b, 0)), (-
1/(b*d*sec(c + d*x)), Eq(a, 0)), (-log(a/b + sec(c + d*x))/(a*d) + log(tan(
c + d*x)**2 + 1)/(2*a*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(35) = 70.
time = 0.67, size = 114, normalized size = 3.26

$$\frac{\log\left(\frac{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{d|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(
```

```
d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(d*a
bs(a))
```

Mupad [B]
time = 1.47, size = 71, normalized size = 2.03

$$\frac{\operatorname{atan}\left(\frac{a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+b\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((a*sin(c/2 + (d*x)/2)^2)/(a*cos(c/2 + (d*x)/2)^2*1i + b*cos(c/2 + (d*
x)/2)^2*1i + b*sin(c/2 + (d*x)/2)^2*1i))*2i)/(a*d)
```

3.291 $\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{\log(\cos(c+dx))}{ad}+\frac{\log(1-\sec(c+dx))}{2(a+b)d}+\frac{\log(1+\sec(c+dx))}{2(a-b)d}-\frac{b^2\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)d}$$

```
[Out] ln(cos(d*x+c))/a/d+1/2*ln(1-sec(d*x+c))/(a+b)/d+1/2*ln(1+sec(d*x+c))/(a-b)/
d-b^2*ln(a+b*sec(d*x+c))/a/(a^2-b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3970, 908}

$$-\frac{b^2\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\log(1-\sec(c+dx))}{2d(a+b)}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)*d) + Log[1 + Sec
[c + d*x]]/(2*(a - b)*d) - (b^2*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)}\,dx,x,b\sec(c+dx)\right)}{d}$$
$$= -\frac{b^2\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)}+\frac{1}{ab^2x}+\frac{1}{a(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)b^2(b+x)}\right)\,dx,x,b\sec(c+\right.}{d}$$
$$= \frac{\log(\cos(c+dx))}{ad}+\frac{\log(1-\sec(c+dx))}{2(a+b)d}+\frac{\log(1+\sec(c+dx))}{2(a-b)d}-\frac{b^2\log(a+b\sec}{a\left(a^2-b^2\right.}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.74

$$\frac{a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b^2\log(b+a\cos(c+dx))+a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)*
Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.12, size = 75, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{b^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(-1+\cos(dx+c))}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{-\frac{b^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(-1+\cos(dx+c))}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
risch	$\frac{ix}{a}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}+\frac{2ib^2x}{a(a^2-b^2)}+\frac{2ib^2c}{ad(a^2-b^2)}+\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}-\frac{b^2\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln(-1+cos(d*x+c))+1/
(2*a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 68, normalized size = 0.72

$$-\frac{\frac{2\,b^2\log(a\cos(dx+c)+b)}{a^3-ab^2}-\frac{\log(\cos(dx+c)+1)}{a-b}-\frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b)/(a^3 - a*b^2) - log(cos(d*x + c) + 1)/(
a - b) - log(cos(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 3.53, size = 75, normalized size = 0.80

$$-\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^3-ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/
2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(90) = 180.
time = 0.75, size = 257, normalized size = 2.73

$$-\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}-\frac{(a^2-2\,b^2)\log\left(\frac{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{(a^2-b^2)|a|}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a)
)/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-cos(d*
x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B]
time = 1.75, size = 93, normalized size = 0.99

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\ (a+b)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}+\frac{b^2\ \ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d\ (a\,b^2-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) +
 (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a*b
^2 - a^3))
```

3.292 $\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$-\frac{\log(\cos(c+dx))}{ad}-\frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2d}-\frac{(2a-3b)\log(1+\sec(c+dx))}{4(a-b)^2d}-\frac{b^4\log(a+b\sec(c+dx$$

```
[Out] -ln(cos(d*x+c))/a/d-1/4*(2*a+3*b)*ln(1-sec(d*x+c))/(a+b)^2/d-1/4*(2*a-3*b)*
ln(1+sec(d*x+c))/(a-b)^2/d-b^4*ln(a+b*sec(d*x+c))/a/(a^2-b^2)^2/d+1/4/(a+b)
/d/(1-sec(d*x+c))+1/4/(a-b)/d/(1+sec(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$-\frac{b^4\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sec(c+dx))}+\frac{1}{4d(a-b)(\sec(c+dx)+1)}-\frac{(2a+3b)\log(1-\sec(c+dx))}{4d(a+b)^2}-\frac{(2a-3b)\log(\sec(c+dx)+1)}{4d(a-b)^2}-\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])/(4*(a + b)
^2*d) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a +
b*Sec[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sec[c + d*x])) +
 1/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2} + \frac{2a+3b}{4b^4(a+b)^2(b-x)} + \frac{1}{ab^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)} - \frac{1}{4(a-b)b^3(b+x)^2} \right.\right.}{d}$$

$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2d} - \frac{(2a-3b)\log(1+\sec(c+d}{4(a-b)^2d}$$

Mathematica [A]
time = 1.07, size = 141, normalized size = 0.90

$$-\frac{a(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right)+4a(2a-3b)(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8b^4\log(b+a\cos(c+dx))+4a(a-b)^2(2a+3b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(a-b)(a+b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a(a-b)^2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*(a - b)^2*(a + b)*Csc[(c + d*x)/2]^2 + 4*a*(2*a - 3*b)*(a + b)^2*Lo
g[Cos[(c + d*x)/2]] + 8*b^4*Log[b + a*Cos[c + d*x]] + 4*a*(a - b)^2*(2*a +
3*b)*Log[Sin[(c + d*x)/2]] + a*(a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(a*(a
- b)^2*(a + b)^2*d)
```

Maple [A]
time = 0.18, size = 126, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{b^4\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a}+\frac{1}{(4a+4b)(-1+\cos(dx+c))}+\frac{(-2a-3b)\ln(-1+\cos(dx+c))}{4(a+b)^2}-\frac{1}{(4a-4b)(1+\cos(dx+c))}+\frac{(-2a+3b)\ln(1+\cos(dx+}{4(a-b)^2}}{d}$
default	$\frac{-\frac{b^4\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a}+\frac{1}{(4a+4b)(-1+\cos(dx+c))}+\frac{(-2a-3b)\ln(-1+\cos(dx+c))}{4(a+b)^2}-\frac{1}{(4a-4b)(1+\cos(dx+c))}+\frac{(-2a+3b)\ln(1+\cos(dx+}{4(a-b)^2}}{d}$
risch	$-\frac{ix}{a}+\frac{iac}{d(a^2-2ba+b^2)}+\frac{iax}{a^2-2ba+b^2}-\frac{3ibc}{2d(a^2-2ba+b^2)}+\frac{iac}{d(a^2+2ba+b^2)}+\frac{3ibx}{2(a^2+2ba+b^2)}+\frac{3ibc}{2d(a^2+2ba+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^4/(a+b)^2/(a-b)^2/a*ln(b+a*cos(d*x+c))+1/(4*a+4*b)/(-1+cos(d*x+c))+
1/4/(a+b)^2*(-2*a-3*b)*ln(-1+cos(d*x+c))-1/(4*a-4*b)/(1+cos(d*x+c))+1/4/(a-
b)^2*(-2*a+3*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 144, normalized size = 0.92

$$-\frac{\frac{4\,b^4\log(a\cos(dx+c)+b)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(b\cos(dx+c)-a)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^4*log(a*cos(d*x + c) + b)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)
*log(cos(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(cos(d*x + c) -
 1)/(a^2 + 2*a*b + b^2) + 2*(b*cos(d*x + c) - a)/((a^2 - b^2)*cos(d*x + c)^
2 - a^2 + b^2))/d
```

Fricas [A]
time = 3.19, size = 263, normalized size = 1.68

$$\frac{2a^4 - 2a^2b^2 - 2(a^3b - ab^3)\cos(dx + c) - 4(b^4\cos(dx + c)^2 - b^4)\log(a\cos(dx + c) + b) + (2a^4 + a^3b - 4a^2b^2 - 3ab^3 - (2a^4 + a^3b - 4a^2b^2 - 3ab^3)\cos(dx + c)^2)\log\left(\frac{1}{2}\cos(dx + c) + \frac{1}{2}\right) + (2a^4 - a^3b - 4a^2b^2 + 3ab^3 - (2a^4 - a^3b - 4a^2b^2 + 3ab^3)\cos(dx + c)^2)\log\left(-\frac{1}{2}\cos(dx + c) + \frac{1}{2}\right)}{4\left((a^5 - 2a^3b^2 + ab^4)d\cos(dx + c)^2 - (a^5 - 2a^3b^2 + ab^4)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4 - 2*a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c) - 4*(b^4*cos(d*x +
c)^2 - b^4)*log(a*cos(d*x + c) + b) + (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3
- (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - (2*a^4 - a^3*b - 4*a^2*b^
2 + 3*a*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - 2*a^3*b^
2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c + dx)}{a + b\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(147) = 294.
time = 0.63, size = 403, normalized size = 2.57

$$-\frac{\frac{2(2a+3b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2ab+b^2} - \frac{4(a^3-2ab^2)\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^4-2a^2b^2+b^4} - \frac{\left(a+b+\frac{4a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^2+2ab+b^2)(\cos(dx+c)-1)} - \frac{4(a^4-2a^2b^2+2b^4)\log\left(\frac{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2|a|\right|}{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2|a|\right|}\right)}{(a^4-2a^2b^2+b^4)|a|} - \frac{\cos(dx+c)-1}{(a-b)(\cos(dx+c)+1)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2
+ 2*a*b + b^2) - 4*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(
```

```
cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2))/(a^4 - 2*a^2*b^2 + b^4) - (a + b + 4*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b + b^2)*(cos(d*x + c) - 1)) - 4*(a
^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*abs(a)))/((a^4 - 2*a^2*b^2 + b^4)*abs(a)) - (cos(d*x + c) - 1)/(
(a - b)*(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 1.85, size = 174, normalized size = 1.11

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d\,\left(4\,a-4\,b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a+3\,b\right)}{d\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}-\frac{a-b}{2\,d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a+b\right)\,\left(4\,a-4\,b\right)}-\frac{b^4\,\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d\,\left(a^2-b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - tan(c/2 + (d*x)/2)^2/(2*d*(4*a - 4*b)
) - (log(tan(c/2 + (d*x)/2))*(2*a + 3*b))/(d*(4*a*b + 2*a^2 + 2*b^2)) - (a
- b)/(2*d*tan(c/2 + (d*x)/2)^2*(a + b)*(4*a - 4*b)) - (b^4*log(a + b - a*ta
n(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^2)
```

3.293 $\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{\log(\cos(c+dx))}{ad}+\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16(a+b)^3d}+\frac{(8a^2-21ab+15b^2)\log(1+\sec(c+dx))}{16(a-b)^3d}-\frac{b^6}{}$$

```
[Out] ln(cos(d*x+c))/a/d+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sec(d*x+c))/(a+b)^3/d+1/
16*(8*a^2-21*a*b+15*b^2)*ln(1+sec(d*x+c))/(a-b)^3/d-b^6*ln(a+b*sec(d*x+c))/
a/(a^2-b^2)^3/d-1/16/(a+b)/d/(1-sec(d*x+c))^2+1/16*(-5*a-7*b)/(a+b)^2/d/(1-
sec(d*x+c))-1/16/(a-b)/d/(1+sec(d*x+c))^2+1/16*(-5*a+7*b)/(a-b)^2/d/(1+sec(
d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16d(a+b)^3}+\frac{(8a^2-21ab+15b^2)\log(\sec(c+dx)+1)}{16d(a-b)^3}-\frac{b^6\log(a+b\sec(c+dx))}{ad\,(a^2-b^2)^3}-\frac{5a+7b}{16d(a+b)^2(1-\sec(c+dx))}-\frac{5a-7b}{16d(a-b)^2(\sec(c+dx)+1)}-\frac{1}{16d(a+b)(1-\sec(c+dx))^2}-\frac{1}{16d(a-b)(\sec(c+dx)+1)^2}+\frac{\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sec[c + d*x]])
/(16*(a + b)^3*d) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sec[c + d*x]])/(16*(
a - b)^3*d) - (b^6*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)^3*d) - 1/(16*(a
+ b)*d*(1 - Sec[c + d*x])^2) - (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sec[c + d*x
])) - 1/(16*(a - b)*d*(1 + Sec[c + d*x])^2) - (5*a - 7*b)/(16*(a - b)^2*d*(
1 + Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^6\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3}+\frac{5a+7b}{16b^5(a+b)^2(b-x)^2}+\frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)}+\frac{1}{ab^6x}+\frac{1}{a(a-b)^3(a+b)^3(a}\right.\right.}{d}$$

$$= \frac{\log(\cos(c+dx))}{ad}+\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16(a+b)^3d}+\frac{(8a^2-21ab+15b^2}{16(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.25, size = 625, normalized size = 2.67

$$\frac{2i(a^5-3a^3b^2+3ab^4)(c+dx)(b+a\cos(c+dx))\sec(c+dx)}{(a-b)^3(a+b)^3d(a+b\sec(c+dx))}-\frac{i(-8a^2+21ab-15b^2)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))\sec(c+dx)}{8(-a+b)^3d(a+b\sec(c+dx))}-\frac{i(8a^2+21ab+15b^2)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))\sec(c+dx)}{8(a+b)^3d(a+b\sec(c+dx))}+\frac{(7a+9b)(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{32(a+b)^2d(a+b\sec(c+dx))}-\frac{(b+a\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{64(a+b)d(a+b\sec(c+dx))}+\frac{(-8a^2+21ab-15b^2)(b+a\cos(c+dx))\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{16(-a+b)^3d(a+b\sec(c+dx))}+\frac{b^6(b+a\cos(c+dx))\log(b+a\cos(c+dx))\sec(c+dx)}{a(-a^2+b^2)^3d(a+b\sec(c+dx))}+\frac{(8a^2+21ab+15b^2)(b+a\cos(c+dx))\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{16(a+b)^3d(a+b\sec(c+dx))}+\frac{(7a-9b)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{32(-a+b)^2d(a+b\sec(c+dx))}+\frac{(b+a\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{64(-a+b)d(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*I)*(a^5 - 3*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d
*x])/((a - b)^3*(a + b)^3*d*(a + b*Sec[c + d*x])) - ((I/8)*(-8*a^2 + 21*a*b
 - 15*b^2)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x])/((-a + b
)^3*d*(a + b*Sec[c + d*x])) - ((I/8)*(8*a^2 + 21*a*b + 15*b^2)*ArcTan[Tan[c
 + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x])/((a + b)^3*d*(a + b*Sec[c + d*x
])) + ((7*a + 9*b)*(b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(3
2*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Csc[(c + d*x)/2
]^4*Sec[c + d*x])/(64*(a + b)*d*(a + b*Sec[c + d*x])) + ((-8*a^2 + 21*a*b -
 15*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2]^2]*Sec[c + d*x])/(16*(-a
 + b)^3*d*(a + b*Sec[c + d*x])) + (b^6*(b + a*Cos[c + d*x])*Log[b + a*Cos[c
 + d*x]]*Sec[c + d*x])/(a*(-a^2 + b^2)^3*d*(a + b*Sec[c + d*x])) + ((8*a^2
+ 21*a*b + 15*b^2)*(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2]*Sec[c + d*x
])/(16*(a + b)^3*d*(a + b*Sec[c + d*x])) + ((7*a - 9*b)*(b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Sec[c + d*x])/(32*(-a + b)^2*d*(a + b*Sec[c + d*x])) +
 ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sec[c + d*x])/(64*(-a + b)*d*(a +
 b*Sec[c + d*x]))
```

Maple [A]
time = 0.21, size = 193, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{b^6\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a}-\frac{1}{2(8a+8b)(-1+\cos(dx+c))^2}-\frac{7a+9b}{16(a+b)^2(-1+\cos(dx+c))}+\frac{\left(8a^2+21ba+15b^2\right)\ln(-1+\cos(dx+c))}{16(a+b)^3}-\frac{}{2(8a-8b)(}}{d}$
default	$\frac{-\frac{b^6\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a}-\frac{1}{2(8a+8b)(-1+\cos(dx+c))^2}-\frac{7a+9b}{16(a+b)^2(-1+\cos(dx+c))}+\frac{\left(8a^2+21ba+15b^2\right)\ln(-1+\cos(dx+c))}{16(a+b)^3}-\frac{}{2(8a-8b)(}}{d}$

risch	$\frac{ix}{a}-\frac{ia^2x}{a^3+3b\,a^2+3b^2a+b^3}-\frac{ia^2x}{a^3-3b\,a^2+3b^2a-b^3}+\frac{2ib^6x}{a(a^6-3a^4b^2+3a^2b^4-b^6)}-\frac{15ib^2c}{8d(a^3+3b\,a^2+3b^2a+b^3)}-\frac{}{8(a^3+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^6/(a+b)^3/(a-b)^3/a*ln(b+a*cos(d*x+c))-1/2/(8*a+8*b)/(-1+cos(d*x+c)
)^2-1/16*(7*a+9*b)/(a+b)^2/(-1+cos(d*x+c))+1/16*(8*a^2+21*a*b+15*b^2)/(a+b)
^3*ln(-1+cos(d*x+c))-1/2/(8*a-8*b)/(1+cos(d*x+c))^2-1/16*(-7*a+9*b)/(a-b)^2
/(1+cos(d*x+c))+1/16*(8*a^2-21*a*b+15*b^2)/(a-b)^3*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 289, normalized size = 1.24

$$-\frac{\frac{16\,b^6\log\left(a\cos\left(dx+c\right)+b\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}-\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(\cos\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(\cos\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(5\,a^2b-9\,b^3\right)\cos\left(dx+c\right)^3+6\,a^3-10\,ab^2-4\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^2-\left(3\,a^2b-7\,b^3\right)\cos\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^6*log(a*cos(d*x + c) + b)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 - (8*a^2 - 21*a*b + 15*b^2)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 2*((5*a^2*b - 9*b^3)*cos(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4
*(2*a^3 - 3*a*b^2)*cos(d*x + c)^2 - (3*a^2*b - 7*b^3)*cos(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*cos(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(218) = 436.
time = 4.02, size = 576, normalized size = 2.46

$$\frac{12\,a^6-32\,a^4b^2+20\,a^2b^4+2\left(5\,a^5b-14\,a^3b^3+9\,ab^5\right)\cos\left(dx+c\right)^3-8\left(2\,a^6-5\,a^4b^2+3\,a^2b^4\right)\cos\left(dx+c\right)^2-2\left(3\,a^5b-10\,a^3b^3+7\,ab^5\right)\cos\left(dx+c\right)-16\left(b^6\cos\left(dx+c\right)^4-2\,b^6\cos\left(dx+c\right)^2+b^6\right)\log\left(a\cos\left(dx+c\right)+b\right)+\left(8\,a^6+3\,a^5b-24\,a^4b^2-10\,a^3b^3+24\,a^2b^4+15\,ab^5+\left(8\,a^6+3\,a^5b-24\,a^4b^2-10\,a^3b^3+24\,a^2b^4+15\,ab^5\right)\cos\left(dx+c\right)^4-2\left(8\,a^6+3\,a^5b-24\,a^4b^2-10\,a^3b^3+24\,a^2b^4+15\,ab^5\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(8\,a^6-3\,a^5b-24\,a^4b^2+10\,a^3b^3+24\,a^2b^4-15\,ab^5+\left(8\,a^6-3\,a^5b-24\,a^4b^2+10\,a^3b^3+24\,a^2b^4-15\,ab^5\right)\cos\left(dx+c\right)^4-2\left(8\,a^6-3\,a^5b-24\,a^4b^2+10\,a^3b^3+24\,a^2b^4-15\,ab^5\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{16\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)^4-2\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)^2+\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^6 - 32*a^4*b^2 + 20*a^2*b^4 + 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)
*cos(d*x + c)^3 - 8*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 - 2*(3*a
^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c) - 16*(b^6*cos(d*x + c)^4 - 2*b^6*
cos(d*x + c)^2 + b^6)*log(a*cos(d*x + c) + b) + (8*a^6 + 3*a^5*b - 24*a^4*b
^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 1
0*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^4 - 2*(8*a^6 + 3*a^5*b - 24
*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^2)*log(1/2*cos(
d*x + c) + 1/2) + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 -
```

```
15*a*b^5 + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*
b^5)*cos(d*x + c)^4 - 2*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2
*b^4 - 15*a*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*d*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(218) = 436.
time = 0.79, size = 649, normalized size = 2.77

$$\frac{\frac{4\left(8a^2+21ab+15b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3a^2b+3ab^2+b^3}-\frac{32\left(a^5-3a^3b^2+3ab^4\right)\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\frac{12a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2-2ab+b^2}-\frac{\left(a^2+2ab+b^2+\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{28ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{16b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{126ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{90b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(a^3+3a^2b+3ab^2+b^3)(\cos(dx+c)-1)^2}-\frac{32\left(a^6-3a^4b^2+3a^2b^4-2b^6\right)\log\left(\frac{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2|a|\right|}{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2|a|\right|}\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)|a|}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(8*a^2 + 21*a*b + 15*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 32*(a^5 - 3*a^3*b^2 + 3*a*b^4)*l
og(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))
/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (12*a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 16*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2
 - 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 28*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*b^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 126*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 90*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*(cos(d*x + c) - 1)^2) - 32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - 2*b^6
)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*abs(a)))/d
```

Mupad [B]
time = 2.27, size = 290, normalized size = 1.24

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(8a^2+21ab+15b^2\right)}{d\left(8a^3+24a^2b+24ab^2+8b^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{4d\left(16a-16b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{16b}{(16a-16b)^2}-\frac{3}{16a-16b}\right)}{d}-\frac{\frac{a^2-2ab+b^2}{4(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-3a^3+2a^2b+5ab^2-4b^3\right)}{(a+b)^2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16a^2-32ab+16b^2\right)}-\frac{b^6\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{ad\left(a^2-b^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(21*a*b + 8*a^2 + 15*b^2))/(d*(24*a*b^2 + 24*a^2*b
 + 8*a^3 + 8*b^3)) - tan(c/2 + (d*x)/2)^4/(4*d*(16*a - 16*b)) - log(tan(c/2
 + (d*x)/2)^2 + 1)/(a*d) - (tan(c/2 + (d*x)/2)^2*((16*b)/(16*a - 16*b)^2 -
3/(16*a - 16*b)))/d - ((a^2 - 2*a*b + b^2)/(4*(a + b)) + (tan(c/2 + (d*x)/2
)^2*(5*a*b^2 + 2*a^2*b - 3*a^3 - 4*b^3))/(a + b)^2)/(d*tan(c/2 + (d*x)/2)^4
*(16*a^2 - 32*a*b + 16*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*
tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^3)
```

3.294 $\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=198

$$-\frac{x}{a}+\frac{(8a^4-20a^2b^2+15b^4)\tanh^{-1}(\sin(c+dx))}{8b^5d}-\frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d}-\frac{a(a^2$$

```
[Out] -x/a+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctanh(sin(d*x+c))/b^5/d-2*(a-b)^(5/2)*
(a+b)^(5/2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^5/d-a*(
a^2-2*b^2)*tan(d*x+c)/b^4/d+1/8*(4*a^2-7*b^2)*sec(d*x+c)*tan(d*x+c)/b^3/d-1
/3*a*tan(d*x+c)^3/b^2/d+1/4*sec(d*x+c)*tan(d*x+c)^3/b/d
```

Rubi [A]
time = 0.27, antiderivative size = 271, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3983, 2976, 2738, 214, 3855, 3852, 8, 3853}

$$-\frac{a(a^2-3b^2)\tan(c+dx)}{b^4d}+\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2b^3d}+\frac{(a^4-3a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{b^5d}-\frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d}-\frac{a\tan^3(c+dx)}{3b^2d}-\frac{a\tan(c+dx)}{b^2d}-\frac{x}{a}+\frac{3\tanh^{-1}(\sin(c+dx))}{8bd}+\frac{\tan(c+dx)\sec^3(c+dx)}{4bd}+\frac{3\tan(c+dx)\sec(c+dx)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*b*d) + ((a^2 - 3*b^2)*ArcTanh[Sin[c +
 d*x]])/(2*b^3*d) + ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(b^5*
d) - (2*(a - b)^(5/2)*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a*b^5*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 - 3*b^2)*Tan[c
 + d*x])/(b^4*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*S
ec[c + d*x]*Tan[c + d*x])/(2*b^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*b*d)
 - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)\tan^5(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= \int \left(-\frac{1}{a} - \frac{(a^2-b^2)^3}{ab^5(b+a\cos(c+dx))} + \frac{(a^4-3a^2b^2+3b^4)\sec(c+dx)}{b^5} + \frac{(-a^3+3ab^2}{}\right.$$

$$= -\frac{x}{a} - \frac{a\int \sec^4(c+dx)\,dx}{b^2} + \frac{\int \sec^5(c+dx)\,dx}{b} - \frac{(a(a^2-3b^2))\int \sec^2(c+dx)\,dx}{b^4} +$$

$$= -\frac{x}{a} + \frac{(a^4-3a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{(a^2-3b^2)\sec(c+dx)\tan(c+dx)}{2b^3d}$$

$$= -\frac{x}{a} + \frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(a^4-3a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{b^5d} -$$

$$= -\frac{x}{a} + \frac{3\tanh^{-1}(\sin(c+dx))}{8bd} + \frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(a^4-3a^2b^2+3b}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 907 vs. 2(198) = 396.
time = 6.18, size = 907, normalized size = 4.58

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])))
 - (2*(-a^2 + b^2)^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(
b + a*Cos[c + d*x])*Sec[c + d*x])/(a*b^5*Sqrt[a^2 - b^2]*d*(a + b*Sec[c + d
*x])) + ((-8*a^4 + 20*a^2*b^2 - 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + (
(8*a^4 - 20*a^2*b^2 + 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + ((b + a*Cos
[c + d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^4) + ((12*a^2 - 4*a*b - 27*b^2)*(b + a*Cos[c + d*x])*Sec[c
 + d*x])/(48*b^3*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^2) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(6*b^2*d*(a
+ b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - ((b + a*Cos[c
+ d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(
6*b^2*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-
```

```
12*a^2 + 4*a*b + 27*b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(48*b^3*d*(a +
b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*x)/2]))/(3
*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*
x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(181) = 362$.
time = 0.24, size = 411, normalized size = 2.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^5*(a-b)*(a^5+a^4*b-2*a^3*b^2-2*a^2*b^3+a*b^4+b^5)/a/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/4/b/(tan(1/2
*d*x+1/2*c)+1)^4-1/6*(-2*a-3*b)/b^2/(tan(1/2*d*x+1/2*c)+1)^3-1/8*(4*a^2+4*a
*b-5*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)^2+1/8*(8*a^4-20*a^2*b^2+15*b^4)/b^5*ln
(tan(1/2*d*x+1/2*c)+1)-1/8*(-8*a^3-4*a^2*b+16*a*b^2+7*b^3)/b^4/(tan(1/2*d*x
+1/2*c)+1)-2/a*arctan(tan(1/2*d*x+1/2*c))+1/4/b/(tan(1/2*d*x+1/2*c)-1)^4-1/
6*(-2*a-3*b)/b^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(-4*a^2-4*a*b+5*b^2)/b^3/(tan
(1/2*d*x+1/2*c)-1)^2+1/8/b^5*(-8*a^4+20*a^2*b^2-15*b^4)*ln(tan(1/2*d*x+1/2*
c)-1)-1/8*(-8*a^3-4*a^2*b+16*a*b^2+7*b^3)/b^4/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.37, size = 603, normalized size = 3.05

$$\left[-\frac{48\,b^{5}dx\cos\left(dx+c\right)^{4}-24\,(a^{4}-2\,a^{2}b^{2}+b^{4})\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)^{4}\log\left(\frac{2\,ab\cos(dx+c)-(a^{2}-2\,b^{2})\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right)-3\,(8\,a^{5}-20\,a^{3}b^{2}+15\,ab^{4})\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+3\,(8\,a^{5}-20\,a^{3}b^{2}+15\,ab^{4})\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(8\,a^{2}b^{3}\cos\left(dx+c\right)-6\,ab^{4}+8\,(3\,a^{4}b-7\,a^{2}b^{3})\cos\left(dx+c\right)^{3}-3\,(4\,a^{3}b^{2}-9\,ab^{4})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{48\,ab^{5}d\cos\left(dx+c\right)^{4}},\ -\frac{48\,b^{5}dx\cos\left(dx+c\right)^{4}+48\,(a^{4}-2\,a^{2}b^{2}+b^{4})\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)\cos\left(dx+c\right)^{4}-3\,(8\,a^{5}-20\,a^{3}b^{2}+15\,ab^{4})\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+3\,(8\,a^{5}-20\,a^{3}b^{2}+15\,ab^{4})\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(8\,a^{2}b^{3}\cos\left(dx+c\right)-6\,ab^{4}+8\,(3\,a^{4}b-7\,a^{2}b^{3})\cos\left(dx+c\right)^{3}-3\,(4\,a^{3}b^{2}-9\,ab^{4})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{48\,ab^{5}d\cos\left(dx+c\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(48*b^5*d*x*cos(d*x + c)^4 - 24*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b
^2)*cos(d*x + c)^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^
4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b^3*cos(d*x + c) - 6*a*b^
4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4*a^3*b^2 - 9*a*b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4), -1/48*(48*b^5*d*x*cos(d*x
 + c)^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^4 - 3*(8*
a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2
*b^3*cos(d*x + c) - 6*a*b^4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4
*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(181) = 362.
time = 2.33, size = 746, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*((a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(a
)*abs(-a + b)*abs(b) + (a^5*b + a^4*b^2 - 2*a^3*b^3 - 2*a^2*b^4 + a*b^5 + 2
*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arc
tan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 + sqrt(b^12 + (a*b^5 + b^6)*(a*b^5 - b^
6)))/(a*b^5 - b^6))))/((a*b^4 - b^5)*a^2*b^2 + (a*b^6 - b^7)*abs(a)*abs(b))
 + 24*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 + a*b^6 - 2*b^7 - a^5*abs(a)*abs(b) +
3*a^3*b^2*abs(a)*abs(b) - 3*a*b^4*abs(a)*abs(b) + b^5*abs(a)*abs(b))*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 - sqrt
(b^12 + (a*b^5 + b^6)*(a*b^5 - b^6)))/(a*b^5 - b^6))))/(a^2*b^6 - b^6*abs(a
)*abs(b)) - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 + 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
```

```
)/b^5 - 2*(24*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7
- 48*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 21*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*a^3*
tan(1/2*d*x + 1/2*c)^5 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 176*a*b^2*tan(1/
2*d*x + 1/2*c)^5 + 45*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2
*c)^3 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 176*a*b^2*tan(1/2*d*x + 1/2*c)^3
+ 45*b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c) + 12*a^2*b*ta
n(1/2*d*x + 1/2*c) + 48*a*b^2*tan(1/2*d*x + 1/2*c) - 21*b^3*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^4))/d
```

Mupad [B]
time = 4.13, size = 2500, normalized size = 12.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b/cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(16*a*b^2 + 4*a^2*b - 8*a^3 - 7*b^3))/(4*b^4) - (tan(c
/2 + (d*x)/2)^7*(16*a*b^2 - 4*a^2*b - 8*a^3 + 7*b^3))/(4*b^4) - (tan(c/2 +
(d*x)/2)^3*(176*a*b^2 + 12*a^2*b - 72*a^3 - 45*b^3))/(12*b^4) + (tan(c/2 +
(d*x)/2)^5*(176*a*b^2 - 12*a^2*b - 72*a^3 + 45*b^3))/(12*b^4))/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1)) + (2*atan((((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 56
8*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 +
416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 3
84*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*12
8i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*
a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 90
5*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12
+ 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/
a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*
b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 248
83*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^1
3*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*
a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b +
 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a
^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12
 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10
520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^
3 - 704*a^19*b^2))/b^16)/a - ((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568
*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 4
16*a^9*b^15 - 192*a^10*b^14))/b^16 + (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 38
4*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128
i)/(a*b^16))*1i)/a + (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a
^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905
```

*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a + (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)/a)/((((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)*1i)/a + (((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a + (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^...

3.295 $\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{x}{a}+\frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d}-\frac{a\tan(c+dx)}{b^2d}+$$

```
[Out] x/a+1/2*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/b^3/d-2*(a-b)^(3/2)*(a+b)^(3/2)*a
rctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^3/d-a*tan(d*x+c)/b^2
/d+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.24, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3983, 2972, 3136, 2738, 214, 3855}

$$\frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{x}{a}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*(a - b)^(3/2)*
(a + b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b^3*d
) - (a*Tan[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
```

```
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)\tan^3(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\int \frac{(-2a^2+3b^2-ab\cos(c+dx)-2b^2\cos^2(c+dx))\sec}{b+a\cos(c+dx)}}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{(a^2-b^2)^2\int\frac{1}{b+a\cos(c+dx)}\,dx}{ab^3} - (-2$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} -$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(}{\sqrt{a+}}\right.}{ab^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 287 vs. 2(126) = 252.

time = 2.22, size = 287, normalized size = 2.28

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{4c}{a}+\frac{4dx}{a}+\frac{8(a^2-b^2)^{3/2}\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3}-\frac{4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^3}+\frac{6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b}+\frac{4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^3}-\frac{6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b}+\frac{1}{b\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{b\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{4a\tan(c+dx)}{b^2}\right)}{4d(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((4*c)/a + (4*d*x)/a + (8*(a^2 - b^2)^(3
/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3) - (4*a^2*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 + (6*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]])/b + (4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3
- (6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b + 1/(b*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) - 1/(b*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (4*
a*Tan[c + d*x])/b^2))/(4*d*(a + b*Sec[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(113) = 226.

time = 0.20, size = 234, normalized size = 1.86

method	result
derivativedivides	$\frac{-\frac{2(a-b)\left(a^3+b\,a^2-b^2a-b^3\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3a\sqrt{(a+b)(a-b)}}-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(2a^2-3b^2)}{}}{d}$
default	$\frac{-\frac{2(a-b)\left(a^3+b\,a^2-b^2a-b^3\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3a\sqrt{(a+b)(a-b)}}-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(2a^2-3b^2)}{}}{d}$
risch	$\frac{x}{a}-\frac{i\left(b\,e^{3i(dx+c)}+2a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}+2a\right)}{d\,b^2\left(e^{2i(dx+c)}+1\right)^2}+\frac{\sqrt{a^2-b^2}\,a\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{d\,b^3}-\frac{\sqrt{a^2-b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^3*(a-b)*(a^3+a^2*b-a*b^2-b^3)/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/2/b/(tan(1/2*d*x+1/2*c)+1)^2-1/2
*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(2*a^2-3*b^2)/b^3*ln(tan(1/2*d*x+1
/2*c)+1)+2/a*arctan(tan(1/2*d*x+1/2*c))+1/2/b/(tan(1/2*d*x+1/2*c)-1)^2-1/2*
(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)-1)+1/2/b^3*(-2*a^2+3*b^2)*ln(tan(1/2*d*x+1
/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.41, size = 444, normalized size = 3.52

$$\left[\frac{4\,b^{3}dx\cos\left(dx+c\right)^{2}-2\,\left(a^{2}-b^{2}\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\log\left(\frac{2\,ab\cos(dx+c)-(a^{2}-2\,b^{2})\cos(dx+c)^{2}+2\,\sqrt{a^{2}-b^{2}}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right)+\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(2\,a^{2}b\cos\left(dx+c\right)-ab^{2}\right)\sin\left(dx+c\right)}{4\,ab^{3}d\cos\left(dx+c\right)^{2}},\frac{4\,b^{3}dx\cos\left(dx+c\right)^{2}-4\,\left(a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)\cos\left(dx+c\right)^{2}+\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(2\,a^{2}b\cos\left(dx+c\right)-ab^{2}\right)\sin\left(dx+c\right)}{4\,ab^{3}d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^3*d*x*cos(d*x + c)^2 - 2*(a^2 - b^2)^(3/2)*cos(d*x + c)^2*log((2*
a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(
d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d
*x + c) + b^2)) + (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) -
(2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*
x + c) - a*b^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^2), 1/4*(4*b^3*d*x*cos(
d*x + c)^2 - 4*(a^2 - b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^3 - 3*a*b^
2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*
log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*x + c) - a*b^2)*sin(d*x + c))/(a*
b^3*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(113) = 226.

time = 1.06, size = 476, normalized size = 3.78

$$-\frac{\frac{2\left(\left(a^{2}+ab-b^{2}\right)\sqrt{-a^{2}+b^{2}}\,|a||-a+b||b|+\left(a^{3}b+a^{2}b^{2}-ab^{3}-2\,b^{4}\right)\sqrt{-a^{2}+b^{2}}\,|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b^{4}+\sqrt{b^{8}+(ab^{3}+b^{4})(ab^{3}-b^{4})}}{ab^{3}-b^{4}}}}\right)\right)}{(ab^{2}-b^{3})a^{2}b^{2}+(ab^{4}-b^{5})|a||b|}+\frac{2\left(a^{4}b-2\,a^{2}b^{3}-ab^{4}+2\,b^{5}-a^{3}|a||b|+2\,ab^{2}|a||b|-b^{3}|a||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b^{4}-\sqrt{b^{8}+(ab^{3}+b^{4})(ab^{3}-b^{4})}}{ab^{3}-b^{4}}}}\right)\right)}{a^{2}b^{4}-b^{4}|a||b|}-\frac{(2\,a^{2}-3\,b^{2})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^{3}}+\frac{(2\,a^{2}-3\,b^{2})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^{3}}-\frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*((a^2 + a*b - b^2)*sqrt(-a^2 + b^2)*abs(a)*abs(-a + b)*abs(b) + (a^
3*b + a^2*b^2 - a*b^3 - 2*b^4)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*
(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 + sqrt(b^8 + (
a*b^3 + b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/((a*b^2 - b^3)*a^2*b^2 + (a*b
^4 - b^5)*abs(a)*abs(b)) + 2*(a^4*b - 2*a^2*b^3 - a*b^4 + 2*b^5 - a^3*abs(a
)*abs(b) + 2*a*b^2*abs(a)*abs(b) - b^3*abs(a)*abs(b))*(pi*floor(1/2*(d*x +
c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 - sqrt(b^8 + (a*b^3 +
 b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/(a^2*b^4 - b^4*abs(a)*abs(b)) - (2*a
^2 - 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a^2 - 3*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/
2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

Mupad [B]
time = 3.29, size = 2500, normalized size = 19.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*3i)/
(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*
sin(c/2 + (d*x)/2)^2)) + (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*
sin(c/2 + (d*x)/2)^4*3i)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4
- 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)*sin(c
/2 + (d*x)/2)^3)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)^3*sin(c/2 + (
d*x)/2))/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^
12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(
c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)
/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^
8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(
c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*
a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5
*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2))*c
```

os(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*sin(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*2i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(c/2 + (d*x)/2)^4*2i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*6i)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*a*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3)/(b^2*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (2*a*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))/(b^2*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (4*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*4i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (atan(((8*a^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 20*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 22*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 14

```
*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 36
*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 24
*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 47
*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 79*
a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 49*a
^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^...
```

3.296 $\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

```
[Out] -x/a+arctanh(sin(d*x+c))/b/d-2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a/b/d
```

Rubi [A]

time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3979, 4136, 3855, 4004, 3916, 2738, 214}

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}-\frac{x}{a}+\frac{\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(b*d) - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4136

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \int \frac{-1+\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= \frac{\int \sec(c+dx)\,dx}{b} + \frac{\int \frac{-b-a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+\right.\right.}{bd} \\
&= -\frac{x}{a} + \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 115, normalized size = 1.51

$$-\frac{bc+bdx-2\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*c + b*d*x - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]] + a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - a*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [A]
time = 0.13, size = 108, normalized size = 1.42

method	result
derivativedivides	$\frac{-\frac{2(a+b)(a-b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{ba\sqrt{(a+b)(a-b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
default	$\frac{-\frac{2(a+b)(a-b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{ba\sqrt{(a+b)(a-b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
risch	$-\frac{x}{a}-\frac{\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}+\frac{i\sqrt{a^2-b^2}+b}{a}\right)}{dba}+\frac{\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{dba}-\frac{\ln(e^{i(d}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b*(a+b)*(a-b)/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))+1/b*ln(tan(1/2*d*x+1/2*c)+1)-2/a*arctan(tan(1/2*d*x+
1/2*c))-1/b*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.50, size = 253, normalized size = 3.33

$$\left[-\frac{2\,bdx-a\log\left(\sin\left(dx+c\right)+1\right)+a\log\left(-\sin\left(dx+c\right)+1\right)-\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,abd},\ -\frac{2\,bdx-a\log\left(\sin\left(dx+c\right)+1\right)+a\log\left(-\sin\left(dx+c\right)+1\right)+2\,\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)}{2\,abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*d*x - a*log(sin(d*x + c) + 1) + a*log(-sin(d*x + c) + 1) - sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a*b*d), -1/2*(2*b*d*x - a*log(sin(d*x +
 c) + 1) + a*log(-sin(d*x + c) + 1) + 2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/(a*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(67) = 134.
time = 0.62, size = 140, normalized size = 1.84

$$-\frac{\frac{dx+c}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(a^2-b^2)}{\sqrt{-a^2+b^2}\,ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + ar
ctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*
(a^2 - b^2)/(sqrt(-a^2 + b^2)*a*b))/d
```

Mupad [B]
time = 1.63, size = 121, normalized size = 1.59

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}-\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a+b)}\right)\,\sqrt{a^2-b^2}}{a\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (2*atan(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) - (2*atanh((sin(c/2 + (d*x)/2)*(a^2 - b
^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a*b*d)
```

3.297 $\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{x}{a}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a^2-b^2}\,\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{a\left(a^2-b^2\right)^{3/2}d}-\frac{a\cot(c+dx)}{\left(a^2-b^2\right)d}+\frac{b\csc(c+dx)}{\left(a^2-b^2\right)d}$$

```
[Out] -x/a-2*b^3*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(3
/2)/d-a*cot(d*x+c)/(a^2-b^2)/d+b*csc(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3983, 2981, 2686, 8, 3554, 2814, 2738, 214}

$$-\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}+\frac{b^2x}{a\left(a^2-b^2\right)}-\frac{ax}{a^2-b^2}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (b^2*x)/(a*(a^2 - b^2)) - (2*b^3*ArcTanh[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d) - (a*Co
t[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= \frac{a\int \cot^2(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$
$$= \frac{b^2x}{a(a^2-b^2)} - \frac{a\cot(c+dx)}{(a^2-b^2)d} - \frac{a\int 1\,dx}{a^2-b^2} - \frac{b^3\int \frac{1}{b+a\cos(c+dx)}\,dx}{a(a^2-b^2)} + \frac{b\text{Subst}(\int 1\,dx, x, \text{c}}{(a^2-b^2)}$$
$$= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a(a^2-b^2)} - \frac{a\cot(c+dx)}{(a^2-b^2)d} + \frac{b\csc(c+dx)}{(a^2-b^2)d} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+b+(-a+}\right.}{a(a^}$$
$$= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a(a^2-b^2)} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}d} - \frac{a\cot(c+dx)}{(a^2-b^2)d} + \frac{b\,\text{c}}{(}$$

Mathematica [A]

time = 0.45, size = 147, normalized size = 1.39

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2b^3\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\sin(c+dx)+\sqrt{a^2-b^2}\left(-ab+a^2\cos(c+dx)+(a^2-b^2)(c+dx)\sin(c+dx)\right)\right)}{2a(a-b)(a+b)\sqrt{a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-2*b^3*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x] + Sqrt[a^2 - b^2]*(-(a*b) + a^2*Cos[
c + d*x] + (a^2 - b^2)*(c + d*x)*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt[a^
2 - b^2]*d)
```

Maple [A]

time = 0.15, size = 115, normalized size = 1.08

method	result
derivativedivides	$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2b^3\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)(a+b)a\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$$
default	$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2b^3\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)(a+b)a\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$$

risch	$-\frac{x}{a}-\frac{2i(-b\,\mathrm{e}^{i(dx+c)}+a)}{d(a^2-b^2)(\mathrm{e}^{2i(dx+c)}-1)}-\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2}}{a\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a-b)*tan(1/2*d*x+1/2*c)-2/(a-b)/(a+b)*b^3/a/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+
1/2*c)-2/a*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.11, size = 362, normalized size = 3.42

$$\left[-\frac{\sqrt{a^2-b^2}\,b^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2\,a^3b+2\,ab^3+2\,(a^4-2\,a^2b^2+b^4)dx\sin(dx+c)+2\,(a^4-a^2b^2)\cos(dx+c)}{2\,(a^5-2\,a^3b^2+ab^4)d\sin(dx+c)},-\frac{\sqrt{-a^2+b^2}\,b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-a^3b+ab^3+(a^4-2\,a^2b^2+b^4)dx\sin(dx+c)+(a^4-a^2b^2)\cos(dx+c)}{(a^5-2\,a^3b^2+ab^4)d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^3*b +
2*a*b^3 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c) + 2*(a^4 - a^2*b^2)*co
s(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c)), -(sqrt(-a^2 + b^2)*
b^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)
))*sin(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c)
+ (a^4 - a^2*b^2)*cos(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(101) = 202$.
time = 0.52, size = 582, normalized size = 5.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5 - a^2*abs(-a^3
 + a*b^2) + a*b*abs(-a^3 + a*b^2) + b^2*abs(-a^3 + a*b^2))*(pi*floor(1/2*(d
*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 + sqrt((
a^3 + a^2*b - a*b^2 - b^3)*(a^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/
(a^3 - a^2*b - a*b^2 + b^3))))/(a^2*b*abs(-a^3 + a*b^2) - b^3*abs(-a^3 + a*
b^2) + (a^3 - a*b^2)^2) + 2*((a^2 - a*b - b^2)*sqrt(-a^2 + b^2)*abs(-a^3 +
a*b^2)*abs(-a + b) + (a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5)*
sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(ta
n(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 - sqrt((a^3 + a^2*b - a*b^2 - b^3)*(a
^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/(a^3 - a^2*b - a*b^2 + b^3)))
)/((a^3 - a*b^2)^2*(a^2 - 2*a*b + b^2) - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5
)*abs(-a^3 + a*b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d*
x + 1/2*c)))/d
```

Mupad [B]
time = 3.94, size = 1002, normalized size = 9.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (b^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^6*atan(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2))*2i - a^2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*6i + a^4*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*6i + b^3*
atanh((2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) -
 a^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 2*b^13
*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 9*a^2*b^11*
sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 3*a^3*b^10*s
in(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 18*a^4*b^9*si
```

```
n(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 12*a^5*b^8*sin
(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 21*a^6*b^7*sin(
c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 19*a^7*b^6*sin(c
/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 15*a^8*b^5*sin(c/
2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 15*a^9*b^4*sin(c/2
 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 6*a^10*b^3*sin(c/2
+ (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 6*a^11*b^2*sin(c/2 +
 (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a^12*b*sin(c/2 + (d*x
)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^16 -
 3*a^2*b^14 + 18*a^4*b^12 - 46*a^6*b^10 + 65*a^8*b^8 - 55*a^10*b^6 + 28*a^1
2*b^4 - 8*a^14*b^2)))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)*2i)/(a^7*d*
1i + a^3*b^4*d*3i - a^5*b^2*d*3i - a*b^6*d*1i) - (a^6*cos(c + d*x)*1i - a*b
^5*1i - a^5*b*1i + a^3*b^3*2i + a^2*b^4*cos(c + d*x)*1i - a^4*b^2*cos(c + d
*x)*2i)/(a^7*d*sin(c + d*x)*1i - a*b^6*d*sin(c + d*x)*1i + a^3*b^4*d*sin(c
+ d*x)*3i - a^5*b^2*d*sin(c + d*x)*3i)
```

3.298 $\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{x}{a}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{a\left(a^2-b^2\right)^{5/2}d}+\frac{a(a^2-2b^2)\cot(c+dx)}{\left(a^2-b^2\right)^2 d}-\frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d}-\frac{b(a^2-2b^2)\csc(c+dx)}{\left(a^2-b^2\right)^2 d}+$$

```
[Out] x/a-2*b^5*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(5/
2)/d+a*(a^2-2*b^2)*cot(d*x+c)/(a^2-b^2)^2/d-1/3*a*cot(d*x+c)^3/(a^2-b^2)/d-
b*(a^2-2*b^2)*csc(d*x+c)/(a^2-b^2)^2/d+1/3*b*csc(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 256, normalized size of antiderivative = 1.45, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3983, 2981, 2686, 3554, 8, 2814, 2738, 214}

$$-\frac{a\cot^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{ab^2\cot(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}+\frac{b^4x}{a\left(a^2-b^2\right)^2}+\frac{b^3\csc(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5
/2)*(a + b)^(5/2)*d) - (a*b^2*Cot[c + d*x])/((a^2 - b^2)^2*d) + (a*Cot[c +
d*x])/((a^2 - b^2)*d) - (a*Cot[c + d*x]^3)/(3*(a^2 - b^2)*d) + (b^3*Csc[c +
 d*x])/((a^2 - b^2)^2*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*
x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\cot^4(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= \frac{a\int \cot^4(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot^3(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{a\cot^3(c+dx)}{3(a^2-b^2)d} + \frac{(ab^2)\int \cot^2(c+dx)\,dx}{(a^2-b^2)^2} - \frac{b^3\int \cot(c+dx)\csc(c+dx)\,dx}{(a^2-b^2)^2} + \frac{b^4}{}$$

$$= \frac{b^4x}{a(a^2-b^2)^2} - \frac{ab^2\cot(c+dx)}{(a^2-b^2)^2 d} + \frac{a\cot(c+dx)}{(a^2-b^2)d} - \frac{a\cot^3(c+dx)}{3(a^2-b^2)d} - \frac{b\csc(c+dx)}{(a^2-b^2)d} +$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} - \frac{ab^2\cot(c+dx)}{(a^2-b^2)^2 d} + \frac{a\cot(c+dx)}{(a^2-b^2)d} - \frac{a\cot^3(}{3(a^2}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{ab^2\,c}{(a^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 416 vs. 2(177) = 354.
time = 6.20, size = 416, normalized size = 2.35

$$\frac{(c+dx)(b+a\cos(c+dx))\sec(c+dx)}{ad(a+b\sec(c+dx))} + \frac{2b^5\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)}{a\sqrt{a^2-b^2}\,(-a^2+b^2)^2 d(a+b\sec(c+dx))} + \frac{\left(8a\cos\left(\frac{1}{2}(c+dx)\right)+11b\cos\left(\frac{1}{2}(c+dx)\right)\right)(b+a\cos(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{12(a+b)^2d(a+b\sec(c+dx))} - \frac{(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{24(a+b)d(a+b\sec(c+dx))} + \frac{(b+a\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-8a\sin\left(\frac{1}{2}(c+dx)\right)+11b\sin\left(\frac{1}{2}(c+dx)\right)\right)}{12(-a+b)^2d(a+b\sec(c+dx))} - \frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{24(-a+b)d(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])) +
(2*b^5*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c +
d*x])*Sec[c + d*x])/(a*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d*(a + b*Sec[c + d*x]
)) + ((8*a*Cos[(c + d*x)/2] + 11*b*Cos[(c + d*x)/2])*(b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]*Sec[c + d*x])/(12*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b +
 a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(24*(a +
 b)*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]*Sec[c
+ d*x]*(-8*a*Sin[(c + d*x)/2] + 11*b*Sin[(c + d*x)/2]))/(12*(-a + b)^2*d*(a
 + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]
*Tan[(c + d*x)/2])/(24*(-a + b)*d*(a + b*Sec[c + d*x]))
```

Maple [A]
time = 0.20, size = 184, normalized size = 1.04

method	result

derivativedivides	$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+7b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)^2(a-b)^2a\sqrt{(a+b)(a-b)}}-\frac{}{24(a+b)\tan}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+7b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)^2(a-b)^2a\sqrt{(a+b)(a-b)}}-\frac{}{24(a+b)\tan}}{d}$
risch	$\frac{x}{a}-\frac{2i\left(3a^2b\,e^{5i(dx+c)}-6b^3e^{5i(dx+c)}-6a^3e^{4i(dx+c)}+9a\,b^2e^{4i(dx+c)}-2b\,a^2e^{3i(dx+c)}+8b^3e^{3i(dx+c)}+6a^3e^{2i(dx+c)}-12b\right.}{3\left(a^4-2b^2a^2+b^4\right)\left(e^{2i(dx+c)}-1\right)^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a-b)^2*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*b*tan(1/2*d*x+1/2*c)^3-5*a
*tan(1/2*d*x+1/2*c)+7*b*tan(1/2*d*x+1/2*c))-2/(a+b)^2/(a-b)^2*b^5/a/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/24/(a+
b)/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^2*(-5*a-7*b)/tan(1/2*d*x+1/2*c)+2/a*arcta
n(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(168) = 336$.
time = 6.96, size = 742, normalized size = 4.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^5*b - 14*a^3*b^3 + 10*a*b^5 + 2*(4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*
cos(d*x + c)^3 + 3*(b^5*cos(d*x + c)^2 - b^5)*sqrt(a^2 - b^2)*log((2*a*b*co
```

```
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x +
c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + b^2))*sin(d*x + c) - 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 - 6
*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x)*sin
(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2 - (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c)), 1/3*(2*a^5*b - 7*a^3*b^3
 + 5*a*b^5 + (4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*cos(d*x + c)^3 - 3*(b^5*cos(d
*x + c)^2 - b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^5*b - 3*a^3*b^3 + 2*a*
b^5)*cos(d*x + c)^2 - 3*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 3*((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*d*x)*sin(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d
*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(168) = 336$.
time = 0.61, size = 1073, normalized size = 6.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*((a^4 - a^3*b - 2*a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(a
^5 - 2*a^3*b^2 + a*b^4)*abs(-a + b) - (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3
+ 6*a^5*b^4 - 7*a^4*b^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9)*sqrt(-a^2
+ b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x
+ 1/2*c)/sqrt(-(a^4*b - 2*a^2*b^3 + b^5 + sqrt((a^5 + a^4*b - 2*a^3*b^2 - 2
*a^2*b^3 + a*b^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)
 + (a^4*b - 2*a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5))))/((a^5 - 2*a^3*b^2 + a*b^4)^2*(a^2 - 2*a*b + b^2) + (a^6*b - 2*
a^5*b^2 - a^4*b^3 + 4*a^3*b^4 - a^2*b^5 - 2*a*b^6 + b^7)*abs(a^5 - 2*a^3*b^
2 + a*b^4)) + 24*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 7*a^4*b
^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9 + a^4*abs(a^5 - 2*a^3*b^2 + a*b^
```

```
4) - a^3*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^2*abs(a^5 - 2*a^3*b^2 + a
*b^4) + 2*a*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4) + b^4*abs(a^5 - 2*a^3*b^2 + a*
b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(
-(a^4*b - 2*a^2*b^3 + b^5 - sqrt((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b
^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5) + (a^4*b - 2*
a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5))))/(
a^4*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4)
 + b^5*abs(a^5 - 2*a^3*b^2 + a*b^4) - (a^5 - 2*a^3*b^2 + a*b^4)^2) - (a^2*t
an(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/
2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) + 36*a*b*tan(1/2*d*x + 1/2*c) - 21*b^2
*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 21*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan(
1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 11.11, size = 2500, normalized size = 14.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (a^10*((cos(3*c + 3*d*x)*4i)/3 - sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*6i + atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3
*d*x)*2i) + a*((b^9*8i)/3 - b^9*cos(2*c + 2*d*x)*4i) - a^7*((b^3*14i)/3 - b
^3*cos(2*c + 2*d*x)*10i) + a^5*(b^5*10i - b^5*cos(2*c + 2*d*x)*18i) - a^3*(
(b^7*26i)/3 - b^7*cos(2*c + 2*d*x)*14i) + a^9*((b*2i)/3 - b*cos(2*c + 2*d*x
)*2i) + a^8*(b^2*cos(c + d*x)*1i - (b^2*cos(3*c + 3*d*x)*19i)/3 + b^2*sin(c
 + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*30i - b^2*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*10i) - a^2*(b^8*cos(c + d*x
)*1i - (b^8*cos(3*c + 3*d*x)*7i)/3 + b^8*sin(c + d*x)*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*30i - b^8*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
)*sin(3*c + 3*d*x)*10i) - a^6*(b^4*cos(c + d*x)*3i - b^4*cos(3*c + 3*d*x)*1
1i + b^4*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^4
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + a^4*(b
^6*cos(c + d*x)*3i - (b^6*cos(3*c + 3*d*x)*25i)/3 + b^6*sin(c + d*x)*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^6*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + b^10*sin(c + d*x)*atan(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*6i - b^10*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*sin(3*c + 3*d*x)*2i + b^5*atanh((2*b^11*sin(c/2 + (d*x)/2)*(a^10 - b
^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(3/2) - a^21*sin(c/2
 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)
^(1/2) + 2*b^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
10*a^6*b^4 - 5*a^8*b^2)^(1/2) + a^20*b*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*
a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 15*a^2*b^19*sin(c/2
```

+ (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 5*a^3*b^18*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 55*a^4*b^17*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 35*a^5*b^16*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 130*a^6*b^15*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 110*a^7*b^14*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 215*a^8*b^13*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 205*a^9*b^12*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 253*a^10*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 251*a^11*b^10*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 210*a^12*b^9*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 210*a^13*b^8*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 120*a^14*b^7*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 120*a^15*b^6*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 45*a^16*b^5*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 45*a^17*b^4*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 10*a^18*b^3*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 10*a^19*b^2*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^26 + 5*a^2*b^24 - 50*a^4*b^22 + 230*a^6*b^20 - 645*a^8*b^18 + 1231*a^10*b^16 - 1688*a^12*b^14 + 1708*a^14*b^12 - 1286*a^16*b^10 + 715*a^18*b^8 - 286*a^20*b^6 + 78*a^22*b^4 - 13*a^24*b^2)))*sin(3*c + 3*d*x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)*2i - b^5*atanh((2*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(3/2) - a^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*b^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + a^20*b*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 15*a^2*b^19*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 5*a^3*b^18*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 55*a^4*b^17*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 35*a^5*b^16*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 130*a^6*b^15*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 110*a^7*b^14*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 215*a^8*b^13*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - ...

3.299 $\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=255

$$-\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)^3\,(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d}-\frac{2a(3a^4-8a^2b^2+6b^4)\sec(c+dx)}{b^7d}+\frac{(5a^4-}{}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+(a^2-b^2)^3*(7*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^8/d-
2*a*(3*a^4-8*a^2*b^2+6*b^4)*sec(d*x+c)/b^7/d+1/2*(5*a^4-12*a^2*b^2+6*b^4)*s
ec(d*x+c)^2/b^6/d-4/3*a*(a^2-2*b^2)*sec(d*x+c)^3/b^5/d+1/4*(3*a^2-4*b^2)*se
c(d*x+c)^4/b^4/d-2/5*a*sec(d*x+c)^5/b^3/d+1/6*sec(d*x+c)^6/b^2/d+(a^2-b^2)^
4/a/b^8/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{(a^2-b^2)^4}{ab^8d(a+b\sec(c+dx))}+\frac{(a^2-b^2)^3\,(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d}-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d}-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a(3a^4-8a^2b^2+6b^4)\sec(c+dx)}{b^7d}+\frac{(5a^4-12a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d}-\frac{2a\sec^5(c+dx)}{5b^3d}+\frac{\sec^6(c+dx)}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Sec[c
 + d*x]])/(a^2*b^8*d) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x])/(b^7
*d) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 -
 2*b^2)*Sec[c + d*x]^3)/(3*b^5*d) + ((3*a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4
*d) - (2*a*Sec[c + d*x]^5)/(5*b^3*d) + Sec[c + d*x]^6/(6*b^2*d) + (a^2 - b^
2)^4/(a*b^8*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^4}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(-2a(3a^4-8a^2b^2+6b^4)+\frac{b^8}{a^2x}+(5a^4-12a^2b^2+6b^4)\,x-4a(a^2-2b\right.\right.}{}$$

$$= -\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)^3\,(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d}-\frac{2a(3a^4-8a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 528 vs. 2(255) = 510.

time = 6.30, size = 528, normalized size = 2.07

$$-\frac{(-a+b)^4(a+b)^4(b+a\cos(c+dx))\sec^2(c+dx)}{a^2b^7d(a+b\sec(c+dx))^2}+\frac{(-7a^6+20a^4b^2-18a^2b^4+4b^6)\,(b+a\cos(c+dx))^2\log(\cos(c+dx))\sec^2(c+dx)}{b^8d(a+b\sec(c+dx))^2}+\frac{(7a^8-20a^6b^2+18a^4b^4-4a^2b^6-b^8)\,(b+a\cos(c+dx))^2\log(b+a\cos(c+dx))\sec^2(c+dx)}{a^2b^8d(a+b\sec(c+dx))^2}-\frac{2a(3a^4-8a^2b^2+6b^4)\,(b+a\cos(c+dx))^2\sec^3(c+dx)}{b^7d(a+b\sec(c+dx))^2}+\frac{(5a^4-12a^2b^2+6b^4)\,(b+a\cos(c+dx))^2\sec^4(c+dx)}{2b^6d(a+b\sec(c+dx))^2}+\frac{4a(-a^2+2b^2)\,(b+a\cos(c+dx))^2\sec^5(c+dx)}{3b^5d(a+b\sec(c+dx))^2}+\frac{(3a^2-4b^2)\,(b+a\cos(c+dx))^2\sec^6(c+dx)}{4b^4d(a+b\sec(c+dx))^2}-\frac{2a(b+a\cos(c+dx))^2\sec^7(c+dx)}{5b^3d(a+b\sec(c+dx))^2}+\frac{(b+a\cos(c+dx))^2\sec^8(c+dx)}{6b^2d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((-a + b)^4*(a + b)^4*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(a^2*b^7*d*(a
+ b*Sec[c + d*x])^2)) + ((-7*a^6 + 20*a^4*b^2 - 18*a^2*b^4 + 4*b^6)*(b + a*
Cos[c + d*x])^2*Log[Cos[c + d*x]]*Sec[c + d*x]^2)/(b^8*d*(a + b*Sec[c + d*x
])^2) + ((7*a^8 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*(b + a*Cos[c +
 d*x])^2*Log[b + a*Cos[c + d*x]]*Sec[c + d*x]^2)/(a^2*b^8*d*(a + b*Sec[c +
d*x])^2) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*(b + a*Cos[c + d*x])^2*Sec[c +
d*x]^3)/(b^7*d*(a + b*Sec[c + d*x])^2) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*(b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^4)/(2*b^6*d*(a + b*Sec[c + d*x])^2) + (4*a*
(-a^2 + 2*b^2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^5)/(3*b^5*d*(a + b*Sec[c
 + d*x])^2) + ((3*a^2 - 4*b^2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^6)/(4*b^
4*d*(a + b*Sec[c + d*x])^2) - (2*a*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^7)/(
5*b^3*d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^8)/(
6*b^2*d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 287, normalized size = 1.13

method	result
derivativedivides	$\frac{-\frac{a^8-4a^6b^2+6a^4b^4-4b^6a^2+b^8}{a^2b^7(b+a\cos(dx+c))}+\frac{\left(7a^8-20a^6b^2+18a^4b^4-4b^6a^2-b^8\right)\ln(b+a\cos(dx+c))}{b^8a^2}-\frac{-3a^2+4b^2}{4b^4\cos(dx+c)^4}-\frac{-5a^4+12b^2a^2-6b^4}{2b^6\cos(dx+c)^2}+\frac{(-}{}}{d}$
default	$\frac{-\frac{a^8-4a^6b^2+6a^4b^4-4b^6a^2+b^8}{a^2b^7(b+a\cos(dx+c))}+\frac{\left(7a^8-20a^6b^2+18a^4b^4-4b^6a^2-b^8\right)\ln(b+a\cos(dx+c))}{b^8a^2}-\frac{-3a^2+4b^2}{4b^4\cos(dx+c)^4}-\frac{-5a^4+12b^2a^2-6b^4}{2b^6\cos(dx+c)^2}+\frac{(-}{}}{d}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)/a^2/b^7/(b+a*cos(d*x+c))+(7*a
^8-20*a^6*b^2+18*a^4*b^4-4*a^2*b^6-b^8)/b^8/a^2*ln(b+a*cos(d*x+c))-1/4*(-3*
a^2+4*b^2)/b^4/cos(d*x+c)^4-1/2*(-5*a^4+12*a^2*b^2-6*b^4)/b^6/cos(d*x+c)^2+
(-7*a^6+20*a^4*b^2-18*a^2*b^4+4*b^6)/b^8*ln(cos(d*x+c))+1/6/b^2/cos(d*x+c)^
6-2/5/b^3*a/cos(d*x+c)^5-4/3*a*(a^2-2*b^2)/b^5/cos(d*x+c)^3-2*a*(3*a^4-8*a^
2*b^2+6*b^4)/b^7/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 321, normalized size = 1.26

$$-\frac{\frac{14\,a^3b^5\cos(dx+c)-10\,a^2b^6+60\,(7\,a^8-20\,a^6b^2+18\,a^4b^4-4\,a^2b^6+b^8)\cos(dx+c)^6+30\,(7\,a^7b-20\,a^5b^3+18\,a^3b^5)\cos(dx+c)^5-10\,(7\,a^6b^2-20\,a^4b^4+18\,a^2b^6)\cos(dx+c)^4+5\,(7\,a^5b^3-20\,a^3b^5)\cos(dx+c)^3-3\,(7\,a^4b^4-20\,a^2b^6)\cos(dx+c)^2}{a^3b^7\cos(dx+c)^7+a^2b^8\cos(dx+c)^6}+\frac{60\,(7\,a^6-20\,a^4b^2+18\,a^2b^4-4\,b^6)\log(\cos(dx+c))}{b^8}-\frac{60\,(7\,a^8-20\,a^6b^2+18\,a^4b^4-4\,a^2b^6-b^8)\log(a\cos(dx+c)+b)}{a^2b^8}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*((14*a^3*b^5*cos(d*x + c) - 10*a^2*b^6 + 60*(7*a^8 - 20*a^6*b^2 + 18*
a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^6 + 30*(7*a^7*b - 20*a^5*b^3 + 18*a
^3*b^5)*cos(d*x + c)^5 - 10*(7*a^6*b^2 - 20*a^4*b^4 + 18*a^2*b^6)*cos(d*x +
 c)^4 + 5*(7*a^5*b^3 - 20*a^3*b^5)*cos(d*x + c)^3 - 3*(7*a^4*b^4 - 20*a^2*b
^6)*cos(d*x + c)^2)/(a^3*b^7*cos(d*x + c)^7 + a^2*b^8*cos(d*x + c)^6) + 60*
(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(cos(d*x + c))/b^8 - 60*(7*a^8
 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^2*
b^8))/d
```

Fricas [A]
time = 2.98, size = 423, normalized size = 1.66

$$-\frac{14\,a^3b^6\cos(dx+c)-10\,a^2b^7+60\,(7\,a^8b-20\,a^6b^3+18\,a^4b^5-4\,a^2b^7+b^9)\cos(dx+c)^6+30\,(7\,a^7b^2-20\,a^5b^4+18\,a^3b^6)\cos(dx+c)^5-10\,(7\,a^6b^3-20\,a^4b^5+18\,a^2b^7)\cos(dx+c)^4+5\,(7\,a^5b^4-20\,a^3b^6)\cos(dx+c)^3-3\,(7\,a^4b^5-20\,a^2b^7)\cos(dx+c)^2-60\,((7\,a^9-20\,a^7b^2+18\,a^5b^4-4\,a^3b^6-ab^8)\cos(dx+c)^7+(7\,a^8b-20\,a^6b^3+18\,a^4b^5-4\,a^2b^7-b^9)\cos(dx+c)^6)\log(a\cos(dx+c)+b)+60\,((7\,a^9-20\,a^7b^2+18\,a^5b^4-4\,a^3b^6)\cos(dx+c)^7+(7\,a^8b-20\,a^6b^3+18\,a^4b^5-4\,a^2b^7)\cos(dx+c)^6)\log(-\cos(dx+c))}{60\,(a^3b^8d\cos(dx+c)^7+a^2b^9d\cos(dx+c)^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(14*a^3*b^6*cos(d*x + c) - 10*a^2*b^7 + 60*(7*a^8*b - 20*a^6*b^3 + 18
*a^4*b^5 - 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 30*(7*a^7*b^2 - 20*a^5*b^4 + 1
8*a^3*b^6)*cos(d*x + c)^5 - 10*(7*a^6*b^3 - 20*a^4*b^5 + 18*a^2*b^7)*cos(d*
x + c)^4 + 5*(7*a^5*b^4 - 20*a^3*b^6)*cos(d*x + c)^3 - 3*(7*a^4*b^5 - 20*a^
2*b^7)*cos(d*x + c)^2 - 60*((7*a^9 - 20*a^7*b^2 + 18*a^5*b^4 - 4*a^3*b^6 -
a*b^8)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4*a^2*b^7 - b^
9)*cos(d*x + c)^6)*log(a*cos(d*x + c) + b) + 60*((7*a^9 - 20*a^7*b^2 + 18*a
^5*b^4 - 4*a^3*b^6)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4
```

```
*a^2*b^7)*cos(d*x + c)^6)*log(-cos(d*x + c)))/(a^3*b^8*d*cos(d*x + c)^7 + a
^2*b^9*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**9/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. 2(245) = 490.
time = 6.04, size = 1696, normalized size = 6.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(7*a^9 - 7*a^8*b - 20*a^7*b^2 + 20*a^6*b^3 + 18*a^5*b^4 - 18*a^4*b
^5 - 4*a^3*b^6 + 4*a^2*b^7 - a*b^8 + b^9)*log(abs(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^8
 - a^2*b^9) + 60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 -
 60*(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(abs(-(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1))/b^8 - 60*(7*a^9 + 9*a^8*b - 18*a^7*b^2 - 26*a^6*b^3
 + 12*a^5*b^4 + 24*a^4*b^5 + 2*a^3*b^6 - 6*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^9*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7*a^8*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 20*a^7*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^6*b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^5*b^4*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 18*a^4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^
6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^2*b^7*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^8) + (1029*a^6 - 720
*a^5*b - 2940*a^4*b^2 + 1760*a^3*b^3 + 2646*a^2*b^4 - 1168*a*b^5 - 588*b^6
+ 6174*a^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3600*a^5*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 18240*a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 9120*a^3*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16956*a^2*b^4*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 6288*a*b^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 3888*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15435*a^6*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7200*a^5*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 46500*a^4*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
```

```
^2 + 18240*a^3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 44730*a^2*b^
4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12960*a*b^5*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 10740*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 20580*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a^5*b*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 62400*a^4*b^2*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 17600*a^3*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 60840*a^2*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 11680*a*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15520*b^6*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 15435*a^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 3600*a^5*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 46500*a^4*b^2*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8160*a^3*b^3*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 44730*a^2*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1
)^4 - 4560*a*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 10740*b^6*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 6174*a^6*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 720*a^5*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 182
40*a^4*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1440*a^3*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 16956*a^2*b^4*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 - 720*a*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 3
888*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1029*a^6*(cos(d*x + c)
- 1)^6/(cos(d*x + c) + 1)^6 - 2940*a^4*b^2*(cos(d*x + c) - 1)^6/(cos(d*x +
c) + 1)^6 + 2646*a^2*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 588*b^
6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/(b^8*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^6))/d
```

Mupad [B]
time = 4.77, size = 760, normalized size = 2.98

$$\frac{\frac{2\left(-105a^7-105a^6b+265a^5b^2+265a^4b^3-191a^3b^4-191a^2b^5+15ab^6+15b^7\right)}{15ab^7}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(-42a^7-7a^6b+113a^5b^2+13a^4b^3-95a^3b^4-5a^2b^5+19ab^6+6b^7\right)}{ab^7}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(-210a^7-105a^6b+523a^5b^2+244a^4b^3-362a^3b^4-145a^2b^5+7ab^6+30b^7\right)}{3ab^7}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(-315a^7-105a^6b+809a^5b^2+223a^4b^3-613a^3b^4-99a^2b^5+91ab^6+45b^7\right)}{3ab^7}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(-525a^7-350a^6b+1290a^5b^2+860a^4b^3-862a^3b^4-598a^2b^5+10ab^6+75b^7\right)}{5ab^7}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(-630a^7-525a^6b+1555a^5b^2+1325a^4b^3-1067a^3b^4-955a^2b^5+45ab^6+90b^7\right)}{15ab^7}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(-7a^7+20a^5b^2-18a^3b^4+4ab^6+b^7\right)}{ab^7}}{d\left((b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+(7a-5b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+(9b-21a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+(35a-5b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+(-35a-5b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+(21a+9b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+(-7a-5b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+a+b\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\left(7a^6-20a^4b^2+18a^2b^4-4b^6\right)}{b^8d}+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(a^2-b^2\right)^3\left(7a^2+b^2\right)}{a^2b^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^9/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*(15*a*b^6 - 105*a^6*b - 105*a^7 + 15*b^7 - 191*a^2*b^5 - 191*a^3*b^4 +
265*a^4*b^3 + 265*a^5*b^2))/(15*a*b^7) - (2*tan(c/2 + (d*x)/2)^10*(19*a*b^6
 - 7*a^6*b - 42*a^7 + 6*b^7 - 5*a^2*b^5 - 95*a^3*b^4 + 13*a^4*b^3 + 113*a^5
*b^2))/(a*b^7) - (4*tan(c/2 + (d*x)/2)^6*(7*a*b^6 - 105*a^6*b - 210*a^7 + 3
0*b^7 - 145*a^2*b^5 - 362*a^3*b^4 + 244*a^4*b^3 + 523*a^5*b^2))/(3*a*b^7) +
 (2*tan(c/2 + (d*x)/2)^8*(91*a*b^6 - 105*a^6*b - 315*a^7 + 45*b^7 - 99*a^2*
b^5 - 613*a^3*b^4 + 223*a^4*b^3 + 809*a^5*b^2))/(3*a*b^7) + (2*tan(c/2 + (d
*x)/2)^4*(10*a*b^6 - 350*a^6*b - 525*a^7 + 75*b^7 - 598*a^2*b^5 - 862*a^3*b
^4 + 860*a^4*b^3 + 1290*a^5*b^2))/(5*a*b^7) - (2*tan(c/2 + (d*x)/2)^2*(45*a
*b^6 - 525*a^6*b - 630*a^7 + 90*b^7 - 955*a^2*b^5 - 1067*a^3*b^4 + 1325*a^4
*b^3 + 1555*a^5*b^2))/(15*a*b^7) + (2*tan(c/2 + (d*x)/2)^12*(4*a*b^6 - 7*a^
7 + b^7 - 18*a^3*b^4 + 20*a^5*b^2))/(a*b^7))/(d*(a + b - tan(c/2 + (d*x)/2)
^14*(a - b) - tan(c/2 + (d*x)/2)^2*(7*a + 5*b) + tan(c/2 + (d*x)/2)^12*(7*a
```

```
 - 5*b) + tan(c/2 + (d*x)/2)^4*(21*a + 9*b) - tan(c/2 + (d*x)/2)^10*(21*a -
 9*b) - tan(c/2 + (d*x)/2)^6*(35*a + 5*b) + tan(c/2 + (d*x)/2)^8*(35*a - 5*
b))) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 -
1)*(7*a^6 - 4*b^6 + 18*a^2*b^4 - 20*a^4*b^2))/(b^8*d) + (log(a + b - a*tan(
c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^3*(7*a^2 + b^2))/(a^
2*b^8*d)
```

3.300 $\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^6d}-\frac{2a(2a^2-3b^2)\sec(c+dx)}{b^5d}+\frac{3(a^2-b^2)\sec^2(}{2b^4d}$$

```
[Out] ln(cos(d*x+c))/a^2/d+(a^2-b^2)^2*(5*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^6/d-2
*a*(2*a^2-3*b^2)*sec(d*x+c)/b^5/d+3/2*(a^2-b^2)*sec(d*x+c)^2/b^4/d-2/3*a*se
c(d*x+c)^3/b^3/d+1/4*sec(d*x+c)^4/b^2/d+(a^2-b^2)^3/a/b^6/d/(a+b*sec(d*x+c)
)
```

Rubi [A]
time = 0.11, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{(a^2-b^2)^3}{ab^6d(a+b\sec(c+dx))}+\frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^6d}-\frac{2a(2a^2-3b^2)\sec(c+dx)}{b^5d}+\frac{3(a^2-b^2)\sec^2(c+dx)}{2b^4d}+\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a\sec^3(c+dx)}{3b^3d}+\frac{\sec^4(c+dx)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*Sec[c +
d*x]])/(a^2*b^6*d) - (2*a*(2*a^2 - 3*b^2)*Sec[c + d*x])/(b^5*d) + (3*(a^2 -
 b^2)*Sec[c + d*x]^2)/(2*b^4*d) - (2*a*Sec[c + d*x]^3)/(3*b^3*d) + Sec[c +
d*x]^4/(4*b^2*d) + (a^2 - b^2)^3/(a*b^6*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(2(2a^3-3ab^2)+\frac{b^6}{a^2x}-3(a^2-b^2)\,x+2ax^2-x^3+\frac{(a^2-b^2)^3}{a(a+x)^2}-\frac{(a^2-b^2)}{a^2(}\right.\right.}{b^6 d}$$

$$= \frac{\log(\cos(c+dx))}{a^2 d}+\frac{(a^2-b^2)^2\,(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^6 d}-\frac{2a(2a^2-3b^2)}{b^5 d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(179) = 358.

time = 6.22, size = 383, normalized size = 2.14

$$\frac{(-a+b)^3(a+b)^3(b+a\cos(c+dx))\sec^2(c+dx)}{a^2b^5d(a+b\sec(c+dx))^2}+\frac{(-5a^4+9a^2b^2-3b^4)\,(b+a\cos(c+dx))^2\log(\cos(c+dx))\sec^2(c+dx)}{b^6d(a+b\sec(c+dx))^2}+\frac{(5a^6-9a^4b^2+3a^2b^4+b^6)\,(b+a\cos(c+dx))^2\log(b+a\cos(c+dx))\sec^2(c+dx)}{a^2b^6d(a+b\sec(c+dx))^2}+\frac{2a(-2a^2+3b^2)\,(b+a\cos(c+dx))^2\sec^3(c+dx)}{b^5d(a+b\sec(c+dx))^2}-\frac{3(-a+b)(a+b)(b+a\cos(c+dx))^2\sec^4(c+dx)}{2b^4d(a+b\sec(c+dx))^2}-\frac{2a(b+a\cos(c+dx))^2\sec^5(c+dx)}{3b^3d(a+b\sec(c+dx))^2}+\frac{(b+a\cos(c+dx))^2\sec^6(c+dx)}{4b^2d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-a + b)^3*(a + b)^3*(b + a*Cos[c + d*x])*Sec[c + d*x]^2)/(a^2*b^5*d*(a +
b*Sec[c + d*x])^2) + ((-5*a^4 + 9*a^2*b^2 - 3*b^4)*(b + a*Cos[c + d*x])^2*L
og[Cos[c + d*x]]*Sec[c + d*x]^2)/(b^6*d*(a + b*Sec[c + d*x])^2) + ((5*a^6 -
 9*a^4*b^2 + 3*a^2*b^4 + b^6)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]
]*Sec[c + d*x]^2)/(a^2*b^6*d*(a + b*Sec[c + d*x])^2) + (2*a*(-2*a^2 + 3*b^2
)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3)/(b^5*d*(a + b*Sec[c + d*x])^2) - (
3*(-a + b)*(a + b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^4)/(2*b^4*d*(a + b*S
ec[c + d*x])^2) - (2*a*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^5)/(3*b^3*d*(a +
 b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^6)/(4*b^2*d*(a +
 b*Sec[c + d*x])^2)
```

Maple [A]

time = 0.15, size = 200, normalized size = 1.12

method	result
derivativedivides	$\frac{-\frac{a^6-3a^4b^2+3a^2b^4-b^6}{a^2b^5(b+a\cos(dx+c))}+\frac{(5a^6-9a^4b^2+3a^2b^4+b^6)\ln(b+a\cos(dx+c))}{b^6a^2}-\frac{-3a^2+3b^2}{2b^4\cos(dx+c)^2}+\frac{(-5a^4+9b^2a^2-3b^4)\ln(\cos(dx+c))}{b^6}+\frac{}{4b^2\,c}}{d}$
default	$\frac{-\frac{a^6-3a^4b^2+3a^2b^4-b^6}{a^2b^5(b+a\cos(dx+c))}+\frac{(5a^6-9a^4b^2+3a^2b^4+b^6)\ln(b+a\cos(dx+c))}{b^6a^2}-\frac{-3a^2+3b^2}{2b^4\cos(dx+c)^2}+\frac{(-5a^4+9b^2a^2-3b^4)\ln(\cos(dx+c))}{b^6}+\frac{}{4b^2\,c}}{d}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}-\frac{2\left(-71a^3b^3e^{4i(dx+c)}-71a^3b^3e^{6i(dx+c)}-182a^4b^2e^{5i(dx+c)}+78a^2b^4e^{5i(dx+c)}+45a^5b\,e^{4i(dx+c)}-118a^4b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^2/b^5/(b+a*cos(d*x+c))+(5*a^6-9*a^4*b
^2+3*a^2*b^4+b^6)/b^6/a^2*ln(b+a*cos(d*x+c))-1/2*(-3*a^2+3*b^2)/b^4/cos(d*x
+c)^2+(-5*a^4+9*a^2*b^2-3*b^4)/b^6*ln(cos(d*x+c))+1/4/b^2/cos(d*x+c)^4-2/3/
b^3*a/cos(d*x+c)^3-2*a*(2*a^2-3*b^2)/b^5/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 227, normalized size = 1.27

$$-\frac{\frac{5\,a^3b^3\cos(dx+c)-3\,a^2b^4+12\,(5\,a^6-9\,a^4b^2+3\,a^2b^4-b^6)\cos(dx+c)^4+6\,(5\,a^5b-9\,a^3b^3)\cos(dx+c)^3-2\,(5\,a^4b^2-9\,a^2b^4)\cos(dx+c)^2}{a^3b^5\cos(dx+c)^5+a^2b^6\cos(dx+c)^4}+\frac{12\,(5\,a^4-9\,a^2b^2+3\,b^4)\log(\cos(dx+c))}{b^6}-\frac{12\,(5\,a^6-9\,a^4b^2+3\,a^2b^4+b^6)\log(a\cos(dx+c)+b)}{a^2b^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((5*a^3*b^3*cos(d*x + c) - 3*a^2*b^4 + 12*(5*a^6 - 9*a^4*b^2 + 3*a^2*
b^4 - b^6)*cos(d*x + c)^4 + 6*(5*a^5*b - 9*a^3*b^3)*cos(d*x + c)^3 - 2*(5*a
^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^2)/(a^3*b^5*cos(d*x + c)^5 + a^2*b^6*cos(d
*x + c)^4) + 12*(5*a^4 - 9*a^2*b^2 + 3*b^4)*log(cos(d*x + c))/b^6 - 12*(5*a
^6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)*log(a*cos(d*x + c) + b)/(a^2*b^6))/d
```

Fricas [A]
time = 4.44, size = 312, normalized size = 1.74

$$-\frac{5\,a^3b^4\cos\left(dx+c\right)-3\,a^2b^5+12\,(5\,a^6b-9\,a^4b^3+3\,a^2b^5-b^7)\cos\left(dx+c\right)^4+6\,(5\,a^5b^2-9\,a^3b^4)\cos\left(dx+c\right)^3-2\,(5\,a^4b^3-9\,a^2b^5)\cos\left(dx+c\right)^2-12\,\left((5\,a^7-9\,a^5b^2+3\,a^3b^4+ab^6)\cos\left(dx+c\right)^5+(5\,a^6b-9\,a^4b^3+3\,a^2b^5+b^7)\cos\left(dx+c\right)^4\right)\log\left(a\cos\left(dx+c\right)+b\right)+12\,\left((5\,a^7-9\,a^5b^2+3\,a^3b^4)\cos\left(dx+c\right)^5+(5\,a^6b-9\,a^4b^3+3\,a^2b^5)\cos\left(dx+c\right)^4\right)\log\left(-\cos\left(dx+c\right)\right)}{12\,\left(a^3b^6d\cos\left(dx+c\right)^5+a^2b^7d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(5*a^3*b^4*cos(d*x + c) - 3*a^2*b^5 + 12*(5*a^6*b - 9*a^4*b^3 + 3*a^2
*b^5 - b^7)*cos(d*x + c)^4 + 6*(5*a^5*b^2 - 9*a^3*b^4)*cos(d*x + c)^3 - 2*(
5*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c)^2 - 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4
+ a*b^6)*cos(d*x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x +
 c)^4)*log(a*cos(d*x + c) + b) + 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4)*cos(d*
x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^4)*log(-cos(d*x +
 c)))/(a^3*b^6*d*cos(d*x + c)^5 + a^2*b^7*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^7\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(173) = 346$.

time = 3.64, size = 1023, normalized size = 5.72

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(5*a^7 - 5*a^6*b - 9*a^5*b^2 + 9*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 +
 a*b^6 - b^7)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^6 - a^2*b^7) - 12*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*(5*a^4 - 9*a^2*b^2 + 3*b
^4)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 12*(5*a^7 +
7*a^6*b - 7*a^5*b^2 - 13*a^4*b^3 - a^3*b^4 + 5*a^2*b^5 + 3*a*b^6 + b^7 + 5*
a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 5*a^6*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 9*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*b^
3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*b^4*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 3*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*b^6*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - b^7*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1))*a^2*b^6) + (125*a^4 - 96*a^3*b - 225*a^2*b^2 + 128*
a*b^3 + 75*b^4 + 500*a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 288*a^3*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 972*a^2*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 416*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 348*b^4*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 750*a^4*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 288*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1494
*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 384*a*b^3*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 + 594*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 500*a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 96*a^3*b*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 972*a^2*b^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 96*a*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 34
8*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 125*a^4*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 225*a^2*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 + 75*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/(b^6*((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

Mupad [B]

time = 3.09, size = 505, normalized size = 2.82

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(20\,a^5+5\,a^4\,b-31\,a^3\,b^2-4\,a^2\,b^3+8\,a\,b^4+4\,b^5\right)}{a\,b^5}-\frac{2\,\left(15\,a^5+15\,a^4\,b-22\,a^3\,b^2-22\,a^2\,b^3+3\,a\,b^4+3\,b^5\right)}{3\,a\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(60\,a^5+45\,a^4\,b-83\,a^3\,b^2-66\,a^2\,b^3+6\,a\,b^4+12\,b^5\right)}{3\,a\,b^5}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(90\,a^5+45\,a^4\,b-127\,a^3\,b^2-56\,a^2\,b^3+6\,a\,b^4+18\,b^5\right)}{3\,a\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a-b\right)\,\left(-5\,a^4-5\,a^3\,b+4\,a^2\,b^2+4\,a\,b^3+b^4\right)}{a\,b^5}}{d\,\left(\left(b-a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\left(5\,a-3\,b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\left(2\,b-10\,a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\left(10\,a+2\,b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(-5\,a-3\,b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)\,\left(5\,a^4-9\,a^2\,b^2+3\,b^4\right)}{b^6\,d}+\frac{\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)\,{\left(a^2-b^2\right)}^2\,\left(5\,a^2+b^2\right)}{a^2\,b^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^6*(8*a*b^4 + 5*a^4*b + 20*a^5 + 4*b^5 - 4*a^2*b^3 -
31*a^3*b^2))/(a*b^5) - (2*(3*a*b^4 + 15*a^4*b + 15*a^5 + 3*b^5 - 22*a^2*b^3
 - 22*a^3*b^2))/(3*a*b^5) + (2*tan(c/2 + (d*x)/2)^2*(6*a*b^4 + 45*a^4*b + 6
0*a^5 + 12*b^5 - 66*a^2*b^3 - 83*a^3*b^2))/(3*a*b^5) - (2*tan(c/2 + (d*x)/2
)^4*(6*a*b^4 + 45*a^4*b + 90*a^5 + 18*b^5 - 56*a^2*b^3 - 127*a^3*b^2))/(3*a
*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a - b)*(4*a*b^3 - 5*a^3*b - 5*a^4 + b^4 +
4*a^2*b^2))/(a*b^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) - tan(c/2 +
(d*x)/2)^2*(5*a + 3*b) + tan(c/2 + (d*x)/2)^4*(10*a + 2*b) + tan(c/2 + (d*x
)/2)^8*(5*a - 3*b) - tan(c/2 + (d*x)/2)^6*(10*a - 2*b))) - log(tan(c/2 + (d
*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(5*a^4 + 3*b^4 - 9*a
^2*b^2))/(b^6*d) + (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/
2)^2)*(a^2 - b^2)^2*(5*a^2 + b^2))/(a^2*b^6*d)
```

3.301 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=121

$$-\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+dx)}{2b^2d}+\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+(a^2-b^2)*(3*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^4/d-2*
a*sec(d*x+c)/b^3/d+1/2*sec(d*x+c)^2/b^2/d+(a^2-b^2)^2/a/b^4/d/(a+b*sec(d*x+
c))
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{\log(\cos(c+dx))}{a^2d}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sec[c +
 d*x]])/(a^2*b^4*d) - (2*a*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^2/(2*b^2*d)
 + (a^2 - b^2)^2/(a*b^4*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2x}+x-\frac{(a^2-b^2)^2}{a(a+x)^2}+\frac{(a^2-b^2)(3a^2+b^2)}{a^2(a+x)}\right)\,dx, x, b\sec(c+dx)\right)}{b^4d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^4d}-\frac{2a\sec(c+d}{b^3d}$$

Mathematica [A]

time = 0.61, size = 187, normalized size = 1.55

$$\frac{-2a\cos(c+dx)\,(a^2(3a^2-2b^2)\log(\cos(c+dx))+(-3a^4+2a^2b^2+b^4)\log(b+a\cos(c+dx)))+b(-2(3a^4-2a^2b^2+b^4+a^2(3a^2-2b^2)\log(\cos(c+dx))+(-3a^4+2a^2b^2+b^4)\log(b+a\cos(c+dx)))-3a^3b\sec(c+dx)+a^2b^2\sec^2(c+dx))}{2a^2b^4d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a*Cos[c + d*x]*(a^2*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]] + (-3*a^4 + 2*a^2
*b^2 + b^4)*Log[b + a*Cos[c + d*x]]) + b*(-2*(3*a^4 - 2*a^2*b^2 + b^4 + a^2
*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]] + (-3*a^4 + 2*a^2*b^2 + b^4)*Log[b + a*C
os[c + d*x]]) - 3*a^3*b*Sec[c + d*x] + a^2*b^2*Sec[c + d*x]^2))/(2*a^2*b^4*
d*(b + a*Cos[c + d*x]))
```

Maple [A]

time = 0.11, size = 127, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{a^4-2b^2a^2+b^4}{a^2b^3(b+a\cos(dx+c))}+\frac{(3a^4-2b^2a^2-b^4)\ln(b+a\cos(dx+c))}{b^4a^2}+\frac{(-3a^2+2b^2)\ln(\cos(dx+c))}{b^4}+\frac{1}{2b^2\cos(dx+c)^2}-\frac{2a}{b^3\cos(dx+c)}}{d}$
default	$\frac{-\frac{a^4-2b^2a^2+b^4}{a^2b^3(b+a\cos(dx+c))}+\frac{(3a^4-2b^2a^2-b^4)\ln(b+a\cos(dx+c))}{b^4a^2}+\frac{(-3a^2+2b^2)\ln(\cos(dx+c))}{b^4}+\frac{1}{2b^2\cos(dx+c)^2}-\frac{2a}{b^3\cos(dx+c)}}{d}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2(3a^4e^{5i(dx+c)}-2b^2a^2e^{5i(dx+c)}+b^4e^{5i(dx+c)}+3b\,a^3e^{4i(dx+c)}+6a^4e^{3i(dx+c)}-6b^2a^2e^{3i(dx+c)}+2b^4e^{3i(dx}}{d\,b^3(e^{2i(dx+c)}+1)^2a^2(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^4-2*a^2*b^2+b^4)/a^2/b^3/(b+a*cos(d*x+c))+(3*a^4-2*a^2*b^2-b^4)/b^
4/a^2*ln(b+a*cos(d*x+c))+(-3*a^2+2*b^2)/b^4*ln(cos(d*x+c))+1/2/b^2/cos(d*x+
c)^2-2/b^3*a/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 149, normalized size = 1.23

$$-\frac{\frac{3\,a^3b\cos(dx+c)-a^2b^2+2\,(3\,a^4-2\,a^2b^2+b^4)\cos(dx+c)^2}{a^3b^3\cos(dx+c)^3+a^2b^4\cos(dx+c)^2}+\frac{2\,(3\,a^2-2\,b^2)\log(\cos(dx+c))}{b^4}-\frac{2\,(3\,a^4-2\,a^2b^2-b^4)\log(a\cos(dx+c)+b)}{a^2b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a^3*b*cos(d*x + c) - a^2*b^2 + 2*(3*a^4 - 2*a^2*b^2 + b^4)*cos(d*x
 + c)^2)/(a^3*b^3*cos(d*x + c)^3 + a^2*b^4*cos(d*x + c)^2) + 2*(3*a^2 - 2*b
^2)*log(cos(d*x + c))/b^4 - 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(a*cos(d*x + c)
+ b)/(a^2*b^4))/d
```

Fricas [A]
time = 2.47, size = 219, normalized size = 1.81

$$-\frac{3\,a^3b^2\cos(dx+c)-a^2b^3+2\,(3\,a^4b-2\,a^2b^3+b^5)\cos(dx+c)^2-2\,\left((3\,a^5-2\,a^3b^2-ab^4)\cos(dx+c)^3+(3\,a^4b-2\,a^2b^3-b^5)\cos(dx+c)^2\right)\log(a\cos(dx+c)+b)+2\,\left((3\,a^5-2\,a^3b^2)\cos(dx+c)^3+(3\,a^4b-2\,a^2b^3)\cos(dx+c)^2\right)\log(-\cos(dx+c))}{2\,\left(a^3b^4d\cos(dx+c)^3+a^2b^5d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*b^2*cos(d*x + c) - a^2*b^3 + 2*(3*a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^2 - 2*((3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x + c)^3 + (3*a^4*b - 2*a
^2*b^3 - b^5)*cos(d*x + c)^2)*log(a*cos(d*x + c) + b) + 2*((3*a^5 - 2*a^3*b
^2)*cos(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c
)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(119) = 238.
time = 1.78, size = 568, normalized size = 4.69

$$\frac{\frac{2\,(3\,a^5-3\,a^4b-2\,a^3b^2+2\,a^2b^3-ab^4+b^5)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^4-a^2b^5}+\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\,(3\,a^2-2\,b^2)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^4}+\frac{9\,a^2-8\,ab-6\,b^2+\frac{18\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{b^4\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}-\frac{2\left(3\,a^5+5\,a^4b-4\,a^2b^3-3\,ab^4-b^5+\frac{3\,a^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^4b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a^3b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,a^2b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{ab^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)a^2b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^5 - 3*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 + b^5)*log(abs(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1)))/(a^3*b^4 - a^2*b^5) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 1))/a^2 - 2*(3*a^2 - 2*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 1))/b^4 + (9*a^2 - 8*a*b - 6*b^2 + 18*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^4*((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2) - 2*(3*a^5 + 5*a^4*b - 4*a^2*b^3 - 3
*a*b^4 - b^5 + 3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^4*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((
a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))*a^2*b^4))/d
```

Mupad [B]
time = 2.13, size = 286, normalized size = 2.36

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-6\,a^3-3\,a^2\,b+a\,b^2+2\,b^3\right)}{a\,b^3}-\frac{2\,\left(-3\,a^3-3\,a^2\,b+a\,b^2+b^3\right)}{a\,b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a-b\right)\,\left(3\,a^2+3\,a\,b+b^2\right)}{a\,b^3}}{d\,\left(\left(b-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(3\,a-b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-3\,a-b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(3\,a^2-2\,b^2\right)}{b^4\,d}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(-3\,a^4+2\,a^2\,b^2+b^4\right)}{a^2\,b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - ((2*tan(c/2 + (d*x)/2)^2*(a*b^2 - 3
*a^2*b - 6*a^3 + 2*b^3))/(a*b^3) - (2*(a*b^2 - 3*a^2*b - 3*a^3 + b^3))/(a*b
^3) + (2*tan(c/2 + (d*x)/2)^4*(a - b)*(3*a*b + 3*a^2 + b^2))/(a*b^3))/(d*(a
 + b - tan(c/2 + (d*x)/2)^2*(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(
c/2 + (d*x)/2)^4*(3*a - b))) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(3*a^2 - 2*b^
2))/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)
*(b^4 - 3*a^4 + 2*a^2*b^2))/(a^2*b^4*d)
```

3.302 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}$$

```
[Out] ln(cos(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^2/d+(a^2-b^2)/a/b^2
/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 + b^2)*Log[a + b*Sec[c + d*x]])/(a^2*b^2*
d) + (a^2 - b^2)/(a*b^2*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2x}+\frac{a^2-b^2}{a(a+x)^2}+\frac{-a^2-b^2}{a^2(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}$$

Mathematica [A]
time = 0.28, size = 62, normalized size = 0.84

$$-\frac{\frac{b-\frac{b^3}{a^2}}{b+a\cos(c+dx)}+\log(\cos(c+dx))-\frac{(a^2+b^2)\log(b+a\cos(c+dx))}{a^2}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
[Out] -(((b - b^3/a^2)/(b + a*Cos[c + d*x]) + Log[Cos[c + d*x]] - ((a^2 + b^2)*Lo
g[b + a*Cos[c + d*x]])/a^2)/(b^2*d))
```

Maple [A]
time = 0.09, size = 72, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{a^2-b^2}{a^2b(b+a\cos(dx+c))}+\frac{(a^2+b^2)\ln(b+a\cos(dx+c))}{b^2a^2}-\frac{\ln(\cos(dx+c))}{b^2}}{d}$
default	$\frac{-\frac{a^2-b^2}{a^2b(b+a\cos(dx+c))}+\frac{(a^2+b^2)\ln(b+a\cos(dx+c))}{b^2a^2}-\frac{\ln(\cos(dx+c))}{b^2}}{d}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}-\frac{2(a^2-b^2)e^{i(dx+c)}}{a^2bd(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{b^2d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+\right.}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-(a^2-b^2)/a^2/b/(b+a*cos(d*x+c))+(a^2+b^2)/b^2/a^2*ln(b+a*cos(d*x+c))
-1/b^2*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.00

$$-\frac{\frac{a^2-b^2}{a^3b\cos(dx+c)+a^2b^2}+\frac{\log(\cos(dx+c))}{b^2}-\frac{(a^2+b^2)\log(a\cos(dx+c)+b)}{a^2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 - b^2)/(a^3*b*cos(d*x + c) + a^2*b^2) + log(cos(d*x + c))/b^2 - (a^2
 + b^2)*log(a*cos(d*x + c) + b)/(a^2*b^2))/d
```

Fricas [A]
time = 1.03, size = 102, normalized size = 1.38

$$-\frac{a^2b-b^3-(a^2b+b^3+(a^3+ab^2)\cos(dx+c))\log(a\cos(dx+c)+b)+(a^3\cos(dx+c)+a^2b)\log(-\cos(dx+c))}{a^3b^2d\cos(dx+c)+a^2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b - b^3 - (a^2*b + b^3 + (a^3 + a*b^2)*cos(d*x + c))*log(a*cos(d*x +
c) + b) + (a^3*cos(d*x + c) + a^2*b)*log(-cos(d*x + c)))/(a^3*b^2*d*cos(d*x
 + c) + a^2*b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(74) = 148.
time = 0.81, size = 313, normalized size = 4.23

$$\frac{\frac{(a^3-a^2b+ab^2-b^3)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^2-a^2b^3}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}-\frac{a^3+3a^2b+3ab^2+b^3+\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)a^2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((a^3 - a^2*b + a*b^2 - b^3)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^2 - a^2*b^3) -
 log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^2*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*(cos(d*x
```

```
+ c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^2))/d
```

Mupad [B]
time = 1.65, size = 124, normalized size = 1.68

$$\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{b^2\,d}-\frac{2\,\left(a+b\right)}{a\,b\,d\,\left(\left(b-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(1/a^2 + 1/b^
2))/d - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 -
1)/(b^2*d) - (2*(a + b))/(a*b*d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.303 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=54

$$-\frac{\log(\cos(c+dx))}{a^2d}-\frac{\log(a+b\sec(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-ln(a+b*sec(d*x+c))/a^2/d+1/a/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3970, 46}

$$-\frac{\log(a+b\sec(c+dx))}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - Log[a + b*Sec[c + d*x]]/(a^2*d) + 1/(a*d*(a
+ b*Sec[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}-\frac{\log(a+b\sec(c+dx))}{a^2d}+\frac{1}{ad(a+b\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$-\frac{b+b\log(b+a\cos(c+dx))+a\cos(c+dx)\log(b+a\cos(c+dx))}{a^2d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b + b*Log[b + a*Cos[c + d*x]] + a*Cos[c + d*x]*Log[b + a*Cos[c + d*x]])/
(a^2*d*(b + a*Cos[c + d*x])))
```

Maple [A]
time = 0.05, size = 49, normalized size = 0.91

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a^2}+\frac{1}{a(a+b\sec(dx+c))}+\frac{\ln(\sec(dx+c))}{a^2}}{d}$	49
default	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a^2}+\frac{1}{a(a+b\sec(dx+c))}+\frac{\ln(\sec(dx+c))}{a^2}}{d}$	49
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2b\,\mathrm{e}^{i(dx+c)}}{a^2d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{a}+1\right)}{a^2d}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(a+b*sec(d*x+c))+1/a/(a+b*sec(d*x+c))+1/a^2*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.76

$$-\frac{\frac{b}{a^3\cos(dx+c)+a^2b}+\frac{\log(a\cos(dx+c)+b)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(b/(a^3*cos(d*x + c) + a^2*b) + log(a*cos(d*x + c) + b)/a^2)/d
```

Fricas [A]
time = 1.47, size = 46, normalized size = 0.85

$$-\frac{(a\cos(dx+c)+b)\log(a\cos(dx+c)+b)+b}{a^3d\cos(dx+c)+a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -((a*cos(d*x + c) + b)*log(a*cos(d*x + c) + b) + b)/(a^3*d*cos(d*x + c) + a
^2*b*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\tilde{\infty} x \tan(c)}{\sec^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log(\tan^2(c+dx)+1)}{2a^2d} & \text{for } b = 0 \\ -\frac{1}{2b^2d\sec^2(c+dx)} & \text{for } a = 0 \\ \frac{\int \frac{\tan(c+dx)}{\cos^2(c+dx)\sec^2(c+dx)-2\cos(c+dx)\sec(c+dx)+1}\,dx}{a^2} & \text{for } b = -a\cos(c+dx) \\ \frac{x\tan(c)}{(a+b\sec(c))^2} & \text{for } d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sec(c+dx)\right)}{2a^3d+2a^2bd\sec(c+dx)} + \frac{a\log(\tan^2(c+dx)+1)}{2a^3d+2a^2bd\sec(c+dx)} + \frac{2a}{2a^3d+2a^2bd\sec(c+dx)} - \frac{2b\log\left(\frac{a}{b}+\sec(c+dx)\right)\sec(c+dx)}{2a^3d+2a^2bd\sec(c+dx)} + \frac{b\log(\tan^2(c+dx)+1)\sec(c+dx)}{2a^3d+2a^2bd\sec(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**2,x)

[Out] Piecewise((zoo*x*tan(c)/sec(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(ta
n(c + d*x)**2 + 1)/(2*a**2*d), Eq(b, 0)), (-1/(2*b**2*d*sec(c + d*x)**2), E
q(a, 0)), (Integral(tan(c + d*x)/(cos(c + d*x)**2*sec(c + d*x)**2 - 2*cos(c
 + d*x)*sec(c + d*x) + 1), x)/a**2, Eq(b, -a*cos(c + d*x))), (x*tan(c)/(a +
 b*sec(c))**2, Eq(d, 0)), (-2*a*log(a/b + sec(c + d*x))/(2*a**3*d + 2*a**2*
b*d*sec(c + d*x)) + a*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a**2*b*d*sec(c
 + d*x)) + 2*a/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) - 2*b*log(a/b + sec(c +
 d*x))*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) + b*log(tan(c + d*
x)**2 + 1)*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(54) = 108.
time = 0.51, size = 238, normalized size = 4.41

$$-\frac{\frac{(a-b)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-a^2b} - \frac{a^2-2\,ab-b^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{(a^3-a^2b)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] -((a - b)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a^2*b) - (a^2 - 2*a*b - b^2 + a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 - a^2*b)*(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))) - log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d

Mupad [B]
time = 1.48, size = 257, normalized size = 4.76

$$\frac{2\,\mathrm{atanh}\left(\frac{a}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}-\frac{a\,\cos\left(c+d\,x\right)}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}\right)}{a^2\,d}-\frac{b\,\left(a+a\,\cos\left(c+d\,x\right)-2\,a\,\mathrm{atanh}\left(\frac{a}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}-\frac{a\,\cos\left(c+d\,x\right)}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}\right)+2\,a\,\cos\left(c+d\,x\right)\,\mathrm{atanh}\left(\frac{a}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}-\frac{a\,\cos\left(c+d\,x\right)}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}\right)+\frac{2\,\mathrm{atanh}\left(\frac{a}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}-\frac{a\,\cos\left(c+d\,x\right)}{2\,\left(\frac{a}{2}+b+\frac{a\,\cos\left(c+d\,x\right)}{2}\right)}\right)\,\left(a^3\,d-a^3\,d\,\cos\left(c+d\,x\right)\right)}{a^2\,d}\right)}{a^2\,d\,\left(a-b\right)\,\left(b+a\,\cos\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 +
b + (a*cos(c + d*x))/2))))/(a^2*d) - (b*(a + a*cos(c + d*x) - 2*a*atanh(a/(
2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c
 + d*x))/2))) + 2*a*cos(c + d*x)*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2))
 - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2))) + (2*atanh(a/(2*(a/
2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*
x))/2)))*(a^3*d - a^3*d*cos(c + d*x)))/(a^2*d)))/(a^2*d*(a - b)*(b + a*cos(
c + d*x)))
```

3.304 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2(a+b)^2d}+\frac{\log(1+\sec(c+dx))}{2(a-b)^2d}-\frac{b^2(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2\left(a^2-b^2\right)^2d}+\frac{}{a\left(a^2-\right.}$$

```
[Out] ln(cos(d*x+c))/a^2/d+1/2*ln(1-sec(d*x+c))/(a+b)^2/d+1/2*ln(1+sec(d*x+c))/(a
-b)^2/d-b^2*(3*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^2/d+b^2/a/(a^2-b^2
)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3970, 908}

$$\frac{b^2}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b^2(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^2}+\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2d(a+b)^2}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)^2*d) + Log[1 +
 Sec[c + d*x]]/(2*(a - b)^2*d) - (b^2*(3*a^2 - b^2)*Log[a + b*Sec[c + d*x]]
)/(a^2*(a^2 - b^2)^2*d) + b^2/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^2\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)^2(b-x)} + \frac{1}{a^2b^2x} + \frac{1}{a(a-b)(a+b)(a+x)^2} + \frac{3a^2-b^2}{a^2(a-b)^2(a+b)^2(a+x)} - \frac{}{2(a}\right.\right.}{d}$$

$$= \frac{\log(\cos(c+dx))}{a^2d} + \frac{\log(1-\sec(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sec(c+dx))}{2(a-b)^2d} - \frac{b^2(3a^2-b^2}{d}$$

Mathematica [A]
time = 0.37, size = 189, normalized size = 1.37

$$\frac{a\cos(c+dx)\left(a^2(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+(-3a^2b^2+b^4)\log(b+a\cos(c+dx))+a^2(a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\left(a^2(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+(-3a^2b^2+b^4)\log(b+a\cos(c+dx))+(a-b)\left(-b^2(a+b)+a^2(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{a^2(a-b)^2(a+b)^2d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*Cos[c + d*x]*(a^2*(a + b)^2*Log[Cos[(c + d*x)/2]] + (-3*a^2*b^2 + b^4)*L
og[b + a*Cos[c + d*x]] + a^2*(a - b)^2*Log[Sin[(c + d*x)/2]]) + b*(a^2*(a +
 b)^2*Log[Cos[(c + d*x)/2]] + (-3*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]] +
(a - b)*(-(b^2*(a + b)) + a^2*(a - b)*Log[Sin[(c + d*x)/2]])))/(a^2*(a - b)
^2*(a + b)^2*d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.16, size = 114, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{b^3}{a^2(a+b)(a-b)(b+a\cos(dx+c))}-\frac{b^2\left(3a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a^2}+\frac{\ln(-1+\cos(dx+c))}{2(a+b)^2}+\frac{\ln(1+\cos(dx+c))}{2(a-b)^2}}{d}$
default	$\frac{-\frac{b^3}{a^2(a+b)(a-b)(b+a\cos(dx+c))}-\frac{b^2\left(3a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a^2}+\frac{\ln(-1+\cos(dx+c))}{2(a+b)^2}+\frac{\ln(1+\cos(dx+c))}{2(a-b)^2}}{d}$
risch	$\frac{ix}{a^2}-\frac{ix}{a^2-2ba+b^2}-\frac{ic}{d(a^2-2ba+b^2)}-\frac{ix}{a^2+2ba+b^2}-\frac{ic}{d(a^2+2ba+b^2)}+\frac{6ib^2x}{a^4-2b^2a^2+b^4}+\frac{6ib^2c}{d(a^4-2b^2a^2+b^4)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))-b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^
2/a^2*ln(b+a*cos(d*x+c))+1/2/(a+b)^2*ln(-1+cos(d*x+c))+1/2/(a-b)^2*ln(1+cos
(d*x+c)))
```

Maxima [A]
time = 0.30, size = 142, normalized size = 1.03

$$-\frac{\frac{2\,b^3}{a^4b-a^2b^3+(a^5-a^3b^2)\cos(dx+c)}+\frac{2\,(3\,a^2b^2-b^4)\log(a\cos(dx+c)+b)}{a^6-2\,a^4b^2+a^2b^4}-\frac{\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c)) + 2*(3*a^2*b^2
 - b^4)*log(a*cos(d*x + c) + b)/(a^6 - 2*a^4*b^2 + a^2*b^4) - log(cos(d*x +
 c) + 1)/(a^2 - 2*a*b + b^2) - log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A]
time = 3.34, size = 234, normalized size = 1.70

$$-\frac{2\,a^2b^3-2\,b^5+2\,(3\,a^2b^3-b^5+(3\,a^3b^2-ab^4)\cos(dx+c))\log(a\cos(dx+c)+b)-(a^4b+2\,a^3b^2+a^2b^3+(a^5+2\,a^4b+a^3b^2)\cos(dx+c))\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(a^4b-2\,a^3b^2+a^2b^3+(a^5-2\,a^4b+a^3b^2)\cos(dx+c))\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,((a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)+(a^6b-2\,a^4b^3+a^2b^5)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^2*b^3 - b^5 + (3*a^3*b^2 - a*b^4)*cos(d*x
+ c))*log(a*cos(d*x + c) + b) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4
*b + a^3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b - 2*a^3*b^
2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 +
 a^2*b^5)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(134) = 268.
time = 0.51, size = 303, normalized size = 2.20

$$-\frac{\frac{2\,(3\,a^2b^2-b^4)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-2\,a^4b^2+a^2b^4}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{2\left(3\,a^2b^2+4\,ab^3+b^4+\frac{3\,a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^5+a^4b-a^3b^2-a^2b^3)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}+\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b^2 - b^4)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 2*a^4*b^2 + a^2*
b^4) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2
) - 2*(3*a^2*b^2 + 4*a*b^3 + b^4 + 3*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^5 + a^4*b - a^3*b^
2 - a^2*b^3)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1))/a^2)/d
```

Mupad [B]
time = 1.98, size = 160, normalized size = 1.16

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a^2+2\,a\,b+b^2\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}-\frac{b^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)\,\left(3\,a^2-b^2\right)}{a^2\,d\,{\left(a^2-b^2\right)}^2}-\frac{2\,b^3}{a\,d\,\left(a+b\right)\,{\left(a-b\right)}^2\,\left(\left(b-a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(2*a*b + a^2 + b^2)) - log(tan(c/2 + (d*x)/2)^2
+ 1)/(a^2*d) - (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/
2)^2)*(3*a^2 - b^2))/(a^2*d*(a^2 - b^2)^2) - (2*b^3)/(a*d*(a + b)*(a - b)^2
*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.305 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$-\frac{\log(\cos(c+dx))}{a^2d}-\frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3d}-\frac{(a-2b)\log(1+\sec(c+dx))}{2(a-b)^3d}-\frac{b^4(5a^2-b^2)\log(a+b\sec(}{a^2\left(a^2-b^2\right)^3d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-1/2*(a+2*b)*ln(1-sec(d*x+c))/(a+b)^3/d-1/2*(a-2*b)*ln
(1+sec(d*x+c))/(a-b)^3/d-b^4*(5*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^3
/d+1/4/(a+b)^2/d/(1-sec(d*x+c))+1/4/(a-b)^2/d/(1+sec(d*x+c))+b^4/a/(a^2-b^2
)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{b^4}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b^4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\log(\cos(c+dx))}{a^2d}+\frac{1}{4d(a+b)^2(1-\sec(c+dx))}+\frac{1}{4d(a-b)^2(\sec(c+dx)+1)}-\frac{(a+2b)\log(1-\sec(c+dx))}{2d(a+b)^3}-\frac{(a-2b)\log(\sec(c+dx)+1)}{2d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - ((a + 2*b)*Log[1 - Sec[c + d*x]])/(2*(a + b)
^3*d) - ((a - 2*b)*Log[1 + Sec[c + d*x]])/(2*(a - b)^3*d) - (b^4*(5*a^2 - b
^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1 -
Sec[c + d*x])) + 1/(4*(a - b)^2*d*(1 + Sec[c + d*x])) + b^4/(a*(a^2 - b^2)^
2*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)^2(b-x)^2} + \frac{a+2b}{2b^4(a+b)^3(b-x)} + \frac{1}{a^2b^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)^2} + \frac{-}{a^2(a-b)}\right.\right.}{d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3d} - \frac{(a-2b)\log(1+\sec(c+}{2(a-b)^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.30, size = 351, normalized size = 1.78

$$\frac{(b+a\cos(c+dx))\left(-\frac{8b^5}{a^2(a-b)^2(a+b)^2} - \frac{16i(a^4-3a^2b^2-2b^4)(c+dx)(b+a\cos(c+dx))}{(a-b)^3(a+b)^3} + \frac{8i(a-2b)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))}{(a-b)^3} + \frac{8i(a+2b)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))}{(a+b)^3} - \frac{(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{4(a-2b)(b+a\cos(c+dx))\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3} + \frac{8b^4(-5a^2+b^2)(b+a\cos(c+dx))\log(b+a\cos(c+dx))}{a^2(a^2-b^2)^3} - \frac{4(a+2b)(b+a\cos(c+dx))\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3} - \frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)\sec^2(c+dx)}{8d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((-8*b^5)/(a^2*(a - b)^2*(a + b)^2) - ((16*I)*(a^4 -
3*a^2*b^2 - 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x]))/((a - b)^3*(a + b)^3) +
((8*I)*(a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]))/(a - b)^3 + ((8
*I)*(a + 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]))/(a + b)^3 - ((b +
a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a - 2*b)*(b + a*Cos[c +
 d*x])*Log[Cos[(c + d*x)/2]^2])/(-a + b)^3 + (8*b^4*(-5*a^2 + b^2)*(b + a*C
os[c + d*x])*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)^3) - (4*(a + 2*b)*(b
 + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2])/(a + b)^3 - ((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a - b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])
^2)
```

Maple [A]
time = 0.20, size = 164, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{b^5}{a^2(a+b)^2(a-b)^2(b+a\cos(dx+c))} - \frac{b^4(5a^2-b^2)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a^2} + \frac{1}{4(a+b)^2(-1+\cos(dx+c))} + \frac{(-a-2b)\ln(-1+\cos(dx+c))}{2(a+b)^3} - \frac{}{4(}}{d}$
default	$\frac{-\frac{b^5}{a^2(a+b)^2(a-b)^2(b+a\cos(dx+c))} - \frac{b^4(5a^2-b^2)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a^2} + \frac{1}{4(a+b)^2(-1+\cos(dx+c))} + \frac{(-a-2b)\ln(-1+\cos(dx+c))}{2(a+b)^3} - \frac{}{4(}}{d}$
risch	$\frac{iax}{a^3+3b\,a^2+3b^2a+b^3} + \frac{10ib^4x}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{ix}{a^2} - \frac{2ibx}{a^3-3b\,a^2+3b^2a-b^3} + \frac{2ibx}{a^3+3b\,a^2+3b^2a+b^3} + \frac{iac}{d(a^3+3b\,a^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^5/a^2/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))-b^4*(5*a^2-b^2)/(a+b)^3/(a-b
)^3/a^2*ln(b+a*cos(d*x+c))+1/4/(a+b)^2/(-1+cos(d*x+c))+1/2*(-a-2*b)/(a+b)^3
*ln(-1+cos(d*x+c))-1/4/(a-b)^2/(1+cos(d*x+c))+1/2*(-a+2*b)/(a-b)^3*ln(1+cos
(d*x+c)))
```

Maxima [A]
time = 0.29, size = 303, normalized size = 1.54

$$-\frac{\frac{2\left(5\,a^{2}b^{4}-b^{6}\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}}+\frac{\left(a-2\,b\right)\log\left(\cos\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(a+2\,b\right)\log\left(\cos\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{a^{4}b+a^{2}b^{3}+2\,b^{5}-2\left(a^{4}b+b^{5}\right)\cos\left(dx+c\right)^{2}+\left(a^{5}-a^{3}b^{2}\right)\cos\left(dx+c\right)}{a^{6}b-2\,a^{4}b^{3}+a^{2}b^{5}-\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)\cos\left(dx+c\right)^{3}-\left(a^{6}b-2\,a^{4}b^{3}+a^{2}b^{5}\right)\cos\left(dx+c\right)^{2}+\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)\cos\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a^8 - 3*a^6*b^2 + 3*a^4*
b^4 - a^2*b^6) + (a - 2*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + (a + 2*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (
a^4*b + a^2*b^3 + 2*b^5 - 2*(a^4*b + b^5)*cos(d*x + c)^2 + (a^5 - a^3*b^2)*
cos(d*x + c))/(a^6*b - 2*a^4*b^3 + a^2*b^5 - (a^7 - 2*a^5*b^2 + a^3*b^4)*co
s(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 + (a^7 - 2*a^5*
b^2 + a^3*b^4)*cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(187) = 374$.
time = 3.62, size = 693, normalized size = 3.52

$$\frac{a^{6}b+a^{2}b^{5}-2\,b^{7}-2\left(a^{6}b-a^{4}b^{3}+a^{2}b^{5}-b^{7}\right)\cos\left(dx+c\right)^{2}+\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)\cos\left(dx+c\right)+2\left(5\,a^{2}b^{5}-b^{7}-\left(5\,a^{3}b^{4}-ab^{6}\right)\cos\left(dx+c\right)^{3}-\left(5\,a^{2}b^{5}-b^{7}\right)\cos\left(dx+c\right)^{2}+\left(5\,a^{3}b^{4}-ab^{6}\right)\cos\left(dx+c\right)\right)\log\left(a\cos\left(dx+c\right)+b\right)+\left(a^{6}b+a^{5}b^{2}-3\,a^{4}b^{3}-5\,a^{3}b^{4}-2\,a^{2}b^{5}-\left(a^{7}+a^{6}b-3\,a^{5}b^{2}-5\,a^{4}b^{3}-2\,a^{3}b^{4}\right)\cos\left(dx+c\right)^{3}-\left(a^{6}b+a^{5}b^{2}-3\,a^{4}b^{3}-5\,a^{3}b^{4}-2\,a^{2}b^{5}\right)\cos\left(dx+c\right)^{2}+\left(a^{7}+a^{6}b-3\,a^{5}b^{2}-5\,a^{4}b^{3}-2\,a^{3}b^{4}\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^{6}b-a^{5}b^{2}-3\,a^{4}b^{3}+5\,a^{3}b^{4}-2\,a^{2}b^{5}-\left(a^{7}-a^{6}b-3\,a^{5}b^{2}+5\,a^{4}b^{3}-2\,a^{3}b^{4}\right)\cos\left(dx+c\right)^{3}-\left(a^{6}b-a^{5}b^{2}-3\,a^{4}b^{3}+5\,a^{3}b^{4}-2\,a^{2}b^{5}\right)\cos\left(dx+c\right)^{2}+\left(a^{7}-a^{6}b-3\,a^{5}b^{2}+5\,a^{4}b^{3}-2\,a^{3}b^{4}\right)\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(\left(a^{9}-3\,a^{7}b^{2}+3\,a^{5}b^{4}-a^{3}b^{6}\right)d\cos\left(dx+c\right)^{3}+\left(a^{8}b-3\,a^{6}b^{3}+3\,a^{4}b^{5}-a^{2}b^{7}\right)d\cos\left(dx+c\right)^{2}-\left(a^{9}-3\,a^{7}b^{2}+3\,a^{5}b^{4}-a^{3}b^{6}\right)d\cos\left(dx+c\right)-\left(a^{8}b-3\,a^{6}b^{3}+3\,a^{4}b^{5}-a^{2}b^{7}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^6*b + a^2*b^5 - 2*b^7 - 2*(a^6*b - a^4*b^3 + a^2*b^5 - b^7)*cos(d*x
+ c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c) + 2*(5*a^2*b^5 - b^7 - (5
*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (5*a^2*b^5 - b^7)*cos(d*x + c)^2 + (5*a^
3*b^4 - a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (a^6*b + a^5*b^2 - 3
*a^4*b^3 - 5*a^3*b^4 - 2*a^2*b^5 - (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2
*a^3*b^4)*cos(d*x + c)^3 - (a^6*b + a^5*b^2 - 3*a^4*b^3 - 5*a^3*b^4 - 2*a^2
*b^5)*cos(d*x + c)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4)*co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*
a^3*b^4 - 2*a^2*b^5 - (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos
(d*x + c)^3 - (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*a^3*b^4 - 2*a^2*b^5)*cos(d*x
 + c)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos
(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 -
(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(187) = 374.
time = 0.54, size = 656, normalized size = 3.33

$$-\frac{\frac{4\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{8\,(5\,a^2b^4-b^6)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6}-\frac{a^5-a^4b-a^3b^2+a^2b^3+\frac{3\,a^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^4b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^3b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20\,ab^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,a^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4\,a^4b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{2\,a^3b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{12\,a^2b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4\,ab^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4\,b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{(a^6-2\,a^4b^2+a^2b^4)\left(\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{\cos(dx+c)-1}{(a^2-2\,ab+b^2)(\cos(dx+c)+1)}-\frac{8\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) + 8*(5*a^2*b^4 - b^6)*log(abs(-a - b - a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - (a^5 - a^4*b - a^3*b^2 + a^2*b^3 +
3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 3*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*
b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a*b^4*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*a^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 2*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 12*a^
2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b^4*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) -
1)/((a^2 - 2*a*b + b^2)*(cos(d*x + c) + 1)) - 8*log(abs(-(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/a^2)/d
```

Mupad [B]
time = 2.48, size = 313, normalized size = 1.59

$$\frac{\frac{a^2-2\,a\,b+b^2}{2\,(a+b)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,(a^5-4\,a^4\,b+6\,a^3\,b^2-4\,a^2\,b^3+a\,b^4-16\,b^5)}{2\,a\,{(a+b)}^2\,(a-b)}}{d\,\left((4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+(-4\,a^3+4\,a^2\,b+4\,a\,b^2-4\,b^3)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d\,{(a-b)}^2}+\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(a+2\,b)}{d\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}-\frac{b^4\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)\,(5\,a^2-b^2)}{a^2\,d\,{(a^2-b^2)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(2*(a + b)) - (tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^4*b +
 a^5 - 16*b^5 - 4*a^2*b^3 + 6*a^3*b^2))/(2*a*(a + b)^2*(a - b)))/(d*(tan(c/
2 + (d*x)/2)^2*(4*a*b^2 + 4*a^2*b - 4*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^4*(
12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^
2) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2))*(a +
2*b))/(d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^4*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(5*a^2 - b^2))/(a^2*d*(a^2 - b^2)^3)
```

3.306 $\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=278

$$\frac{\log(\cos(c+dx))}{a^2d}+\frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8(a+b)^4d}+\frac{(4a^2-13ab+12b^2)\log(1+\sec(c+dx))}{8(a-b)^4d}-\frac{b^6(}{}$$

```
[Out] ln(cos(d*x+c))/a^2/d+1/8*(4*a^2+13*a*b+12*b^2)*ln(1-sec(d*x+c))/(a+b)^4/d+1
/8*(4*a^2-13*a*b+12*b^2)*ln(1+sec(d*x+c))/(a-b)^4/d-b^6*(7*a^2-b^2)*ln(a+b*
sec(d*x+c))/a^2/(a^2-b^2)^4/d-1/16/(a+b)^2/d/(1-sec(d*x+c))^2+1/16*(-5*a-9*
b)/(a+b)^3/d/(1-sec(d*x+c))-1/16/(a-b)^2/d/(1+sec(d*x+c))^2+1/16*(-5*a+9*b)
/(a-b)^3/d/(1+sec(d*x+c))+b^6/a/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8d(a+b)^4}+\frac{(4a^2-13ab+12b^2)\log(\sec(c+dx)+1)}{8d(a-b)^4}+\frac{b^6}{ad\left(a^2-b^2\right)^3\left(a+b\sec(c+dx)\right)}-\frac{b^6(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^4}+\frac{\log(\cos(c+dx))}{a^2d}-\frac{5a+9b}{16d(a+b)^3(1-\sec(c+dx))}-\frac{5a-9b}{16d(a-b)^3(\sec(c+dx)+1)}-\frac{1}{16d(a+b)^2(1-\sec(c+dx))^2}-\frac{1}{16d(a-b)^2(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((4*a^2 + 13*a*b + 12*b^2)*Log[1 - Sec[c + d*x]
])/(8*(a + b)^4*d) + ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Sec[c + d*x]])/(8*(
a - b)^4*d) - (b^6*(7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^
4*d) - 1/(16*(a + b)^2*d*(1 - Sec[c + d*x])^2) - (5*a + 9*b)/(16*(a + b)^3*
d*(1 - Sec[c + d*x])) - 1/(16*(a - b)^2*d*(1 + Sec[c + d*x])^2) - (5*a - 9*
b)/(16*(a - b)^3*d*(1 + Sec[c + d*x])) + b^6/(a*(a^2 - b^2)^3*d*(a + b*Sec[
c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^6\mathrm{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\mathrm{Subst}\left(\int \left(\frac{1}{8b^4(a+b)^2(b-x)^3} + \frac{5a+9b}{16b^5(a+b)^3(b-x)^2} + \frac{4a^2+13ab+12b^2}{8b^6(a+b)^4(b-x)} + \frac{1}{a^2b^6x} + \frac{1}{a(a-b)^3(a+\cdots}\right.\right.}{\cdots}$$

$$= \frac{\log(\cos(c+dx))}{a^2d} + \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8(a+b)^4d} + \frac{(4a^2-13ab+1\cdots}{8\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.15, size = 473, normalized size = 1.70

$$\frac{(b+a\cos(c+dx))\left(\frac{64b^7}{a^2(-a+b)^3(a+b)^3} + \frac{128i(a^6-4a^4b^2+6a^2b^4+3b^6)(c+dx)(b+a\cos(c+dx))}{(a-b)^4(a+b)^4} - \frac{16i(4a^2-13ab+12b^2)\mathrm{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))}{(a-b)^4} - \frac{16i(4a^2+13ab+12b^2)\mathrm{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))}{(a+b)^4} + \frac{2(7a+11b)(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} - \frac{(b+a\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{8(4a^2-13ab+12b^2)(b+a\cos(c+dx))\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^4} + \frac{64(-7a^2b^6+b^8)(b+a\cos(c+dx))\log(b+a\cos(c+dx))}{a^2(a^2-b^2)^4} + \frac{8(4a^2+13ab+12b^2)(b+a\cos(c+dx))\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^4} + \frac{2(7a-11b)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3} - \frac{(b+a\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)\sec^2(c+dx)}{64d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*b^7)/(a^2*(-a + b)^3*(a + b)^3) + ((128*I)*(a^6
- 4*a^4*b^2 + 6*a^2*b^4 + 3*b^6)*(c + d*x)*(b + a*Cos[c + d*x]))/((a - b)^4
*(a + b)^4) - ((16*I)*(4*a^2 - 13*a*b + 12*b^2)*ArcTan[Tan[c + d*x]]*(b + a
*Cos[c + d*x]))/(a - b)^4 - ((16*I)*(4*a^2 + 13*a*b + 12*b^2)*ArcTan[Tan[c
+ d*x]]*(b + a*Cos[c + d*x]))/(a + b)^4 + (2*(7*a + 11*b)*(b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^
4)/(a + b)^2 + (8*(4*a^2 - 13*a*b + 12*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c
 + d*x)/2]^2])/(a - b)^4 + (64*(-7*a^2*b^6 + b^8)*(b + a*Cos[c + d*x])*Log[
b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)^4) + (8*(4*a^2 + 13*a*b + 12*b^2)*(b
+ a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]^2])/(a + b)^4 + (2*(7*a - 11*b)*(b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 - ((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 230, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{b^7}{a^2(a+b)^3(a-b)^3(b+a\cos(dx+c))} - \frac{b^6\left(7a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4a^2} - \frac{1}{16(a+b)^2(-1+\cos(dx+c))^2} - \frac{7a+11b}{16(a+b)^3(-1+\cos(dx+c))} + \frac{(4a^{\cdots}}{\cdots}}{d}$
default	$\frac{-\frac{b^7}{a^2(a+b)^3(a-b)^3(b+a\cos(dx+c))} - \frac{b^6\left(7a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4a^2} - \frac{1}{16(a+b)^2(-1+\cos(dx+c))^2} - \frac{7a+11b}{16(a+b)^3(-1+\cos(dx+c))} + \frac{(4a^{\cdots}}{\cdots}}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^7/a^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-b^6*(7*a^2-b^2)/(a+b)^4/(a-b
)^4/a^2*ln(b+a*cos(d*x+c))-1/16/(a+b)^2/(-1+cos(d*x+c))^2-1/16*(7*a+11*b)/(
a+b)^3/(-1+cos(d*x+c))+1/8*(4*a^2+13*a*b+12*b^2)/(a+b)^4*ln(-1+cos(d*x+c))-
1/16/(a-b)^2/(1+cos(d*x+c))^2-1/16*(-7*a+11*b)/(a-b)^3/(1+cos(d*x+c))+1/8*(
4*a^2-13*a*b+12*b^2)/(a-b)^4*ln(1+cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(262) = 524$.
time = 0.31, size = 558, normalized size = 2.01

$$-\frac{\frac{8\left(7a^2b^6-b^8\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^{10}-4a^8b^2+6a^6b^4-4a^4b^6+a^2b^8}-\frac{\left(4a^2-13ab+12b^2\right)\log\left(\cos\left(dx+c\right)+1\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{\left(4a^2+13ab+12b^2\right)\log\left(\cos\left(dx+c\right)-1\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{2\left(3a^6b-6a^4b^3-5a^2b^5-4b^7+\left(5a^6b-13a^4b^3-4b^7\right)\cos\left(dx+c\right)^4-\left(4a^7-11a^5b^2+7a^3b^4\right)\cos\left(dx+c\right)^3-\left(7a^6b-17a^4b^3-6a^2b^5-8b^7\right)\cos\left(dx+c\right)^2+3\left(a^7-3a^5b^2+2a^3b^4\right)\cos\left(dx+c\right)\right)}{a^8b-3a^6b^3+3a^4b^5-a^2b^7+\left(a^9-3a^7b^2+3a^5b^4-a^3b^6\right)\cos\left(dx+c\right)^5+\left(a^8b-3a^6b^3+3a^4b^5-a^2b^7\right)\cos\left(dx+c\right)^4-2\left(a^9-3a^7b^2+3a^5b^4-a^3b^6\right)\cos\left(dx+c\right)^3-2\left(a^8b-3a^6b^3+3a^4b^5-a^2b^7\right)\cos\left(dx+c\right)^2+\left(a^9-3a^7b^2+3a^5b^4-a^3b^6\right)\cos\left(dx+c\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(7*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^10 - 4*a^8*b^2 + 6*a^6
*b^4 - 4*a^4*b^6 + a^2*b^8) - (4*a^2 - 13*a*b + 12*b^2)*log(cos(d*x + c) +
1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + 13*a*b + 12*b^2)*
log(cos(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^
6*b - 6*a^4*b^3 - 5*a^2*b^5 - 4*b^7 + (5*a^6*b - 13*a^4*b^3 - 4*b^7)*cos(d*
x + c)^4 - (4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^3 - (7*a^6*b - 17*
a^4*b^3 - 6*a^2*b^5 - 8*b^7)*cos(d*x + c)^2 + 3*(a^7 - 3*a^5*b^2 + 2*a^3*b^
4)*cos(d*x + c))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 + (a^9 - 3*a^7*b^
2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^5 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
a^2*b^7)*cos(d*x + c)^4 - 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x
 + c)^3 - 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^9
 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(262) = 524$.
time = 4.11, size = 1378, normalized size = 4.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^8*b - 18*a^6*b^3 + 2*a^4*b^5 + 2*a^2*b^7 + 8*b^9 + 2*(5*a^8*b - 18
*a^6*b^3 + 13*a^4*b^5 - 4*a^2*b^7 + 4*b^9)*cos(d*x + c)^4 - 2*(4*a^9 - 15*a
^7*b^2 + 18*a^5*b^4 - 7*a^3*b^6)*cos(d*x + c)^3 - 2*(7*a^8*b - 24*a^6*b^3 +
```

```
11*a^4*b^5 - 2*a^2*b^7 + 8*b^9)*cos(d*x + c)^2 + 6*(a^9 - 4*a^7*b^2 + 5*a^
5*b^4 - 2*a^3*b^6)*cos(d*x + c) - 8*(7*a^2*b^7 - b^9 + (7*a^3*b^6 - a*b^8)*
cos(d*x + c)^5 + (7*a^2*b^7 - b^9)*cos(d*x + c)^4 - 2*(7*a^3*b^6 - a*b^8)*c
os(d*x + c)^3 - 2*(7*a^2*b^7 - b^9)*cos(d*x + c)^2 + (7*a^3*b^6 - a*b^8)*co
s(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 1
4*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7 + (4*a^9 + 3*a^8*b - 16*a^
7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^5 +
 (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 +
 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3
+ 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^3 - 2*(4*a^8*b + 3*a^7
*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7)*cos(
d*x + c)^2 + (4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a
^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (4*a^8*b -
 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7
 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 1
2*a^3*b^6)*cos(d*x + c)^5 + (4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4
+ 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 - 3*a^8*b
 - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x
+ c)^3 - 2*(4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35
*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^2 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*
a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*co
s(d*x + c)^5 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^4 - 2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(
d*x + c)^3 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^2 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*
x + c) + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(262) = 524.
time = 0.59, size = 795, normalized size = 2.86

$$\frac{\frac{8\,(4\,a^2+13\,ab+12\,b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{64\,(7\,a^2b^6-b^8)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8}-\frac{\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{32\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{2\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{\left(a^2+2\,ab+b^2+\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{32\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{156\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{144\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)^2}+\frac{64\left(7\,a^3b^6+5\,a^2b^7-3\,ab^8-b^9+\frac{7\,a^3b^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{7\,a^2b^7(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{ab^8(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^9(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}-\frac{64\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(4*a^2 + 13*a*b + 12*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 64*(7*a^2*b^6 - b^8)
*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*
b^8) - (12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 32*a*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a
^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 156*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 144*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^
2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) + 64*(
7*a^3*b^6 + 5*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^3*b^6*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 7*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^8*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x + c) - 1)/(cos(d*x + c) +
1))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a + b + a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))
 - 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

Mupad [B]
time = 2.97, size = 471, normalized size = 1.69

$$\frac{\frac{a^3-3a^2b+3ab^2-b^3}{4(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-13a^4+20a^3b+18a^2b^2-44ab^3+19b^4\right)}{4(a+b)^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(3a^7-10a^6b+5a^5b^2+20a^4b^3-35a^3b^4+22a^2b^5-5ab^6+32b^7\right)}{a(a+b)^3(a-b)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(16a^4-64a^3b+96a^2b^2-64ab^3+16b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16a^4-32a^3b+32ab^3-16b^4\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d(a-b)^2}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{16a^2+32ab-48b^2}{512(a-b)^4}-\frac{7}{32(a-b)^2}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(4a^2+13ab+12b^2\right)}{d\left(4a^4+16a^3b+24a^2b^2+16ab^3+4b^4\right)}-\frac{b^6\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(7a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(4*(a + b)) + (tan(c/2 + (d*x)/2)^2*(20*a^
3*b - 44*a*b^3 - 13*a^4 + 19*b^4 + 18*a^2*b^2))/(4*(a + b)^2) + (tan(c/2 +
(d*x)/2)^4*(3*a^7 - 10*a^6*b - 5*a*b^6 + 32*b^7 + 22*a^2*b^5 - 35*a^3*b^4 +
 20*a^4*b^3 + 5*a^5*b^2))/(a*(a + b)^3*(a - b)))/(d*(tan(c/2 + (d*x)/2)^6*(
16*a^4 - 64*a^3*b - 64*a*b^3 + 16*b^4 + 96*a^2*b^2) - tan(c/2 + (d*x)/2)^4*
(32*a*b^3 - 32*a^3*b + 16*a^4 - 16*b^4))) - tan(c/2 + (d*x)/2)^4/(64*d*(a -
 b)^2) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (tan(c/2 + (d*x)/2)^2*((32
*a*b + 16*a^2 - 48*b^2)/(512*(a - b)^4) - 7/(32*(a - b)^2)))/d + (log(tan(c
/2 + (d*x)/2))*(13*a*b + 4*a^2 + 12*b^2))/(d*(16*a*b^3 + 16*a^3*b + 4*a^4 +
 4*b^4 + 24*a^2*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2
 + (d*x)/2)^2)*(7*a^2 - b^2))/(a^2*d*(a^2 - b^2)^4)
```

3.307 $\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{x}{a^2}-\frac{a(4a^2-5b^2)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{2(a-b)^{3/2}(a+b)^{3/2}(4a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d}+\frac{(a}{ab^4}$$

```
[Out] -x/a^2-a*(4*a^2-5*b^2)*arctanh(sin(d*x+c))/b^5/d+2*(a-b)^(3/2)*(a+b)^(3/2)*
(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/b^5/d+(
a^2-b^2)^2*sin(d*x+c)/a/b^4/d/(b+a*cos(d*x+c))+(3*a^2-2*b^2)*tan(d*x+c)/b^4
/d-a*sec(d*x+c)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

Rubi [A]
time = 0.31, antiderivative size = 283, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3983, 2976, 2743, 12, 2738, 214, 3855, 3852, 8, 3853}

$$-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}-\frac{2a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{4(a-b)^{3/2}(a+b)^{3/2}(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d}+\frac{3(a^2-b^2)\tan(c+dx)}{b^4d}+\frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(a\cos(c+dx)+b)}-\frac{x}{a^2}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)\sec(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (a*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(2*a^2 - 3*b^2)*ArcTanh
[Sin[c + d*x]])/(b^5*d) - (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d) + (4*(a - b)^(3/2)*(a + b)^(
3/2)*(2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*b^5*d) + ((a^2 - b^2)^2*Sin[c + d*x])/(a*b^4*d*(b + a*Cos[c + d*x])) + Ta
n[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Tan[c + d*x])/(b^4*d) - (a*Sec[c + d*x]
*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} + \frac{(a^2-b^2)^3}{a^2b^4(b+a\cos(c+dx))^2} + \frac{2(2a^6-3a^4b^2+b^6)}{a^2b^5(b+a\cos(c+dx))} + \frac{2(-2a^3+3ab}{b}\right.$$

$$= -\frac{x}{a^2} - \frac{(2a)\int \sec^3(c+dx)\,dx}{b^3} + \frac{\int \sec^4(c+dx)\,dx}{b^2} - \frac{(2a(2a^2-3b^2))\int \sec(c+}{b^5}$$

$$= -\frac{x}{a^2} - \frac{2a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))} - \frac{a\sec(c}{}$$

$$= -\frac{x}{a^2} - \frac{a\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{4(a-b)^3}{}$$

$$= -\frac{x}{a^2} - \frac{a\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{4(a-b)^3}{}$$

$$= -\frac{x}{a^2} - \frac{a\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{2(a-b)^3}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 865 vs. $2(200) = 400$.
time = 6.26, size = 865, normalized size = 4.32

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(a^2*d*(a + b*Sec[c + d
*x])^2)) - (2*(-a^2 + b^2)^2*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(a^2*b^5*Sqrt[a
^2 - b^2]*d*(a + b*Sec[c + d*x])^2) + ((4*a^3 - 5*a*b^2)*(b + a*Cos[c + d*x
])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2)/(b^5*d*(a + b
*Sec[c + d*x])^2) + ((-4*a^3 + 5*a*b^2)*(b + a*Cos[c + d*x])^2*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2)/(b^5*d*(a + b*Sec[c + d*x])^2)
 + ((-6*a + b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(12*b^3*d*(a + b*Sec[
c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + ((b + a*Cos[c + d*x]
)^2*Sec[c + d*x]^2*Sin[(c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])^2*(Cos[(
```

```
c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2
*Sin[(c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3) + ((6*a - b)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^2)/(12*b^
3*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^2*(9*a^2*Sin[(c + d*x)/2] - 7*b^2*Sin[(c +
d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(9*a^2*Sin[(c + d*x)/2] - 7*
b^2*Sin[(c + d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])^2*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(a^4*Sin[c + d*x]
 - 2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(a*b^4*d*(a + b*Sec[c + d*x]
)^2)
```

Maple [A]
time = 0.25, size = 365, normalized size = 1.82 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^2/b^5*((a^5*b-2*a^3*b^3+a*b^5)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+
1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(4*a^6-7*a^4*b^2+2*a^2*b^4+b^6)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/3/b^
2/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-2*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)^2-(3*a^2
+a*b-2*b^2)/b^4/(tan(1/2*d*x+1/2*c)+1)-a*(4*a^2-5*b^2)/b^5*ln(tan(1/2*d*x+1
/2*c)+1)-2/a^2*arctan(tan(1/2*d*x+1/2*c))-1/3/b^2/(tan(1/2*d*x+1/2*c)-1)^3-
1/2*(2*a+b)/b^3/(tan(1/2*d*x+1/2*c)-1)^2-(3*a^2+a*b-2*b^2)/b^4/(tan(1/2*d*x
+1/2*c)-1)+a*(4*a^2-5*b^2)/b^5*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(189) = 378.
time = 2.58, size = 843, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 + 3*((4*a^5 -
3*a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c
)^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*
x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*
((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) + 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b -
 13*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(a^3*b^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3), -
1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 - 6*((4*a^5 - 3*
a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c)^
3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a
^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((4*a^6 - 5*a^4*b^2)*cos(
d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) +
 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b - 13*a^3*b^3 + 3*a*b^5)*co
s(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^3*b
^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(189) = 378.
time = 2.31, size = 411, normalized size = 2.06

$$-\frac{\frac{3(dx+c)}{a^2}+\frac{3(4a^3-5ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^5}-\frac{3(4a^3-5ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^5}+\frac{6\left(a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)ab^4}-\frac{6(4a^6-7a^4b^2+2a^2b^4+b^6)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^2b^5}+\frac{2\left(9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+16b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3b^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 + 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 - 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*(a
^4*tan(1/2*d*x + 1/2*c) - 2*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x
+ 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a*
```

```
b^4) - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^5) + 2*(9*a^2*tan(1/2*d*x
 + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*b^2*tan(1/2*d*x + 1/2*c)^5 -
 18*a^2*tan(1/2*d*x + 1/2*c)^3 + 16*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(
1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) - 6*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^4))/d
```

Mupad [B]
time = 4.82, size = 2500, normalized size = 12.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5*(10*a*b^3 - 6*a^3*b + 36*a^4 + 9*b^4 - 37*a^2*b^2)
)/(3*a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(6*a^3*b - 10*a*b^3 + 36*a^4 + 9*b^4
- 37*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(2*a^3*b - 2*a*b^3 + 4*a^4
 + b^4 - 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^7*(a - b)*(3*a*b^2 - 2
*a^2*b - 4*a^3 + b^3))/(a*b^4))/(d*(a + b + tan(c/2 + (d*x)/2)^8*(a - b) -
tan(c/2 + (d*x)/2)^2*(4*a + 2*b) - tan(c/2 + (d*x)/2)^6*(4*a - 2*b) + 6*a*t
an(c/2 + (d*x)/2)^4)) - (2*atan((((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13
*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*
b^15 - 12*a^12*b^14))/(a^3*b^16) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*
b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^1
6))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^
21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 -
60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11
 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(
4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 +
74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 +
 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*
b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 - (8
192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19
+ 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 58
8*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 +
2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b
^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))/a^2 - ((((((((8192*(9*a^5*
b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 +
3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (tan(c/2 + (d*x)/2
)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11
*b^18)*8192i)/(a^6*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 -
6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*
b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^1
```

3*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))/a^2)/(((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))*1i)/a^2 + (((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))...

3.308 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{x}{a^2}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}+\frac{(2a^2-b^2)\sin(c}{ab^2d(b+a\cos(c}$$

```
[Out] x/a^2-2*a*arctanh(sin(d*x+c))/b^3/d+(2*a^2-b^2)*sin(d*x+c)/a/b^2/d/(b+a*cos
(d*x+c))+2*(2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*
(a-b)^(1/2)*(a+b)^(1/2)/a^2/b^3/d+tan(d*x+c)/b/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3983, 2969, 3136, 2738, 214, 3855}

$$\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}+\frac{x}{a^2}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*Sqrt[a - b]*Sqrt[a + b]*(2
*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d
) + ((2*a^2 - b^2)*Sin[c + d*x])/(a*b^2*d*(b + a*Cos[c + d*x])) + Tan[c + d
*x]/(b*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
```

```
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx \\
&= \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} + \frac{\int \frac{(-2a^2-ab\cos(c+dx)+b^2\cos^2(c+\cdots}{b+a\cos(c+dx)}}{ab^2} \\
&= \frac{x}{a^2} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} - \frac{(2a)\int \sec(c+dx)\,dx}{b^3} \cdots \\
&= \frac{x}{a^2} - \frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx\cdots} \\
&= \frac{x}{a^2} - \frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{2\sqrt{a-b}\,\sqrt{a+b}\,(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\cdots}{\sqrt{a\cdots}}\right.}{a^2b^3d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(150) = 300.
time = 1.70, size = 327, normalized size = 2.18

$$\frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{(c+dx)(b+a\cos(c+dx))}{a^2}+\frac{2(-2a^4+a^2b^2+b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{a^2b^3\sqrt{a^2-b^2}}+\frac{2a(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^3}-\frac{2a(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^3}+\frac{(b+a\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)}{b^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{(b+a\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)}{b^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{(a^2-b^2)\sin(c+dx)}{ab^2}\right)}{d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(((c + d*x)*(b + a*Cos[c + d*x]))/a^2
+ (2*(-2*a^4 + a^2*b^2 + b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^3*Sqrt[a^2 - b^2]) + (2*a*(b + a*Cos[c
 + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 - (2*a*(b + a*Cos[c
+ d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3 + ((b + a*Cos[c + d*x
])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b + a*
Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
 + ((a^2 - b^2)*Sin[c + d*x])/(a*b^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.18, size = 219, normalized size = 1.46

method	result

derivativedivides	$-\frac{\frac{2\left(\frac{\left(b\,a^3-b^3a\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^4-b^2a^2-b^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2b^3}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$
default	$-\frac{\frac{2\left(\frac{\left(b\,a^3-b^3a\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^4-b^2a^2-b^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2b^3}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$
risch	$\frac{x}{a^2}+\frac{2i\left(b\,a^2\mathrm{e}^{3i(dx+c)}-b^3\mathrm{e}^{3i(dx+c)}+2a^3\mathrm{e}^{2i(dx+c)}-b^2a\,\mathrm{e}^{2i(dx+c)}+3a^2b\,\mathrm{e}^{i(dx+c)}-b^3\mathrm{e}^{i(dx+c)}+2a^3-b^2a\right)}{a^2b^2d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2a\ln\left(\mathrm{e}^{i(dx-\right.}}{b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^2/b^3*((a^3*b-a*b^3)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b
*tan(1/2*d*x+1/2*c)^2-a-b)-(2*a^4-a^2*b^2-b^4)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/b^2/(tan(1/2*d*x+1/2*c)+1)
-2*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2/a^2*arctan(tan(1/2*d*x+1/2*c))-1/b^2/(t
an(1/2*d*x+1/2*c)-1)+2*a/b^3*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.99, size = 584, normalized size = 3.89

$$\left[\frac{2\,ab^3dx\cos(dx+c)^2+2\,b^4dx\cos(dx+c)+\left(\left(2\,a^3+ab^2\right)\cos(dx+c)^2+\left(2\,a^2b+b^3\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\left(b\cos(dx+c)+a\right)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\left(a^4\cos(dx+c)^2+a^3b\cos(dx+c)\right)\log\left(\sin(dx+c)+1\right)+2\left(a^4\cos(dx+c)^2+a^3b\cos(dx+c)\right)\log\left(-\sin(dx+c)+1\right)+2\left(a^2b^2+\left(2\,a^3b-ab^3\right)\cos(dx+c)\right)\sin(dx+c)}{2\left(a^3b^3d\cos(dx+c)^2+a^2b^4d\cos(dx+c)\right)},\ \frac{ab^3dx\cos(dx+c)^2+b^4dx\cos(dx+c)+\left(\left(2\,a^3+ab^2\right)\cos(dx+c)^2+\left(2\,a^2b+b^3\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\cos(dx+c)+a\right)}{\left(a^2-b^2\right)\sin(dx+c)}\right)-\left(a^4\cos(dx+c)^2+a^3b\cos(dx+c)\right)\log\left(\sin(dx+c)+1\right)+\left(a^4\cos(dx+c)^2+a^3b\cos(dx+c)\right)\log\left(-\sin(dx+c)+1\right)+\left(a^2b^2+\left(2\,a^3b-ab^3\right)\cos(dx+c)\right)\sin(dx+c)}{a^3b^3d\cos(dx+c)^2+a^2b^4d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^3*d*x*cos(d*x + c)^2 + 2*b^4*d*x*cos(d*x + c) + ((2*a^3 + a*b^2
)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x + c)
 + 1) + 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) + 1)
+ 2*(a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos
(d*x + c)^2 + a^2*b^4*d*cos(d*x + c)), (a*b^3*d*x*cos(d*x + c)^2 + b^4*d*x*
cos(d*x + c) + ((2*a^3 + a*b^2)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x
 + c) + 1) + (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) +
1) + (a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*co
s(d*x + c)^2 + a^2*b^4*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(141) = 282.
time = 1.13, size = 294, normalized size = 1.96

$$\frac{\frac{dx+c}{a^2}-\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}+\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^3}-\frac{2\left(2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)ab^2}+\frac{2\left(2\,a^4-a^2b^2-b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^2b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + 2*a*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a^2*tan(1/2*d*x + 1/2*c)^3 - a*b*tan(1
/2*d*x + 1/2*c)^3 - b^2*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*tan(1/2*d*x + 1/2*c)
 - a*b*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*a*
b^2) + 2*(2*a^4 - a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a
^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^3))/d
```

Mupad [B]
time = 3.47, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^2,x)

[Out] (2*atan((((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2 - ((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2)/((((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*1i)/a^2 + ((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3

```
*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14
*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 - (
8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 -
 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5
 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*1i)/a^2 - (16384*(2*a
*b^9 - 16*a^9*b + 16*a^10 - 2*b^10 - 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 +
 26*a^5*b^5 + 12*a^6*b^4 - 36*a^7*b^3 + 8*a^8*b^2))/(a^3*b^8))))/(a^2*d) +
((2*tan(c/2 + (d*x)/2)*(a*b + 2*a^2 - b^2))/(a*b^2) - (2*tan(c/2 + (d*x)/2)
^3*(a - b)*(2*a + b))/(a*b^2))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2
*a*tan(c/2 + (d*x)/2)^2)) + (atan((((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)
^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((
(2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/
2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^
6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (8
192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*
b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^
11*b^10))/(a^4*b^8)) - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 +
10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8
- 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192
*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 1
8*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^
3 + 24*a^12*b^2))/(a^3*b^8)) + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12
 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a
^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/
(a^4*b^8))*1i - ((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((
2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a...
```

3.309 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{x}{a^2}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}\,d}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

```
[Out] -x/a^2+2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)^
(1/2)/(a+b)^(1/2)+tan(d*x+c)/a/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3979, 4146, 12, 3868, 2738, 214}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) + Tan[c + d*x]/(a*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
```

```
] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx\\
&= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}-\frac{\int \frac{a^2-b^2}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}-\frac{\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}\\
&= -\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}+\frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}\\
&= -\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}+\frac{2\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}\\
&= -\frac{x}{a^2}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 80, normalized size = 0.94

$$-\frac{c+dx+\frac{2b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{a\sin(c+dx)}{b+a\cos(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((c + d*x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqr
t[a^2 - b^2] - (a*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*d))
```

Maple [A]
time = 0.13, size = 115, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}+\frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}+\frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$-\frac{x}{a^2}+\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{a^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}-\frac{b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}+\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b}{a\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^2*(-a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2
*c)^2-a-b)+b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

time = 1.90, size = 383, normalized size = 4.51

$$\left[-\frac{2\,(a^3-ab^2)dx\cos\left(dx+c\right)+2\,(a^2b-b^3)dx-(ab\cos\left(dx+c\right)+b^2)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\,(a^3-ab^2)\sin\left(dx+c\right)}{2\,((a^5-a^3b^2)d\cos\left(dx+c\right)+(a^4b-a^2b^3)d)},-\frac{(a^3-ab^2)dx\cos\left(dx+c\right)+(a^2b-b^3)dx-(ab\cos\left(dx+c\right)+b^2)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^3-ab^2)\sin\left(dx+c\right)}{(a^5-a^3b^2)d\cos\left(dx+c\right)+(a^4b-a^2b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*d*x*cos(d*x + c) + 2*(a^2*b - b^3)*d*x - (a*b*cos(d*
x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^3 - a*b^2)*sin(
d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d), -((a^3 -
a*b^2)*d*x*cos(d*x + c) + (a^2*b - b^3)*d*x - (a*b*cos(d*x + c) + b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) +
 (a^4*b - a^2*b^3)*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]

time = 0.65, size = 144, normalized size = 1.69

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a^2}-\frac{dx+c}{a^2}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2
)*a^2) - (d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a))/d
```

Mupad [B]

time = 2.14, size = 551, normalized size = 6.48

$$-\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d}-\frac{b^2\,\left(a\,\sin\left(c+d\,x\right)+2\,\mathrm{atanh}\left(\frac{2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2-b^2\right)}^{3/2}-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+2\,b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}-3\,a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+a^3\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+a^4\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a\,b^2-a^3\right)\,\left(b\,\left(a^2-b^2\right)+a\,b^2-a^2\,b-a^3+b^3\right)}\right)\,\sqrt{a^2-b^2}\right)-a^3\,\sin\left(c+d\,x\right)+2\,a\,b\,\cos\left(c+d\,x\right)\,\mathrm{atanh}\left(\frac{2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2-b^2\right)}^{3/2}-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+2\,b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}-3\,a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+a^3\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+a^4\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a\,b^2-a^3\right)\,\left(b\,\left(a^2-b^2\right)+a\,b^2-a^2\,b-a^3+b^3\right)}\right)\,\sqrt{a^2-b^2}}{a^2\,d\,\left(a^2-b^2\right)\,\left(b+a\,\cos\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] - (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - (b^2*(a*sin(c +
 d*x) + 2*atanh((2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 +
 (d*x)/2)*(a^2 - b^2)^(1/2) + 2*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) -
3*a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 +
 (d*x)/2)*(a*b^2 - a^3)*(b*(a^2 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2
- b^2)^(1/2)) - a^3*sin(c + d*x) + 2*a*b*cos(c + d*x)*atanh((2*b^3*sin(c/2
+ (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2
*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^2*b^3*sin(c/2 + (d*x)/2)*(a
^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^4*b*sin(
c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(b*(a^2
 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2 - b^2)^(1/2))/(a^2*d*(a^2 - b^2
)*(b + a*cos(c + d*x)))
```

3.310 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$-\frac{x}{a^2}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}-\frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx}{2(a+b)^2d(1-\cos}$$

```
[Out] -x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^
(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2-b^2
)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.31, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3983, 2976, 2727, 2743, 12, 2738, 214}

$$-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} + \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{}{a^2\,(a}\right.$$

$$= -\frac{x}{a^2} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{b^4\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\,(a^2-b^2)} + \frac{(2b^3(2a}{}$$

$$= -\frac{x}{a^2} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{}{a\,(a^2-b}$$

$$= -\frac{x}{a^2} - \frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+}$$

$$= -\frac{x}{a^2} - \frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+}$$

$$= -\frac{x}{a^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan}{\sqrt{a+}}\right.}{a^2(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A]
time = 1.82, size = 209, normalized size = 0.92

$$\frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-\frac{2(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{4b^3(-4a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{a^2(a^2-b^2)^{5/2}} - \frac{(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{2b^4\sin(c+dx)}{a(a-b)^2(a+b)^2} + \frac{(b+a\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)}{2d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-2*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(5/2)) - ((b + a*Cos[c + d*x])*
Cot[(c + d*x)/2])/(a + b)^2 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2)
+ ((b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b)^2))/(2*d*(a + b*Sec[c + d
*x])^2)
```

Maple [A]
time = 0.22, size = 184, normalized size = 0.81

method	result

derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ba+2b^2}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(4a^2-b^2)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)^2(a-b)^2a^2}-\frac{}{2(a+b)^2\,\mathrm{ta}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ba+2b^2}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(4a^2-b^2)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)^2(a-b)^2a^2}-\frac{}{2(a+b)^2\,\mathrm{ta}}}{d}$
risch	$-\frac{x}{a^2}-\frac{2i\left(-2a^4b\,e^{3i(dx+c)}-b^5e^{3i(dx+c)}+a^5e^{2i(dx+c)}-3a^3b^2e^{2i(dx+c)}-a\,b^4e^{2i(dx+c)}+2a^2b^3e^{i(dx+c)}+b^5e^{i(dx+c)}+a^5+\right.}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)(a^2-b^2)^2a^2\left(e^{2i(dx+c)}-1\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*b^3/(a+b)^2/(a-b)^2/a^2*(-a*b
*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(4*
a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)-2/a^2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.59, size = 705, normalized size = 3.11

$$\left[\frac{4a^5b^2-2a^3b^4-2ab^6-\left(4a^2b^4-b^6+(4a^3b^3-ab^5)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2(a^7-ab^6)\cos(dx+c)^2+2(a^6b-2a^4b^3+a^2b^5)\cos(dx+c)-2\left((a^7-3a^5b^2+3a^3b^4-ab^6)dx\cos(dx+c)+(a^6b-3a^4b^3+3a^2b^5-b^7)dx\right)\sin(dx+c)}{2\left((a^9-3a^7b^2+3a^5b^4-a^3b^6)d\cos(dx+c)+(a^8b-3a^6b^3+3a^4b^5-a^2b^7)d\right)\sin(dx+c)}, \frac{2a^5b^2-a^3b^4-ab^6-\left(4a^2b^4-b^6+(4a^3b^3-ab^5)\cos(dx+c)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-(a^7-ab^6)\cos(dx+c)^2+(a^6b-2a^4b^3+a^2b^5)\cos(dx+c)-\left((a^7-3a^5b^2+3a^3b^4-ab^6)dx\cos(dx+c)+(a^6b-3a^4b^3+3a^2b^5-b^7)dx\right)\sin(dx+c)}{\left((a^9-3a^7b^2+3a^5b^4-a^3b^6)d\cos(dx+c)+(a^8b-3a^6b^3+3a^4b^5-a^2b^7)d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(
a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
- 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.53, size = 332, normalized size = 1.46

$$-\frac{\frac{4\,(4\,a^2b^3-b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-2\,a^4b^2+a^2b^4)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4+a^3b+a^2b^2-ab^3}{(a^5-2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{2\,(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1
/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 +
 4*b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^
3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*(d*x + c)/a^2)/d
```

Mupad [B]
time = 6.46, size = 2500, normalized size = 11.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^2,x)

[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^3 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)/a^2 + (tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)/a^2)/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 + 1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11 + 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 - ((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16

- 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 822
4*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*
b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24
*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 +
 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a
^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b
^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688
*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/
2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20
- 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760
*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19
*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 +
5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2
)*1i)/a^2)*1i)/a^2)*1i)/a^2 + ((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 6
4*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 +
6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^
12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^
10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a
^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + ((32*a^28 - 32*a^27
*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11
*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
- 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880
*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 +
 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 576...

3.311 $\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=360

$$\frac{x}{a^2}-\frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d}-\frac{4b^5(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d}-\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c$$

```
[Out] x/a^2-2*b^7*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^(
7/2)/(a+b)^(7/2)/d-4*b^5*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))/a^2/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))^2-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/4*(3*a+5*b)*sin(d*x+
c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2-1/4*
(3*a-5*b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+
cos(d*x+c))+b^6*sin(d*x+c)/a/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.42, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3983, 2976, 2729, 2727, 2743, 12, 2738, 214}

$$-\frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^6\sin(c+dx)}{ad\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{4b^5(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+\frac{x}{a^2}+\frac{(3a+5b)\sin(c+dx)}{4d(a+b)^3(1-\cos(c+dx))}-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)}-\frac{(3a-5b)\sin(c+dx)}{4d(a-b)^3(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))^2}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*b^7*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a
 - b)^(7/2)*(a + b)^(7/2)*d) - (4*b^5*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c +
 d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])^2) - Sin[c + d*x]/(12*(a + b)^2*d*
(1 - Cos[c + d*x])) + ((3*a + 5*b)*Sin[c + d*x])/(4*(a + b)^3*d*(1 - Cos[c
+ d*x])) + Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) - ((3*a - 5*b
)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(12*(a -
b)^2*d*(1 + Cos[c + d*x])) + (b^6*Sin[c + d*x])/(a*(a^2 - b^2)^3*d*(b + a*C
os[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{1}{a^2} + \frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{3a-5b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{}{4(a-}\right. \\
&= \frac{x}{a^2} + \frac{(3a-5b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(3}{} \\
&= \frac{x}{a^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{}{12(a-b)} \\
&= \frac{x}{a^2} - \frac{4b^5(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx}{} \\
&= \frac{x}{a^2} - \frac{4b^5(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx}{} \\
&= \frac{x}{a^2} - \frac{2b^7\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b^5(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(\right.}{\sqrt{a+b}}\right.}{a^2(a-b)^{7/2}(a+b)^{7/2}d}
\end{aligned}
$$

Mathematica [A]
time = 2.57, size = 303, normalized size = 0.84

$$
\frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{24(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{48b^5(-6a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{a^2(a^2-b^2)^{7/2}} + \frac{4(4a+7b)(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} - \frac{(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{24b^6\sin(c+dx)}{a(a-b)^3(a+b)^3} + \frac{4(-4a+7b)(b+a\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3} + \frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)}{24d(a+b\sec(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((24*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (48*b^5*(-6*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(7/2)) + (4*(4*a + 7*b)*(b + a
*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((b + a*Cos[c + d*x])*Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*b^6*Sin[c + d*x])/(a*(a - b)^3
*(a + b)^3) + (4*(-4*a + 7*b)*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b
)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a - b)^2)
)/(24*d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.26, size = 260, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2-2ba+b^2\right)(a-b)}+\frac{2b^5\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(6a}{}\right.}{(a+b)^3(a-b)}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2-2ba+b^2\right)(a-b)}+\frac{2b^5\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(6a}{}\right.}{(a+b)^3(a-b)}}{d}$
risch	$\frac{x}{a^2}+\frac{2i\left(-53a^3b^4e^{4i(dx+c)}-9a\,b^6e^{4i(dx+c)}-14a^6b\,e^{3i(dx+c)}-9b^7e^{5i(dx+c)}+9b^7e^{3i(dx+c)}-2a^7e^{2i(dx+c)}+6a^7e^{6i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*b*tan(1/2*d*
x+1/2*c)^3-5*a*tan(1/2*d*x+1/2*c)+9*b*tan(1/2*d*x+1/2*c))+2*b^5/(a+b)^3/(a-
b)^3/a^2*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2
*c)^2-a-b)-(6*a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2)))-1/24/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^3*(-5*a-
9*b)/tan(1/2*d*x+1/2*c)+2/a^2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(324) = 648$.
time = 1.36, size = 1481, normalized size = 4.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(8*a^7*b^2 - 40*a^5*b^4 + 26*a^3*b^6 + 6*a*b^8 + 2*(4*a^9 - 13*a^7*b^2
 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x + c)^4 - 2*(2*a^8*b - 11*a^6*b^
3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 - 3*(6*a^2*b^6 - b^8 - (6*a^3*b^
5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*cos(d*x + c)^2 + (6*a^3*b^5 -
 a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 6*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 2*(
a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 6*((a^9 - 4*a^7*
b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d)*sin(d*x + c)), 1/3*(4*a^7*b^2 - 20*a^5*b^4 + 13*a^3*b^6 +
 3*a*b^8 + (4*a^9 - 13*a^7*b^2 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x +
 c)^4 - (2*a^8*b - 11*a^6*b^3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 + 3*
(6*a^2*b^6 - b^8 - (6*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*c
os(d*x + c)^2 + (6*a^3*b^5 - a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x +
 c) - 3*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2
+ (a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 3*((a^9 - 4*a
^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6
*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 +
 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a
^4*b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^
4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5
 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.56, size = 487, normalized size = 1.35

$$-\frac{\frac{48\,b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{7}-3\,a^{5}b^{2}+3\,a^{3}b^{4}-ab^{6}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)}-\frac{48\left(6\,a^{2}b^{5}-b^{7}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)\sqrt{-a^{2}+b^{2}}}-\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+72\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-126\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-27\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}-6\,a^{5}b+15\,a^{4}b^{2}-20\,a^{3}b^{3}+15\,a^{2}b^{4}-6\,ab^{5}+b^{6}}-\frac{24\left(dx+c\right)}{a^{2}}-\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+27\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*b^6*tan(1/2*d*x + 1/2*c)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(6*a^2*b
^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 3*a^6
*b^2 + 3*a^4*b^4 - a^2*b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)^3
 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*
b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*
x + 1/2*c) + 72*a^3*b*tan(1/2*d*x + 1/2*c) - 126*a^2*b^2*tan(1/2*d*x + 1/2*
c) + 96*a*b^3*tan(1/2*d*x + 1/2*c) - 27*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) - 24*(d*x + c
)/a^2 - (15*a*tan(1/2*d*x + 1/2*c)^2 + 27*b*tan(1/2*d*x + 1/2*c)^2 - a - b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 6.97, size = 2500, normalized size = 6.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(3
*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(11*a^3*b - 31*a*b^3 - 8*a^4 + 13*b^4 +
 15*a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(11*a^5*b - 9*a*b^5 - 5
*a^6 + 16*b^6 + 31*a^2*b^4 - 34*a^3*b^3 + 6*a^4*b^2))/(a*(a + b)^3))/(d*(ta
n(c/2 + (d*x)/2)^5*(8*a^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan
(c/2 + (d*x)/2)^3*(16*a*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x
)/2)*((16*a*b + 8*a^2 - 24*b^2)/(64*(a - b)^4) - 3/(4*(a - b)^2)))/d + (2*a
tan(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32
- 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^
9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 114240*a^13
*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 + 96000*a^17*
b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 + 412640*a^21
*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12 + 243456*a^2
5*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6
 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*b^2) - ((32*a^3
```

8 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 4992*a^10*b^28 +
2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^14*b^24 + 774
72*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^18*b^20 + 344
960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^22*b^16 + 4319
04*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^26*b^12 + 171
072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30*b^8 + 20064*
a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 512*a^35*b^3 -
512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32
+ 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*
a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a
^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a
^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a
^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*
a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b
^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2)*1i)/a^2)*
1i)/a^2)/a^2 + (tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128
*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28
- 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 1
14240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 + 9
6000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 + 4
12640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12 +
243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 - 448
0*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*b^2)
- ((32*a^37*b - 32*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a
^10*b^28 - 2624*a^11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*
b^24 - 77472*a^15*b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*
b^20 - 344960*a^19*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b
^16 - 431904*a^23*b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*
b^12 - 171072*a^27*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^
8 - 20064*a^31*b^7 + 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a
^35*b^3 + 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7*b^33 + 128
*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^2
8 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24
+ 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20
+ 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15
- 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11
- 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13
440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2)
*1i)/a^2)*1i)/a^2)/a^2)/(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3
*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 1536
0*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^
12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^
16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^
20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a
^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^2

9*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384
*a^34*b^2) - ((32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^
30 - 4992*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 -
57920*a^14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 1
07008*a^18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 7
6032*a^22*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 1
67552*a^26*b^12 + 171072*a^27*b^11 - 113664*a^2...

3.312 $\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=761

$$\frac{ae^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d}-\frac{(a^2-b^2)\,e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d}-\frac{ae^{5/2}\text{ArcTan}}{}$$

```
[Out] 1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)-
1/2*(a^2-b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^2/
d*2^(1/2)-1/2*a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/
d*2^(1/2)+1/2*(a^2-b^2)*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/
2))/a/b^2/d*2^(1/2)-1/4*a*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e
^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a^2-b^2)*e^(5/2)*ln(e^(1/2)-2^(1/2)*(
e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)+1/4*a*e^(5/2)*ln(e^
(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a
^2-b^2)*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a/b^2/d*2^(1/2)+2*e^2*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a
-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)-2*e^2*EllipticPi(sin(d*x+c)^(1/2
)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)
^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)+2*e^2*c
os(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4
*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/b/d/sin(2*d*x+2*c)^(1/2)+2*e*cos(d*x
+c)*(e*tan(d*x+c))^(3/2)/b/d
```

Rubi [A]
time = 0.80, antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3976, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 3975, 2812, 2809, 2985, 2984, 504, 1227, 551}

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a
^2 - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
```

```
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a^2 - b^2)*e^(5/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) + (2*Sqrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]
*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 +
Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*S
qrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b
]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[
e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(b*d*Sqrt[Sin[2*c + 2*d*x]]) + (2
*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
```

```
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2809

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3975

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3976

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[-e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[e^2*((a^2 - b^2)/b^2), Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = -\frac{e^2\int (a-b\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}}{b^2}$$

$$= -\frac{\left(ae^2\right)\int \sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{e^2\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} + \frac{\left(\left(a^2-b^2\right.\right.}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(2e^2\right)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} - \frac{(ae}{}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} - \frac{\left(2ae^3\right)\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d}$$

$$= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} + \frac{\left(ae^3\right)\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d}$$

$$= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+}{bd}$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,b^2d} + \frac{\left(a^2-b^2\right)e^{5/2}\log}{}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d} - \frac{\left(a^2-b^2\right)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan}}{\sqrt{}}\right.}{\sqrt{2}\,ab^2d}$$

$$= \frac{ae^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d} - \frac{\left(a^2-b^2\right)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan}}{\sqrt{}}\right.}{\sqrt{2}\,ab^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 56.62, size = 1846, normalized size = 2.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Cot[c + d*x]*(e*Tan[c + d*x])^(5/2))/(b*d*(a + b*Se
c[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Tan[c + d*x])^(5/2)*((
4*a*Sec[c + d*x]^2*((-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a
^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2
+ b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
qrt[Tan[c + d*x]] + b*Tan[c + d*x]] - Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]])/(4*Sqrt[2]*Sqrt[
b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(3*a^2 - 3*b^2))*(a + b*
Sqrt[1 + Tan[c + d*x]^2]))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)
) - (b*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b
^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(
3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (
6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
 d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3
+ 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a
^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))*(a + b*S
qrt[1 + Tan[c + d*x]^2]))/(4*(a^3 - a*b^2)*(b + a*Cos[c + d*x])*(1 + Tan[c
+ d*x]^2)) + (Cos[2*(c + d*x)]*Sec[c + d*x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((42 + 42*I)*(a^2
- 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)]
)/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2
 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*
(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (168*a*b
^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (24*a*b^2*AppellF1[7/4, 1/2, 1, 11/
4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(
a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/Sqrt[1 + Tan[c + d*x]^2])*(a + b*Sq
rt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos[c + d*x])*(-1 + Tan[c + d*x]^2)*S
qrt[1 + Tan[c + d*x]^2])))/(b*d*(a + b*Sec[c + d*x])*Tan[c + d*x]^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3802 vs. 2(646) = 1292.
time = 0.51, size = 3803, normalized size = 5.00

method	result	size
default	Expression too large to display	3803

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a-b)*(-1+cos(d*x+c))^2*(-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*b^2-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*b^2-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-
1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),
1/2*2^(1/2))*a^2+(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b
))^(1/2)),1/2*2^(1/2))*b^2-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b
+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2+(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^2+I*cos(d*x+c)*EllipticPi(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1
+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*b^2+cos(d*x+c)*(-(-1+cos(d*
x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*(a^2-b
^2)^(1/2)*a+cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b
)*(a-b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*b-cos(d*x+c)*(-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)
^(1/2)*a-cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
```

```
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)
*(a-b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*b+2*cos(d*x+c)*(-(-1+cos(d*x+c)
-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b-4*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2*2^(1/2))*a*b-I*cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*b^2-(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b
+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a-(-(-1+cos(d*x+c)-sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2
)*b+2*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-(-1+
cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b-4*(-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticE((-(-1+cos(d*x+c)-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b-I*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*b^2+I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*b^2-cos(d*x+c)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*b^2-cos(d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(tan(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \tan (c+dx))^{\frac{5}{2}}}{a+b \sec (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(5/2)/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)\,\left(e\,\tan(c+d\,x)\right)^{5/2}}{b+a\,\cos (c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

3.313 $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=740

$$\frac{ae^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d}-\frac{(a^2-b^2)\,e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ab^2d}-\frac{ae^{3/2}\text{ArcTan}}{}$$

```
[Out] 1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)-
1/2*(a^2-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^2/
d*2^(1/2)-1/2*a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/
d*2^(1/2)+1/2*(a^2-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/
2))/a/b^2/d*2^(1/2)+1/4*a*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e
^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a^2-b^2)*e^(3/2)*ln(e^(1/2)-2^(1/2)*(
e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)-1/4*a*e^(3/2)*ln(e^
(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a
^2-b^2)*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a/b^2/d*2^(1/2)-2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),
b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d*x+c)^(1/2)/a/b/d/(-c
os(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/
(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d
*x+c)^(1/2)/a/b/d/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)-e^2*(sin(c+1/4*P
i+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(
d*x+c)*sin(2*d*x+2*c)^(1/2)/b/d/(e*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.67, antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 19, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3976, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 3977, 2812, 2808, 2986, 1227, 551}

$$-\frac{2\sqrt{2}e^2\sqrt{a^2-b^2}\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\text{ArcSin}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}e^2\sqrt{a^2-b^2}\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\text{ArcSin}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}-\frac{e^{3/2}(a^2-b^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}+\frac{e^{3/2}(a^2-b^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ab^2d}-\frac{e^{3/2}(a^2-b^2)\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ab^2d}+\frac{e^{3/2}(a^2-b^2)\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ab^2d}+\frac{ae^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}-\frac{ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}b^2d}+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}b^2d}-\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}b^2d}+\frac{e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)F\left(c+dx-\frac{\pi}{4}\middle|2\right)}{bd\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) + (a*e^(3/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a
^2 - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
```

```
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) - (2*Sqrt[2]*Sqrt[a^2 - b^2]*e^2*EllipticPi[b/(a - Sqrt[a^2
 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[
c + d*x]])/(a*b*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*Sq
rt[a^2 - b^2]*e^2*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c +
d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*b*d*Sqrt[-Cos[c +
 d*x]]*Sqrt[e*Tan[c + d*x]]) + (e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*
x]*Sqrt[Sin[2*c + 2*d*x]])/(b*d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2808

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
```

```
+ f*x]]), Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3976

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[-e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[e^2*((a^2 - b^2)/b^2), Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rule 3977

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx &= -\frac{e^2\int \frac{a-b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{b^2}\\
&= -\frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{}{\sqrt{e\,t}}}{ab^2}\\
&= -\frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{b^2d}+\frac{\left(\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e}\right.}{ab^2}\\
&= -\frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}+\frac{\left(2\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,d\right.}{ab^2d}\\
&= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}-\frac{\left(ae^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e}\right.}{b^2d}\\
&= \frac{e^2F\left(c-\frac{\pi}{4}+dx\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}+\frac{\left(ae^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2}{-e-\sqrt{2}\sqrt{e}\,x}\right.}{2\sqrt{2}}\\
&= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,b^2d}-\frac{\left(a^2-b^2\right)e^{3/2}\log\left(\right.}{}\\
&= \frac{ae^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,b^2d}-\frac{\left(a^2-b^2\right)e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(}}{\sqrt{e}}\right.}{\sqrt{2}\,ab^2d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.20, size = 202, normalized size = 0.27

$$
\frac{2e\cot\left(\frac{1}{2}(c+dx)\right)\left(\Pi\left(-i;\text{ArcSin}\left(\sqrt{-\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)+\Pi\left(i;\text{ArcSin}\left(\sqrt{-\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\text{ArcSin}\left(\sqrt{-\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\text{ArcSin}\left(\sqrt{-\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)\right)\sqrt{\frac{-1+\cos(c+dx)-\sin(c+dx)}{1+\cos(c+dx)}}\sqrt{1-\tan\left(\frac{1}{2}(c+dx)\right)}\sqrt{e\tan(c+dx)}}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*e*Cot[(c + d*x)/2]*(EllipticPi[-I, ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1]
+ EllipticPi[I, ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1] - EllipticPi[-(Sqrt[a
- b]/Sqrt[a + b]), ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1] - EllipticPi[Sqrt[a
```

```
 - b]/Sqrt[a + b], ArcSin[Sqrt[-Tan[(c + d*x)/2]]], -1])*Sqrt[(-1 + Cos[c +
 d*x] - Sin[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[1 - Tan[(c + d*x)/2]]*Sqrt[e
*Tan[c + d*x]])/(a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1829 vs. $2(636) = 1272$.
time = 0.24, size = 1830, normalized size = 2.47

method	result	size
default	Expression too large to display	1830

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(1+cos(d*x+c))^2*(e*sin(d*x+c)/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*cos(
d*x+c)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(2*I*EllipticPi
((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2
-b^2)^(1/2)*a*b-2*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b+EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
a^2-b^2)^(3/2)+2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^3+2*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/
2))*b^3-2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/
(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^3-2*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))
*b^3+I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2+I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-I*EllipticPi((
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b
^2)^(1/2)*a^2-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2+2*EllipticPi((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b+2*Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*(a^2-b^2)^(1/2)*a*b-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+I*EllipticPi((-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+2*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)
*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b+2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2-Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*(a^2-b^2)^(1/2)*a^2-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-EllipticPi((-(-1+cos(d*x+c
```

```
)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2-
EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))*(a^2-b^2)^(1/2)*b^2-2*(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin
(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2
+2*(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2
),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^2-2*(a^2-b^2)^(1/2)*Ellipt
icPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-
b))^(1/2)),1/2*2^(1/2))*a^2+2*(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))
*b^2-2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b
+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b-2*EllipticPi((-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^
2)/sin(d*x+c)^4*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^
2)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(3/2)*integrate(tan(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\tan\left(c+d\,x\right)\right)}^{3/2}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

3.314 $\int \frac{\sqrt{e \tan(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=415

$$-\frac{\sqrt{e}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\log\left(\sqrt{e}+\sqrt{e}\right.}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(1/2
)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a
/d*2^(1/2)+2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a-b)^(1/2
)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/d/(a-b)^(1
/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)-2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x
+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c
))^(1/2)/a/d/(a-b)^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3975, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2812, 2809, 2985, 2984, 504, 1227, 551}

$$\frac{2\sqrt{2}\,b\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}\,\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\mathrm{ArcSin}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(c+dx)}\,\sqrt{e\tan(c+dx)}\,\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\mathrm{ArcSin}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{\sin(c+dx)}}-\frac{\sqrt{e}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad}+\frac{\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}-\frac{\sqrt{e}\,\log\left(\sqrt{e}\,\tan(c+dx)+\sqrt{2}\,\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*Sqrt[2]*b*Sqrt[Cos[c + d*
x]]*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1
 + Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*S
qrt[Sin[c + d*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b]
/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[e
*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[Sin[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2809

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
```

```
b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3975

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} - \frac{b\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}\,dx}{a}$$

$$= \frac{e\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{\left(b\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{}{(b\cdots}}{a}$$

$$= \frac{(2e)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\cdots\right.}{a\sqrt{\sin(\cdots}}$$

$$= -\frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{e\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx\cdots}\right.}{ad}$$

$$= \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}\,x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}\cdots}{-e+\sqrt{2}\sqrt{e}\cdots}\right.}{2\cdots}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\cdots\right.}{\sqrt{2}\,ad}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad} + \frac{\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\cdots\right.}{\sqrt{2}\,ad}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 15.80, size = 224, normalized size = 0.54

$$-\frac{4(b+a\cos(c+dx))\csc(c+dx)\left(-i\Pi\left(-i;\text{ArcSin}\left(\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)+i\Pi\left(i;\text{ArcSin}\left(\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)+\frac{b\left(-\Pi\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}};\text{ArcSin}\left(\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)+\Pi\left(\frac{\sqrt{a-b}}{\sqrt{a+b}};\text{ArcSin}\left(\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-1\right)\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}\sqrt{e\tan(c+dx)}}{ad\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-4*(b + a*Cos[c + d*x])*Csc[c + d*x]*((-I)*EllipticPi[-I, ArcSin[Sqrt[Tan[
(c + d*x)/2]]], -1] + I*EllipticPi[I, ArcSin[Sqrt[Tan[(c + d*x)/2]]], -1] +
```

```
(b*(-EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Tan[(c + d*x)/2]]]
, -1] + EllipticPi[Sqrt[a - b]/Sqrt[a + b], ArcSin[Sqrt[Tan[(c + d*x)/2]]],
 -1]))/(Sqrt[a - b]*Sqrt[a + b]))*Sqrt[Tan[(c + d*x)/2]]*Sqrt[e*Tan[c + d*x
]])/(a*d*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x]))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(328) = 656$.
time = 0.24, size = 874, normalized size = 2.11

method	result	size
default	Expression too large to display	874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2)*(e*sin(d*x+c)/cos(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(d*x+c))/sin
(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(I*EllipticPi((-(-1+cos(d*x
+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-I*EllipticPi((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*a+I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*b-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*a+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*b-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-(a^2-b^2)^(1/2)*EllipticPi((-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)
),1/2*2^(1/2))+(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(
d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))+EllipticPi((-(
-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2))
,1/2*2^(1/2))*a-b*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2)
,(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))+EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))
*a-b*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b
+((a+b)*(a-b))^(1/2)),1/2*2^(1/2)))/sin(d*x+c)^3*2^(1/2)*b/((a^2-b^2)^(1/2)
-a+b)/((a^2-b^2)^(1/2)+a-b)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan{(c+dx)}}}{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sqrt(e*tan(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\sqrt{e\,\tan\left(c+d\,x\right)}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

3.315 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal. Leaf size=422

$$-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}+\frac{\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}-\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)\right.}{2\sqrt{2}\,a}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(1/2
)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/
2)/e^(1/2)-2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a-(a^
2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1/2)/(-cos(d*x+c))
^(1/2)/(e*tan(d*x+c))^(1/2)+2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c
))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1
/2)/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3977, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2812, 2808, 2986, 1227, 551}

$$-\frac{2\sqrt{2}\,b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\mathrm{ArcSin}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\,b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\mathrm{ArcSin}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,ad\sqrt{e}}-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad\sqrt{e}}+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,ad\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) - (2*Sqrt[2]*b*EllipticPi[b/(a - Sqrt[
a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[S
in[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]
) + (2*Sqrt[2]*b*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d
*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*
Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2808

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]]), Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3977

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} - \frac{b\int \frac{\sqrt{e\cot(}{b+a\cos}}{a\sqrt{e\cot(c+dx)}}$$

$$= \frac{(2e)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(b\sqrt{\sin(c+dx)}\right.}{a\sqrt{-\cos}}$$

$$= \frac{\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x,\right.}{ad}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} + \frac{\text{Subst}\left(\int \frac{}{e+}\right.}{}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,ad\sqrt{e}} + \frac{\log\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,ad\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\tan}}{\sqrt{e}}\right.}{\sqrt{2}\,ad\sqrt{e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.22, size = 246, normalized size = 0.58

$$\frac{4\left(a(a-b)F\left(\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)+(-a^2+b^2)\Pi\left(-i;\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)-a^2\Pi\left(i;\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)+b^2\Pi\left(i;\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)-b^2\Pi\left(-\frac{\sqrt{a+b}}{\sqrt{a-b}};\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)-b^2\Pi\left(\frac{\sqrt{a+b}}{\sqrt{a-b}};\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(c+dx)\right)}}\right)\middle|-1\right)\right)}{a(a^2-b^2)d\sqrt{1+\cot\left(\frac{1}{2}(c+dx)\right)}\sqrt{-1+\tan\left(\frac{1}{2}(c+dx)\right)}\sqrt{e\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (4*(a*(a - b)*EllipticF[ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] + (-a^2 + b^2
)*EllipticPi[-I, ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] - a^2*EllipticPi[I,
ArcSin[1/Sqrt[Tan[(c + d*x)/2]]], -1] + b^2*EllipticPi[I, ArcSin[1/Sqrt[Tan
[(c + d*x)/2]]], -1] - b^2*EllipticPi[-(Sqrt[a + b]/Sqrt[a - b]), ArcSin[1/
Sqrt[Tan[(c + d*x)/2]]], -1] - b^2*EllipticPi[Sqrt[a + b]/Sqrt[a - b], ArcS
```

in[1/Sqrt[Tan[(c + d*x)/2]]], -1]))/(a*(a^2 - b^2)*d*Sqrt[1 + Cot[(c + d*x)/2]]*Sqrt[-1 + Tan[(c + d*x)/2]]*Sqrt[e*Tan[c + d*x]])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2351 vs. $2(343) = 686$.
time = 0.23, size = 2352, normalized size = 5.57

method	result	size
default	Expression too large to display	2352

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b+3*EllipticPi((-
(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^
2)^(1/2)*a^2*b-3*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2+3*EllipticPi((-(-1+cos(d*x+c)-
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+
3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b-3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2-3*I*Ellipti
cPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
a^2-b^2)^(1/2)*a^2*b+3*I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2-2*EllipticPi((-(-1+cos
(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2
^(1/2))*b^4+2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a
-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^4-3*EllipticPi((-(-1+cos(d*x+
c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b
^2+2*(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2+2*(a^2-b^2)^(1/2)*El
lipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)
*(a-b))^(1/2)),1/2*2^(1/2))*a*b^2+I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3+I*EllipticPi(
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-
b^2)^(1/2)*a^3-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-I*EllipticPi((-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-I*Ellipt
icPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
(a^2-b^2)^(1/2)*a^3-I*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3-4*EllipticF((-(-1+cos(d*x+c
)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+2*Ellipt
icF((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(
1/2)*a*b^2-2*(a^2-b^2)^(1/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*
```

```
x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^3-2*(a^2-b^2)^(1
/2)*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+
((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*b^3-2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x
+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b^2
+4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+((a
+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^3+2*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+
c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a^2*b^
2-4*EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+
((a+b)*(a-b))^(1/2)),1/2*2^(1/2))*a*b^3-2*EllipticF((-(-1+cos(d*x+c)-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+2*EllipticF((-(-1+co
s(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3+Ell
ipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*(a^2-b^2)^(3/2)*a-EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-EllipticPi((-(-1+cos(d*x+c)-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3+Elli
pticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*(a^2-b^2)^(1/2)*b^3+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-EllipticPi((-(-1+cos(d*x+c)-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-Ellip
ticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*(a^2-b^2)^(1/2)*a^3+EllipticPi((-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(-1+cos(d*x+c)-sin(
d*x+c))/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^2/sin(d*x+c)^2/cos
(d*x+c)/(e*sin(d*x+c)/cos(d*x+c))^(1/2)*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2
-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/(a-b)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((b*sec(d*x + c) + a)*sqrt(tan(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\tan(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*tan(c + d*x))*(a + b*sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\sqrt{e\tan(c+d\,x)}\,(b+a\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

3.316 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=863

$$\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}}-\frac{b^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a\,(a^2-b^2)\,de^{3/2}}-\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(\cdot}}{\sqrt{e}}\right.}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(3/2)*2^
(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/d/
e^(3/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b
^2)/d/e^(3/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2)
)/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/
2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)-2^(1/
2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1
/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/
d/e^(3/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*t
an(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-2*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(
e*tan(d*x+c))^(1/2)+2*b^3*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),
-(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a
/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-2*b^3*EllipticPi(sin(d*x+c)
^(1/2)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(
1/2)*(e*tan(d*x+c))^(1/2)/a/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-
2*b*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(
c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(2*d*x+2*c)^
(1/2)-2*b*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/(a^2-b^2)/d/e^3
```

Rubi [A]
time = 0.86, antiderivative size = 863, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 23, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3978, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 3975, 2812, 2809, 2985, 2984, 504, 1227, 551}

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(3/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
```

```
 - b^2)*d*e^(3/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) + (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (2*(a - b*Sec[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*
EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + C
os[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e
^2*Sqrt[Sin[c + d*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[
a - b]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*
Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e^2*Sqrt[Sin[c + d*x
]]) + (2*b*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/
((a^2 - b^2)*d*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (2*b*Cos[c + d*x]*(e*Tan[c + d
*x])^(3/2))/((a^2 - b^2)*d*e^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2809

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3975

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3978

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2-b^2} + \frac{b^2\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{(a^2-b^2)\,e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} + \frac{2\int \left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)\sqrt{\ }}{(a^2-b^2)\,e}$$

$$= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{a\int \sqrt{e\tan(c+dx)}\,dx}{(a^2-b^2)\,e^2} - b$$

$$= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx)}{(a^2-b^2)\,de^3}$$

$$= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx)}{(a^2-b^2)\,de^3}$$

$$= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx)}{(a^2-b^2)\,de^3}$$

$$= \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a\,(a^2-b^2)\,de^{3/2}} - \frac{b^2}{}$$

$$= -\frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a\,(a^2-b^2)\,de^{3/2}} + \frac{b^2\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,a\,(a^2-}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}} - \frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}\,a\,(a^2-b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 47.57, size = 1571, normalized size = 1.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(b - a*Cos[c + d*x])*Csc[c + d*x])/
(-a^2 + b^2) + (2*b*Sin[c + d*x])/(-a^2 + b^2))*Tan[c + d*x]^2)/(d*(a + b*S
ec[c + d*x])*(e*Tan[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x]^(3/2)*(-1/12*((-a^2 + 3*b^2)*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2
)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
 - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan
[c + d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^
2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1
- Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt
[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Ta
n[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1
+ I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a
*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/((a^3 - a*b^2)*
(b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)) + (b*Cos[2*(c + d*x)]*Sec[c + d*
x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2
- b^2)^(1/4)) + ((42 + 42*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2
]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*
Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]]
+ I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b
^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*
x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/
4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c +
d*x]^(3/2))/(a^2 - b^2) - (168*a*b^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*
x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (
24*a*b^2*AppellF1[7/4, 1/2, 1, 11/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/
(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/S
qrt[1 + Tan[c + d*x]^2])*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos
[c + d*x])*(-1 + Tan[c + d*x]^2)*Sqrt[1 + Tan[c + d*x]^2])))/((a - b)*(a +
b)*d*(a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6529 vs. 2(746) = 1492.
time = 0.23, size = 6530, normalized size = 7.57

method	result	size
default	Expression too large to display	6530

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((b*sec(d*x + c) + a)*tan(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(e\tan\left(c+d\,x\right)\right)}^{3/2}\,\left(b+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

3.317 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=836

$$\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}}-\frac{b^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a\,(a^2-b^2)\,de^{5/2}}-\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(5/2)*2^
(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/d/
e^(5/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b
^2)/d/e^(5/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2)
)/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)+1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/
2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)-2^(1/
2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1
/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/
d/e^(5/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*t
an(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-2*b^3*EllipticPi((-cos(d*x+c))^(1/
2)/(1+sin(d*x+c))^(1/2),b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a
/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*b^3*Ellip
ticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(
1/2)*sin(d*x+c)^(1/2)/a/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*
x+c))^(1/2)-1/3*b*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(c
os(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/(a^2-b^2)/d/e^2/(
e*tan(d*x+c))^(1/2)-2/3*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.74, antiderivative size = 836, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3978, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 3977, 2812, 2808, 2986, 1227, 551}

$$-\frac{2\sqrt{2}\,\Pi\left(\frac{b}{a-\sqrt{a^2-b^2}};\text{ArcSin}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}\,b^3}{a(a^2-b^2)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{2\sqrt{2}\,\Pi\left(\frac{b}{a+\sqrt{a^2-b^2}};\text{ArcSin}\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right)\middle|-1\right)\sqrt{\sin(c+dx)}\,b^3}{a(a^2-b^2)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)b^2}{\sqrt{2}a(a^2-b^2)de^{5/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)b^2}{\sqrt{2}a(a^2-b^2)de^{5/2}}-\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}-\sqrt{2}\sqrt{e\tan(c+dx)}\right)b^2}{2\sqrt{2}a(a^2-b^2)de^{5/2}}+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)b^2}{2\sqrt{2}a(a^2-b^2)de^{5/2}}+\frac{F\left(c+dx-\frac{\pi}{4}\middle|2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\,b}{3(a^2-b^2)de^2\sqrt{e\tan(c+dx)}}+\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2-b^2)de^{5/2}}-\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}(a^2-b^2)de^{5/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}(a^2-b^2)de^{5/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}(a^2-b^2)de^{5/2}}-\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)de(e\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(5/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) + (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(5/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
```

```
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (2*(a - b*Sec[c + d*x]))/
(3*(a^2 - b^2)*d*e*(e*Tan[c + d*x])^(3/2)) - (2*Sqrt[2]*b^3*EllipticPi[b/(a
 - Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1
]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e
*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin
[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*(a
^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (b*Ellipt
icF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*(a^2 - b^2)*
d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2808

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]]), Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3977

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3978

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2-b^2} + \frac{b^2\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{(a^2-b^2)\,e^2}$$

$$= -\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3(a^2-b^2)\,e^2} +$$

$$= -\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} - \frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{(a^2-b^2)\,e^2} +$$

$$= -\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx, x\right.}{(a^2-b^2)\,d}$$

$$= -\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} + \frac{b^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e}\right.}{a(a^2-b^2)\,de}$$

$$= -\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} + \frac{bF\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sec(c+}{3(a^2-b^2)\,de^2\sqrt{e}}$$

$$= -\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\,\tan(c+dx)-\sqrt{2}\,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,a(a^2-b^2)\,de^{5/2}} + \frac{b^2}{}$$

$$= -\frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,a(a^2-b^2)\,de^{5/2}} + \frac{b^2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{}}{}\right.}{\sqrt{2}\,a(a^2-}$$

$$= \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}} - \frac{b^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e}}{}\right.}{\sqrt{2}\,a(a^2-b^2)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.29, size = 2169, normalized size = 2.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

[Out] ((b + a*Cos[c + d*x])*((2*a)/(3*(a^2 - b^2)) - (2*(-a + b*Cos[c + d*x])*Csc
[c + d*x]^2)/(3*(-a^2 + b^2)))*Sec[c + d*x]*Tan[c + d*x]^3)/(d*(a + b*Sec[c
+ d*x])*(e*Tan[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c
+ d*x]^(5/2)*((2*(3*a^2 - 5*b^2)*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*
x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/
(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2
- b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]]))/(Sqrt[b]*(a^2
- b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]]*Sqrt[1 + Tan[c + d
x]^2])/((5(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*T
an[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2
)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*
x]^2)^2) + (8*a*b*Sec[c + d*x]^2*(a + b*Sqrt[1 + Tan[c + d*x]^2])*((Sqrt[b]
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] +
b*Tan[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]]))/(4*Sqrt[2]*(-a^2 + b^2)^(3/4)) + (
5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/(Sqrt[1 + Tan[c + d*x]^2]*(-5*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)*(-a^2 + b^2*(1 +
Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)) + ((3
*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*x]^
2])*((-20*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + (20*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + ((10 - 10*I)*(a^2 - 2*b^2)*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(
a^2 - b^2)^(3/4)) - ((10 - 10*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (10
*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + (10*Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + ((5 - 5*I)*(a^2 -
2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c
+ d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - ((5 - 5*I)*(a^
2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan
[c + d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (8*b*Appell
F1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan
[c + d*x]^(5/2))/(-a^2 + b^2) - (200*b*(-a^2 + b^2)*AppellF1[1/4, 1/2, 1, 5
/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/
(Sqrt[1 + Tan[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c

```
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, 1/2, 2
, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[5/4, 3/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2)*(-a^2 + b^2*(1 + Tan[c + d*x]^2)))))/(20*(b + a*Cos[c + d
*x])*(1 - Tan[c + d*x]^2)*(1 + Tan[c + d*x]^2))))/(6*(a - b)*(a + b)*d*(a +
 b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16409 vs. $2(725) = 1450$.
time = 0.29, size = 16410, normalized size = 19.63

method	result	size
default	Expression too large to display	16410

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate(1/((b*sec(d*x + c) + a)*tan(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\tan{(c+dx)})^{\frac{5}{2}}(a+b\sec{(c+dx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((e*tan(c + d*x))**(5/2)*(a + b*sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(e\,\tan\left(c+d\,x\right)\right)}^{5/2}\,\left(b+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*tan(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

3.318 $\int \sqrt{a+b\sec(c+dx)}\,\tan^5(c+dx)\,dx$

Optimal. Leaf size=169

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2(3a^2\cdots$$

```
[Out] -2/3*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d+2/5*(3*a^2-2*b^2)*(a+b*sec(
d*x+c))^(5/2)/b^4/d-6/7*a*(a+b*sec(d*x+c))^(7/2)/b^4/d+2/9*(a+b*sec(d*x+c))
^(9/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec
(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 213}

$$\frac{2(3a^2-2b^2)\,(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d}-\frac{6a(a+b\sec(c+dx))^{7/2}}{7b^4d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/(3*b^4*d) +
 (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) - (6*a*(a + b*Sec
[c + d*x])^(7/2))/(7*b^4*d) + (2*(a + b*Sec[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\ &= \frac{2\operatorname{Subst}\left(\int \frac{x^2\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}\\ &= \frac{2\operatorname{Subst}\left(\int \left(b^4-a(a^2-2b^2)x^2+(3a^2-2b^2)x^4-3ax^6+x^8+\right.\right.}{b^4 d}\\ &= \frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2a(a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4 d}+\frac{2(}{}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 6.30, size = 254, normalized size = 1.50

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2(-16a^4+84a^2b^2+315b^4)}{315b^4}-\frac{4a(-4a^2+21b^2)\sec(c+dx)}{315b^3}-\frac{4(a^2+21b^2)\sec^2(c+dx)}{105b^2}+\frac{2a\sec^3(c+dx)}{63b}+\frac{2}{9}\sec^4(c+dx)\right)}{d}-\frac{\sqrt{a\cos(c+dx)}\left(-\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)+\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\sqrt{a+b\sec(c+dx)}\sin^2(c+dx)}{d\sqrt{b+a\cos(c+dx)}\left(1-\cos^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-16*a^4 + 84*a^2*b^2 + 315*b^4))/(315*b^4) -
 (4*a*(-4*a^2 + 21*b^2)*Sec[c + d*x])/(315*b^3) - (4*(a^2 + 21*b^2)*Sec[c +
 d*x]^2)/(105*b^2) + (2*a*Sec[c + d*x]^3)/(63*b) + (2*Sec[c + d*x]^4)/9))/d
 - (Sqrt[a*Cos[c + d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c +
d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x]^2)/(d*Sqrt[b + a*Cos[c + d*x]]*(1 - Cos[c + d*x
]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3267 vs. $2(145) = 290$.
time = 0.84, size = 3268, normalized size = 19.34

method	result	size
default	Expression too large to display	3268

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2520/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos
(d*x+c))^4*(630*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2
/(a-b)^(1/2))*cos(d*x+c)^6*b^6+648*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^6*a^2*b^3+2121*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^6*a*b^4+24*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^7*a^2*b^2+6
4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+
c)^6*a^3*b-336*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(
1/2)*cos(d*x+c)^6*a*b^3+24*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
3/2)*cos(d*x+c)^5*(a-b)^(1/2)*a^2*b^2+192*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^4*a^3*b-968*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^4*a*b^3-144*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^3*a^2*
b^2+120*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*co
s(d*x+c)^2*a*b^3+648*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(
a-b)^(1/2)*cos(d*x+c)^7*a^3*b^2-128*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^6*a^4*b+648*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^6*a^3*b^2+2121*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^7*a*b^4-1
28*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x
+c)^5*a^4*b+648*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^
(1/2)*cos(d*x+c)^5*a^2*b^3-1260*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*cos(d*x+c)^7*(a-b)^(1/2)*a^(3/2)*
b^4+72*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos
(d*x+c)^6*a^2*b^2-1260*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*cos(d*x+c)^6*(a-b)^(1/2)*a^(1/2)*b^5+192*(
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^
5*a^3*b-1008*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/
2)*cos(d*x+c)^5*a*b^3-120*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3
/2)*(a-b)^(1/2)*cos(d*x+c)^4*a^2*b^2+64*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
```

```
(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^3*a^3*b-216*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^3*a*b^3-48*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^2*a^2*b^2
+40*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*
x+c)*a*b^3+280*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
3/2)*b^4+2121*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1
/2)*cos(d*x+c)^5*b^5-128*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*(a-b)^(1/2)*cos(d*x+c)^7*a^5-128*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^6*a^5+2121*((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)*cos(d*x+c)^6*b^5+1197*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^6*b^4-1260*ln(
-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*co
s(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d
*x+c)^7*a^2*b^4+630*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+
c)^2/(a-b)^(1/2))*cos(d*x+c)^7*a*b^5+1260*ln(-2*(-1+cos(d*x+c))*(2*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+
b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^7*a^2*b^4-630*ln(-2*(-
1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d
*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a
-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+
c)^7*a*b^5-1260*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2
/(a-b)^(1/2))*cos(d*x+c)^6*a*b^5+1260*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*co
s(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a*b^5-2625*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*cos(d*x+c)^4*b^4-168*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3...
```

Maxima [A]
time = 0.49, size = 191, normalized size = 1.13

$$\frac{315\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+630\sqrt{a+\frac{b}{\cos(dx+c)}}+\frac{70\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{9}{2}}}{b^4}-\frac{270\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}a}{b^4}+\frac{378\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}a^2}{b^4}-\frac{210\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a^3}{b^4}-\frac{252\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^2}+\frac{420\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^2}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
[Out] 1/315*(315*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos
(d*x + c)) + sqrt(a))) + 630*sqrt(a + b/cos(d*x + c)) + 70*(a + b/cos(d*x +
```

```
c))^(9/2)/b^4 - 270*(a + b/cos(d*x + c))^(7/2)*a/b^4 + 378*(a + b/cos(d*x
+ c))^(5/2)*a^2/b^4 - 210*(a + b/cos(d*x + c))^(3/2)*a^3/b^4 - 252*(a + b/c
os(d*x + c))^(5/2)/b^2 + 420*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

Fricas [A]
time = 3.93, size = 425, normalized size = 2.51

$$\left[\frac{315\sqrt{a}b^4\cos(dx+c)^4\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)+4\left(5ab^3\cos(dx+c)-\left(16a^4-84a^2b^2-315b^4\right)\cos(dx+c)^4+35b^4+2\left(4a^3b-21ab^3\right)\cos(dx+c)^3-6\left(a^2b^2+21b^4\right)\cos(dx+c)^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{630b^4d\cos(dx+c)^4}, \frac{315\sqrt{-a}b^4\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)\cos(dx+c)^4+2\left(5ab^3\cos(dx+c)-\left(16a^4-84a^2b^2-315b^4\right)\cos(dx+c)^4+35b^4+2\left(4a^3b-21ab^3\right)\cos(dx+c)^3-6\left(a^2b^2+21b^4\right)\cos(dx+c)^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{315b^4d\cos(dx+c)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*b^4*cos(d*x + c)^4*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + 4*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a
^2*b^2 - 315*b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x
+ c)^3 - 6*(a^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)))/(b^4*d*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*b^4*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 b))*cos(d*x + c)^4 + 2*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a^2*b^2 - 315*
b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x + c)^3 - 6*(a
^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
b^4*d*cos(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(145) = 290.
time = 1.97, size = 966, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 2/315*(315*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
```

```
 a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(315*(sqrt(a - b)*tan(1/2*d*x
 + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*
a*tan(1/2*d*x + 1/2*c)^2 + a + b))^8*a - 3150*(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan
(1/2*d*x + 1/2*c)^2 + a + b))^7*sqrt(a - b)*a + 210*(sqrt(a - b)*tan(1/2*d*
x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2
*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^6*(39*a^2 - 5*a*b - 32*b^2) - 630*(sqrt
(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*
d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^5*(9*a^2 + 15*a*b - 1
6*b^2)*sqrt(a - b) - 252*(25*a^3 - 37*a^2*b + 80*a*b^2 - 72*b^3)*(sqrt(a -
b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x +
 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4 + 945*a^5 + 3864*a^4*b +
 2562*a^3*b^2 + 2448*a^2*b^3 - 1083*a*b^4 + 224*b^5 + 42*(255*a^3 + 2*a^2*b
 + 655*a*b^2 - 288*b^3)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b))^3*sqrt(a - b) - 18*(175*a^4 - 483*a^3*b + 1113*a^2*b^2 - 773*a*b^3
+ 448*b^4)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c
)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - 1
8*(105*a^4 + 637*a^3*b + 203*a^2*b^2 + 447*a*b^3 - 112*b^4)*(sqrt(a - b)*ta
n(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*
c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*sqrt(a - b))/(sqrt(a - b)*tan(1
/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^
4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^9)*sgn(cos(d*x + c))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2), x)
```

3.319 $\int \sqrt{a+b\sec(c+dx)}\,\tan^3(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}+\frac{2(a+b\sec(c+dx)}{5b^2d}$$

```
[Out] -2/3*a*(a+b*sec(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^2/d+2*arct
anh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 213}

$$\frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\sec(c+dx)}}{d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Sec
[c + d*x]])/d - (2*a*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*Sec[
c + d*x])^(5/2))/(5*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2 d}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}$$

$$= -\frac{2\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right.}{b^2 d}$$

$$= -\frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2 d} + \frac{2(a+b\sec}{5b}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d} -$$

Mathematica [A]
time = 6.20, size = 194, normalized size = 1.94

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2(2a^2+15b^2)}{15b^2}+\frac{2a\sec(c+dx)}{15b}+\frac{2}{5}\sec^2(c+dx)\right)}{d}+\frac{\sqrt{a\cos(c+dx)}\left(-\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)+\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\sqrt{a+b\sec(c+dx)}\,\sin^2(c+dx)}{d\sqrt{b+a\cos(c+dx)}\,(1-\cos^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((-2*(2*a^2 + 15*b^2))/(15*b^2) + (2*a*Sec[c + d*
x])/(15*b) + (2*Sec[c + d*x]^2)/5))/d + (Sqrt[a*Cos[c + d*x]]*(-Log[1 - Sqr
t[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*
x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]^2)/(d*Sqrt
[b + a*Cos[c + d*x]]*(1 - Cos[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2341 vs. 2(84) = 168.
time = 0.32, size = 2342, normalized size = 23.42

method	result	size
default	Expression too large to display	2342

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/60/d*(1+cos(d*x+c))*(-1+cos(d*x+c))^4*(-30*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^3-30*cos(d*x+c)^2*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2+8*cos(d*x+c)^5*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^3+8*cos(d*x+c)^4*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^3+54*cos(d*x+c)^4*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*b^3+54*cos(d*x+c)^3*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*b^3+30*cos(d*x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^2-15*cos(d*x+c)^5*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^3-30*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^2+15*cos(d*x+c)^5*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^3+18*cos(d*x+c)^4*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2+30*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^3+6*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*b^2+6*cos(d*x+c)^3*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*b^2-36*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*b^2-15*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*b^4+15*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*b^4-12*((

```
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(1/2)*b^2-30*cos(d
*x+c)^5*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c
)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*
a*cos(d*x+c)+2*b)*(a-b)^(1/2)*a^(3/2)*b^2-30*cos(d*x+c)^4*ln(4*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*(a-b)^(
1/2)*a^(1/2)*b^3-12*cos(d*x+c)^3*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(3/2)*a*b-4*cos(d*x+c)*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b+8*cos(d*x+c)^3*(a-b)^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b+8*cos(d*x+c)^4*(a-b)^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b+54*cos(d*x+c)^4*(a-
b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a*b^2+54*cos(
d*x+c)^5*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a
*b^2-4*cos(d*x+c)^4*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(3/2)*a*b-12*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(3/2)*(a-b)^(1/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/sin(d*
x+c)^8/cos(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)/(a
-b)^(1/2)/b^2
```

Maxima [A]
time = 0.51, size = 108, normalized size = 1.08

$$-\frac{15\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{b}{\cos(dx+c)}}-\frac{6\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^2}+\frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(
d*x + c)) + sqrt(a))) + 30*sqrt(a + b/cos(d*x + c)) - 6*(a + b/cos(d*x + c)
)^(5/2)/b^2 + 10*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

Fricas [A]
time = 4.28, size = 311, normalized size = 3.11

$$\left[\frac{15\sqrt{a}\,b^2\cos(dx+c)^2\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)+4\left(ab\cos(dx+c)-(2a^2+15b^2)\cos(dx+c)^2+3b^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{30\,b^2d\cos(dx+c)^2},\ -\frac{15\sqrt{-a}\,b^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)\cos(dx+c)^2-2\left(ab\cos(dx+c)-(2a^2+15b^2)\cos(dx+c)^2+3b^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{15\,b^2d\cos(dx+c)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^2*cos(d*x + c)^2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(
d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*co
```

```
s(d*x + c) + b)/cos(d*x + c))) + 4*(a*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos
(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(b^2*d*cos(d*
x + c)^2), -1/15*(15*sqrt(-a)*b^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^2 - 2*(a
*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec{(c + dx)}}\,\tan^{3}{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(84) = 168.
time = 0.89, size = 539, normalized size = 5.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
[Out] -2/15*(15*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(15*(sqrt(a - b)*tan(1/2*d*x +
 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*
tan(1/2*d*x + 1/2*c)^2 + a + b))^4*a - 30*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)
^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^2 + a + b))^3*(a + 2*b)*sqrt(a - b) + 20*(sqrt(a - b)*tan(1/2
*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4
- 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(4*a*b - 3*b^2) - 15*a^3 - 10*a^2*
b - 35*a*b^2 + 12*b^3 + 10*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan
(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
 + a + b))*(3*a^2 - a*b + 6*b^2)*sqrt(a - b))/(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan
(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^5)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\tan\left(c+d\,x\right)}^3\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

3.320 $\int \sqrt{a+b\sec(c+dx)}\,\tan(c+dx)\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 52, 65, 213}

$$\frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(51) = 102.
time = 0.25, size = 137, normalized size = 2.69

$$\frac{\left(2\sqrt{b+a\cos(c+dx)}+\sqrt{a\cos(c+dx)}\,\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)-\sqrt{a\cos(c+dx)}\,\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{b+a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] ((2*Sqrt[b + a*Cos[c + d*x]] + Sqrt[a*Cos[c + d*x]]*Log[1 - Sqrt[b + a*Cos[
c + d*x]]/Sqrt[a*Cos[c + d*x]]] - Sqrt[a*Cos[c + d*x]]*Log[1 + Sqrt[b + a*C
os[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[b + a
*Cos[c + d*x]])
```

Maple [A]
time = 0.07, size = 42, normalized size = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a+b*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1
/2)))
```

Maxima [A]
time = 0.49, size = 67, normalized size = 1.31

$$\frac{\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
 + sqrt(a))) + 2*sqrt(a + b/cos(d*x + c)))/d
```

Fricas [A]
time = 3.57, size = 192, normalized size = 3.76

$$\left[\frac{\sqrt{a}\,\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2+4\left(2\,a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)+4\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{2\,d},\ \frac{\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+b}\right)+2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))) + 4*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*
```

```
sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
+ c) + b)) + 2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec{(c+dx)}}\,\tan{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(43) = 86.
time = 0.58, size = 185, normalized size = 3.63

$$\frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2b}{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}-\sqrt{a-b}}\right)\mathrm{sgn}(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)
+ sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*b/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2
 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d
*x + 1/2*c)^2 + a + b) - sqrt(a - b)))*sgn(cos(d*x + c))/d
```

Mupad [**B**]
time = 1.86, size = 47, normalized size = 0.92

$$\frac{2\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{d}-\frac{2\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + b/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + b/cos(c + d*x))^(1
/2)/a^(1/2)))/d
```

3.321 $\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))*(a+b)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 912, 1301, 212, 213}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
```

```
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{b^2\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right.}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)}+\frac{a+b}{2b^2(a+b-x^2)}+\frac{-a+b}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{c}\right.}{d}$$

$$= \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{(a-b)\text{Subst}\left(\int \right.}{}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.57, size = 224, normalized size = 2.11

$$\frac{\left(2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)+i\sqrt{-a+b}\,\text{ArcTan}\left(\frac{a+a\cos(c+dx)+i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)+i\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-a\cos(c+dx)-i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\right)\sqrt{-a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{\sqrt{a}\,d\sqrt{b+a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((2*Sqrt[a]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] + I*Sq
rt[-a + b]*ArcTan[(a + a*Cos[c + d*x] + I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b +
```

```
a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])] + I*Sqrt[a + b]*ArcTanh[(a - a*Cos
[c + d*x] - I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sq
rt[a + b])])*Sqrt[-(a*Cos[c + d*x])]*Sqrt[a + b*Sec[c + d*x]])/(Sqrt[a]*d*S
qrt[b + a*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(88) = 176$.
time = 0.23, size = 575, normalized size = 5.42

method	result
default	$\frac{\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{4}\,\cos(dx+c)\left(\sqrt{a+b}\,\ln\left(-\frac{2\left(2\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(1+\cos(dx+c))^2}}\,\sqrt{a+b}+2a\cos(dx+c)+b\cos(\right.}{-1+\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)*cos(d*x+c)*((a+b)^(1/2)*l
n(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+
b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*(a-b)^(1/2)-2*a^(1/2)*ln(4
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*
a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)
+2*b)*(a-b)^(1/2)-ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)
^2/(a-b)^(1/2))*a+ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)
^2/(a-b)^(1/2))*b)*(-1+cos(d*x+c))/sin(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)/(a-b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

time = 5.17, size = 2132, normalized size = 20.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2
*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 -
4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 +
2*cos(d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*
x + c)^2 - 2*cos(d*x + c) + 1)))/d, 1/4*(2*sqrt(-a - b)*arctan(2*sqrt(-a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
+ c) + b)) + 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2
 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 +
 b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x +
c) + b)) - 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 -
 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b
^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)
^2 - 2*cos(d*x + c) + 1)))/d, -1/2*(sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a)*log(-8*a^2*cos(
d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x +
 c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/d, -1/4*(4*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^
2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/d, -1/4*(4*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + b)) - 2*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/co
```

```
s(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a - b)*log(-(
(8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 +
b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*
a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*
(4*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d
*x + c)/(2*a*cos(d*x + c) + b)) + 2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/d, -1/2*(2*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + sqrt(-a + b)*arc
tan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((
2*a - b)*cos(d*x + c) + b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)))/d
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec{(c + dx)}}\cot{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + d\,x)\,\sqrt{a + \frac{b}{\cos{(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

3.322 $\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}\,d}$$

[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+a*arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+a*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)-1/2*cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.21, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3970, 912, 1329, 1192, 12, 1107, 212, 1184, 213}

$$-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]

[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (a*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) - (3*b*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) + (a*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) + (3*b*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]])/(2*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{b^4\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{(2b^4)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{(2b^2)\,\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{(2a}{}$$

$$= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} + \frac{(2}{}$$

$$= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} + \frac{aS}{}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a-}}\right.}{\sqrt{a-b}\,d}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a-}}\right.}{\sqrt{a-b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.24, size = 331, normalized size = 1.54

$$-\frac{\sqrt{-a\cos(c+dx)}\left(8a\sqrt{-a+b}\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)-i\sqrt{a}(4a-3b)\sqrt{a+b}\,\text{ArcTan}\left(\frac{a+a\cos(c+dx)+i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)+i\sqrt{a}\sqrt{-a+b}(4a+3b)\tanh^{-1}\left(\frac{a-a\cos(c+dx)-i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)+\frac{2a\sqrt{-a+b}\sqrt{a+b}\sqrt{b+a\cos(c+dx)}\cot^2(c+dx)}{\sqrt{-a\cos(c+dx)}}\right)\sqrt{a+b\sec(c+dx)}}{4a\sqrt{-a+b}\sqrt{a+b}\,d\sqrt{b+a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -1/4*(Sqrt[-(a*Cos[c + d*x])]*(8*a*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[Sqrt[b +
 a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - I*Sqrt[a]*(4*a - 3*b)*Sqrt[a +
b]*ArcTan[(a + a*Cos[c + d*x] + I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c
+ d*x]])/(Sqrt[a]*Sqrt[-a + b])] + I*Sqrt[a]*Sqrt[-a + b]*(4*a + 3*b)*ArcTa
nh[(a - a*Cos[c + d*x] - I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]]
```

```
)/(Sqrt[a]*Sqrt[a + b])] + (2*a*Sqrt[-a + b]*Sqrt[a + b]*Sqrt[b + a*Cos[c +
 d*x]]*Cot[c + d*x]^2)/Sqrt[-(a*Cos[c + d*x])])*Sqrt[a + b*Sec[c + d*x]])/(
a*Sqrt[-a + b]*Sqrt[a + b]*d*Sqrt[b + a*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2843 vs. $2(177) = 354$.
time = 0.20, size = 2844, normalized size = 13.23

method	result	size
default	Expression too large to display	2844

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))*(-16*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)*(a-b)^(3/2)*(a+b)^(1/2)-4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*4^(1/2)*(a-b)^(3/2)*(a+b)^(1/2)*a+8*cos(d*x+c)^2*ln(4
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*
a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)
+2*b)*4^(1/2)*(a-b)^(3/2)*a^(1/2)*(a+b)^(1/2)*b-8*ln(4*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)*(a-b)^(
3/2)*a^(3/2)*(a+b)^(1/2)+4*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*co
s(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*(a+b)^(1/2)*a^3+3*cos(d*x+c)^
2*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*
4^(1/2)*(a+b)^(1/2)*b^3+7*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/
2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))
*4^(1/2)*(a-b)^(3/2)*a*b+3*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/s
in(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*(a+b)^(1/2)*a^2*b+4*ln(-(-1+cos(d*x+c))*(2
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2
)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*co
s(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*(a+b)^(1/2)*a*b^
2+4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*4^(1/2)
*(a-b)^(3/2)*(a+b)^(1/2)*b+8*cos(d*x+c)^2*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)*(a-b)^(3/2)*a^(
3/2)*(a+b)^(1/2)-7*cos(d*x+c)^2*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+
```

```
b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*
x+c)))*4^(1/2)*(a-b)^(3/2)*a*b-3*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))*(2*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c
)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*(a+b)^(1/2)*a^2*b-4*cos
(d*x+c)^2*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)
^(1/2))*4^(1/2)*(a+b)^(1/2)*a*b^2-8*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)*(a-b)^(3/2)*a^(1/2)*(
a+b)^(1/2)*b+4*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*4^(1/2)*(a-b)^(3/2)*(a+b)^(1/2)*a-4*cos(d*x+c)^2*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*4^(1/2)*(a-b)^(3/2)*(a+b)^(1/2)*b+4*ln(-2*
(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1
/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*(a-b)^(3/2)*a^2+3*ln(-2
*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(
1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*(a-b)^(3/2)*b^2-4*ln(-
(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos
(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*
(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2
)*(a+b)^(1/2)*a^3-3*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+
c)^2/(a-b)^(1/2))*4^(1/2)*(a+b)^(1/2)*b^3-4*cos(d*x+c)^2*ln(-2*(2*cos(d*x+c
)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(
d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c
))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*(a-b)^(3/2)*a^2-3*cos(d*x+c)^2*ln(-
2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^
(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*(a-b)^(3/2)*b^2-16*co
s(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)
*(a+b)^(1/2)-32*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(
3/2)*(a-b)^(3/2)*(a+b)^(1/2))*((b+a*cos(d*x+c))...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(177) = 354$.
time = 8.15, size = 3523, normalized size = 16.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2
 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x +
c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*co
s(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*cos(d
*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 -
(a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)^2 - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3
*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x +
 c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) -
 b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b
+ 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*(
(2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*
(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 2*((4*
a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arcta
n(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*
a - b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x +
 c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) +
((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*lo
g(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^
2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2
*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^
2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + ((4*a^2 + a*b - 3*b^2)*cos(d*x
 + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*
```

```
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b))
- ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)
*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)
/((2*a + b)*cos(d*x + c) + b)) + 4*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(
d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d
*x + c)^2 - a^2 + b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^2
)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(
d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*c
os(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt
(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x
 + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^
2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b
 + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^
2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
 b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*
sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos
(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x +
 c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(
-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec (c + dx)}\, \cot^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(177) = 354.

time = 0.99, size = 514, normalized size = 2.39

$$\frac{\left(\frac{16\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,(4\,a+3\,b)\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\frac{(4\,a-3\,b)\log\left(\left|\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}\right)(a-b)-\sqrt{a-b}\,a\right|\right)}{\sqrt{a-b}}+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}-\frac{2\left(\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}\right)a-(a+b)\sqrt{a-b}\right)}{\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}\right)^{2}-a-b}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*
d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
+ b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 3*b)*arctan(-(sqrt(a - b)
*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1
/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) +
 (4*a - 3*b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d
*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
 b))*(a - b) - sqrt(a - b)*a))/sqrt(a - b) + sqrt(a*tan(1/2*d*x + 1/2*c)^4
- b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - 2*((sqrt
(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*
d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*a - (a + b)*sqrt(a -
b))/((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 -
b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - a - b))
*sgn(cos(d*x + c))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

3.323 $\int \sqrt{a+b\sec(c+dx)}\,\tan^2(c+dx)\,dx$

Optimal. Leaf size=344

$$-\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{3b^2d}$$

```
[Out] -2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*(a+b*sec(d*x+c))^
(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.26, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4142, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2\sqrt{a+b}\,(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}+\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a + 2*b
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4142

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^
m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[
a*A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m,
0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,\tan^2(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\,\left(-1+\sec^2(c+dx)\right)\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} - b\sec(c+dx) +}{\sqrt{a+b\sec(c}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2} - b\right)\sec(c}{\sqrt{a+b\sec(c+dx}}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{3b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{3b^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.54, size = 692, normalized size = 2.01

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*((-I)*a*(a - b)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*b*EllipticF[I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - (6*I)*a*b*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + a*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2)))/(3*b*Sqrt[(
-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Ta
n[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b - b*Tan[(c + d*x)/2]^4 + a*(
-1 + Tan[(c + d*x)/2]^2)^2)) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*x]
)/(3*b) + (2*Tan[c + d*x])/3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(309) = 618$.
time = 0.26, size = 1109, normalized size = 3.22

method	result	size
default	Expression too large to display	1109

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] -2/3/d*(-1+cos(d*x+c))^2*(4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-6*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)^3*a^2+cos(d*x+c)^3*a*b-cos(d*x+c)^2*a^2+b*cos(d*x+c)^2*a+cos(d*x+c)^2*b^2-2*cos(d*x+c)*a*b-b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(c + dx \right)}} \tan^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.324 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}\,(a$$

$$\sqrt{a+b}\,d$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)*(
b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3865}

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+d}{a+b\sec(c+d}}}{\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.26, size = 151, normalized size = 1.21

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-a+b)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2a\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{a+b\sec(c+dx)}}{d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.18, size = 215, normalized size = 1.72

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}(1+\cos(dx+c))^2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)a-\text{Elli}\right.}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{b}{\cos(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2), x)
```

3.325 $\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{a+b}\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

```
[Out] cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/d+2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-
b)/(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2
)*(b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)-cot(d*x+c)*(a+b*s
ec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.15, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3981, 3865, 3960, 3917}

$$\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/d +
(2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c +
 d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x]
))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x]))
/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \int \left(-\sqrt{a+b\sec(c+dx)}+\csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\right)\,dx\\ &= -\int \sqrt{a+b\sec(c+dx)}\,dx+\int \csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\\ &= -\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(-\right.\right.}{}\\ &= \frac{\sqrt{a+b}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1}{}}}{d}\end{aligned}$$

Mathematica [A]
time = 3.69, size = 154, normalized size = 0.63

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\cot(c+dx)-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\left((-2a+b)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+4a\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}}{b+a\cos(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(-Cot[c + d*x] - (2*Cos[(c + d*x)/2]^2*((-2*a + b
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*a*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sqrt[(1 + Sec[c + d*x])^(-1)]*
Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/(b + a*Cos[c + d*x
])))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(226) = 452$.
time = 0.35, size = 628, normalized size = 2.55

method	result
default	$-\frac{(-1+\cos(dx+c))^2\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)a\sin(dx+c)\cos(dx+c)-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*sin(d*x+c)*cos(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)*cos(d*x+c)-4*a*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)+2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-4*a
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x
+c)+cos(d*x+c)^2*a+b*cos(d*x+c))*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{(c + dx)}} \cot^2{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{cot}(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.326 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d}$$

```
[Out] 2/3*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d-6/5*a*(a+b*sec(d*x+c))^(5/2)
/b^4/d+2/7*(a+b*sec(d*x+c))^(7/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^
(1/2))/d/a^(1/2)-2*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1167, 213}

$$\frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d}-\frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}-\frac{6a(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sec[c + d*x]])/(b^4*d) + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c +
d*x])^(3/2))/(3*b^4*d) - (6*a*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) + (2*(a
 + b*Sec[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\ &= \frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}\\ &= \frac{2\text{Subst}\left(\int \left(-a^3+2ab^2+(3a^2-2b^2)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)dx, x, \sqrt{a+b\text{s}\right.}{b^4 d}\\ &= -\frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4 d}+\frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4 d}-\frac{6a}{}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4 d}+\frac{2(3a}{}\end{aligned}$$

Mathematica [A]
time = 6.34, size = 248, normalized size = 1.68

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{8a(-12a^2+35b^2)}{105b^4}-\frac{4(-12a^2+35b^2)\sec(c+dx)}{105b^3}-\frac{12a\sec^2(c+dx)}{35b^2}+\frac{2\sec^3(c+dx)}{7b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}\left(-\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)+\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\sin(c+dx)\tan(c+dx)}{ad\,(1-\cos^2(c+dx))\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((8*a*(-12*a^2 + 35*b^2))/(105*b^4) - (4
*(-12*a^2 + 35*b^2)*Sec[c + d*x])/(105*b^3) - (12*a*Sec[c + d*x]^2)/(35*b^2
) + (2*Sec[c + d*x]^3)/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[a*Cos[c
 + d*x]]*Sqrt[b + a*Cos[c + d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a
```

```
*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*S
in[c + d*x]*Tan[c + d*x])/(a*d*(1 - Cos[c + d*x]^2)*Sqrt[a + b*Sec[c + d*x]
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4996 vs. $2(128) = 256$.
time = 0.41, size = 4997, normalized size = 33.76

method	result	size
default	Expression too large to display	4997

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/420/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(-1+cos(
d*x+c))^4*(-105*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)
^2/(a-b)^(1/2))*cos(d*x+c)^6*a^7+105*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d
*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^7-100*cos(d*x+c)^2*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a*b^3-72*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)*(a-b)^(3/2)*a^2*b^
2-192*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-
b)^(3/2)*a^4*b+524*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*(a-b)^(3/2)*a^2*b^3-210*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*cos(d*x+c)^6*(a-b)^(3/2)*a^(3/2
)*b^4-192*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*(a-b)^(3/2)*a^4*b+524*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(3/2)*a^3*b^2+524*cos(d*x+c)^5*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(3/2)*a^2*b^3+36*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^6*(a-b)^(3/2)*a^2*b^2+96*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^5*(a-b)^(3/2)*a^3*b-
280*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^5*(a-b)
^(3/2)*a*b^3+36*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)*(a-b)^(3/2)*a^2*b^2+288*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a^3*b-780*cos(d*x+c)^3*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a*b^3-216*cos(d*x+c)^2*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a^2*b^2+180*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)*(a-b)^(3/2)*a*
b^3+524*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(
a-b)^(3/2)*a^3*b^2-192*cos(d*x+c)^6*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(3/2)*a^5+210*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c)
```

```
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*
x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^6*b+105*ln(-2*(-1+cos(d*x+
c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b
)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^5*b
^2-420*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^
(1/2))*cos(d*x+c)^6*a^4*b^3+315*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c
))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+
c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^3*b^4-105*ln(-2*(-1+cos(d*x+
c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b
)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^2*b
^5-210*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1
/2))*cos(d*x+c)^6*a^6*b-105*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/
sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^5*b^2+420*ln(-(-1+cos(d*x+c))*(2*(
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+
2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(
d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^4*b^3-315*l
n(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*
cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos
(d*x+c)^6*a^3*b^4+105*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*
x+c)^2/(a-b)^(1/2))*cos(d*x+c)^6*a^2*b^5-105*ln(-2*(-1+cos(d*x+c))*(2*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((
b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+
c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*co...
```

Maxima [A]

time = 0.49, size = 175, normalized size = 1.18

$$\frac{\frac{105\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{30\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}}{b^4}-\frac{126\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{b^4}+\frac{210\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}{b^4}-\frac{210\sqrt{a+\frac{b}{\cos(dx+c)}}\,a^3}{b^4}-\frac{140\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{b^2}+\frac{420\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}{b^2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c
)) + sqrt(a)))/sqrt(a) + 30*(a + b/cos(d*x + c))^(7/2)/b^4 - 126*(a + b/cos
(d*x + c))^(5/2)*a/b^4 + 210*(a + b/cos(d*x + c))^(3/2)*a^2/b^4 - 210*sqrt(
a + b/cos(d*x + c))*a^3/b^4 - 140*(a + b/cos(d*x + c))^(3/2)/b^2 + 420*sqrt
(a + b/cos(d*x + c))*a/b^2)/d
```

Fricas [A]
time = 4.08, size = 381, normalized size = 2.57

$$\left[\frac{105\,\sqrt{a}\,b^{4}\cos\left(dx+c\right)^{3}\log\left(-8\,a^{2}\cos\left(dx+c\right)^{2}-8\,ab\cos\left(dx+c\right)-b^{2}+4\left(2\,a\cos\left(dx+c\right)^{2}+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\right)-4\left(18\,a^{2}b^{2}\cos\left(dx+c\right)-15\,ab^{3}+4\left(12\,a^{4}-35\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{3}-2\left(12\,a^{3}b-35\,ab^{3}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}}{210\,ab^{4}d\cos\left(dx+c\right)^{3}},\ \frac{105\,\sqrt{-a}\,b^{4}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+b}\right)\cos\left(dx+c\right)^{3}-2\left(18\,a^{2}b^{2}\cos\left(dx+c\right)-15\,ab^{3}+4\left(12\,a^{4}-35\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{3}-2\left(12\,a^{3}b-35\,ab^{3}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}}{105\,ab^{4}d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*b^4*cos(d*x + c)^3*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) - 4*(18*a^2*b^2*cos(d*x + c) - 15*a*b^3 +
4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*b - 35*a*b^3)*cos(d*x +
c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b^4*d*cos(d*x + c)^3), 1/
105*(105*sqrt(-a)*b^4*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^3 - 2*(18*a^2*b^2*co
s(d*x + c) - 15*a*b^3 + 4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*
b - 35*a*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b
^4*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}{\left(c+dx\right)}}{\sqrt{a+b\sec{\left(c+dx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. 2(128) = 256.
time = 2.12, size = 699, normalized size = 4.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
 + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(105*(sqrt(a - b)*tan(1/2*d*x +
 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*
tan(1/2*d*x + 1/2*c)^2 + a + b))^6 - 840*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^
2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b))^5*sqrt(a - b) + 35*(sqrt(a - b)*tan(1/2*d*x + 1/2*
c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1
/2*d*x + 1/2*c)^2 + a + b))^4*(27*a - 23*b) + 280*(sqrt(a - b)*tan(1/2*d*x
+ 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a
*tan(1/2*d*x + 1/2*c)^2 + a + b))^3*(3*a + 4*b)*sqrt(a - b) - 21*(sqrt(a -
b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x +
 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(65*a^2 - 2*a*b - 15*b^2
) + 315*a^3 + 707*a^2*b - 7*a*b^2 - 55*b^3 - 56*(sqrt(a - b)*tan(1/2*d*x +
1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*t
an(1/2*d*x + 1/2*c)^2 + a + b))*(19*a*b + 5*b^2)*sqrt(a - b))/(sqrt(a - b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/
2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^7)/(d*sgn(cos(d
*x + c)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^5}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2), x)
```

3.327 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}$$

```
[Out] 2/3*(a+b*sec(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/
d/a^(1/2)-2*a*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1167, 213}

$$\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a + b
*Sec[c + d*x]])/(b^2*d) + (2*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^2d}\\ &= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}\\ &= -\frac{2\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d}\\ &= -\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{\vphantom{a}}\right.}{d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2(a+b\sec(c+}{3b^2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(79) = 158.
time = 1.17, size = 194, normalized size = 2.46

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{4a}{3b^2}+\frac{2\sec(c+dx)}{3b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{a\cos(c+dx)}\,\sqrt{b+a\cos(c+dx)}\left(-\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)+\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\sin(c+dx)\tan(c+dx)}{ad\left(1-\cos^2(c+dx)\right)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-4*a)/(3*b^2) + (2*Sec[c + d*x])/(3*b)
))/(d*Sqrt[a + b*Sec[c + d*x]]) + (Sqrt[a*Cos[c + d*x]]*Sqrt[b + a*Cos[c +
d*x]]*(-Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sq
rt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])*Sin[c + d*x]*Tan[c + d*x])/(a
*d*(1 - Cos[c + d*x]^2)*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3002 vs. $2(67) = 134$.

time = 0.27, size = 3003, normalized size = 38.01

method	result	size
default	Expression too large to display	3003

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^3*(3*cos
(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)
^(1/2))*a*b^4+3*cos(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-
b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b-6*cos(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*
((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)
+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos
(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^2+6*cos(d*x+c)^3*ln
(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^
2*b^3-3*cos(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d
*x+c)^2/(a-b)^(1/2))*a*b^4-4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)*(a-b)^(3/2)*a*b+8*cos(d*x+c)^4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)*(a-b)^(3/2)*a^3+8*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(3/2)*a^3-3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))
*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(
1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a
*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^5+3*cos(d*x+c)^4*ln
(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^
5+6*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^
2/(a-b)^(1/2))*a^4*b-6*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c)
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*
x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^2+3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)
```

```
)*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^
(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^3-6*cos(d*x+c)
^4*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2
))*a^4*b+6*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/si
n(d*x+c)^2/(a-b)^(1/2))*a^3*b^2-3*cos(d*x+c)^4*ln(-2*(-1+cos(d*x+c))*(2*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*
((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*
x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^3-3*cos(d*x+c)^3*ln(-(
-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(
d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(
a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b+6
*cos(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(
a-b)^(1/2))*a^3*b^2-6*cos(d*x+c)^3*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x
+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^3+8*cos(d*x+c)^2*((b+a*cos(d*x+c))*c
os(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(3/2)*a^2*b-6*cos(d*x+c)^4*ln(4*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b
)*(a-b)^(3/2)*a^(3/2)*b^2-6*cos(d*x+c)^3*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*(a-b)^(3/2)*a^(1/2)*b^3-
12*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^
(3/2)*a*b-4*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/
2)*(a-b)^(3/2)*a*b-12*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(3/2)*(a-b)^(3/2)*a*b+8*cos(d*x+c)^3*((b+a...
```

Maxima [A]

time = 0.49, size = 92, normalized size = 1.16

$$-\frac{\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\,\left(a+\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{3}{2}}}{b^2}+\frac{6\,\sqrt{a+\frac{b}{\cos\left(dx+c\right)}}\,a}{b^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
+ sqrt(a)))/sqrt(a) - 2*(a + b/cos(d*x + c))^(3/2)/b^2 + 6*sqrt(a + b/cos(d
*x + c))*a/b^2)/d
```

Fricas [A]
time = 3.55, size = 273, normalized size = 3.46

$$\left[\frac{3\sqrt{a}\,b^{2}\cos\left(dx+c\right)\log\left(-8\,a^{2}\cos\left(dx+c\right)^{2}-8\,ab\cos\left(dx+c\right)-b^{2}-4\left(2\,a\cos\left(dx+c\right)^{2}+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\right)-4\left(2\,a^{2}\cos\left(dx+c\right)-ab\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}}{6\,ab^{2}d\cos\left(dx+c\right)},\ -\frac{3\sqrt{-a}\,b^{2}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+b}\right)\cos\left(dx+c\right)+2\left(2\,a^{2}\cos\left(dx+c\right)-ab\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}}{3\,ab^{2}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*b^2*cos(d*x + c)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x
+ c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c))) - 4*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*b^2*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d
*x + c) + b))*cos(d*x + c) + 2*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(67) = 134.
time = 0.96, size = 265, normalized size = 3.35

$$-\frac{2\left(\frac{3\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-2\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\left(3\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-2\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a+b}\right)^{2}-3\,a-b\right)}{\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-2\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a+b}-\sqrt{a-b}\right)^{3}}\right)}{3\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*
x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(3*(sqrt(a - b)*tan(1/2*d*x + 1/2*
c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1
/2*d*x + 1/2*c)^2 + a + b))^2 - 3*a - b)/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^
2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^3)/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

3.328 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3970, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(31) = 62.
time = 0.22, size = 108, normalized size = 3.48

$$\frac{\sqrt{b+a\cos(c+dx)}\left(\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)-\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)}{d\sqrt{a\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cos[c + d*x]]*(Log[1 - Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c +
d*x]]] - Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]]))/(d*Sqrt[a
*Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Maxima [A]
time = 0.50, size = 49, normalized size = 1.58

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(a
)))/(sqrt(a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.
time = 2.26, size = 145, normalized size = 4.68

$$\left[\frac{\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2+4\left(2\,a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)}{2\sqrt{a}\,d},\ \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+b}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
sqrt(a)*d), sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(25) = 50.
time = 0.65, size = 102, normalized size = 3.29

$$\frac{2 \arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/
2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) + s
qrt(a - b))/sqrt(-a))/(sqrt(-a)*d*sgn(cos(d*x + c)))
```

Mupad [B]
time = 1.63, size = 27, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*atanh((a + b/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.329 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 912, 1184, 212, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
```

```
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^2(a-x^2)}+\frac{1}{2b^2(a+b-x^2)}-\frac{1}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\sec(c+
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+
}{d} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{\tanh^{-1}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.07, size = 251, normalized size = 2.37

$$-\frac{i\left(-\sqrt{a+b}\left(2i\sqrt{-a+b}\,\text{ArcTan}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)+\sqrt{a}\,\text{ArcTan}\left(\frac{a+a\cos(c+dx)+i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\right)+\sqrt{a}\sqrt{-a+b}\tanh^{-1}\left(\frac{a-a\cos(c+dx)-i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\right)\sqrt{b+a\cos(c+dx)}}{\sqrt{-a+b}\sqrt{a+b}\,d\sqrt{-a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((-I)*(-(Sqrt[a + b]*((2*I)*Sqrt[-a + b]*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sq
rt[-(a*Cos[c + d*x])]] + Sqrt[a]*ArcTan[(a + a*Cos[c + d*x] + I*Sqrt[-(a*Co
s[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])])) + Sqrt[a]*
Sqrt[-a + b]*ArcTanh[(a - a*Cos[c + d*x] - I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b
 + a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[a + b])])*Sqrt[b + a*Cos[c + d*x]])/(Sqrt
[-a + b]*Sqrt[a + b]*d*Sqrt[-(a*Cos[c + d*x])]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(88) = 176$.
time = 0.22, size = 691, normalized size = 6.52

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{4}\,\cos(dx+c)\left(2(a-b)^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(4\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(1+\cos(dx+c))^{2}}}\,\cos(dx+c)\sqrt{a}+4\sqrt{a}\,\sqrt{\frac{(b+a\cos(\cdots}{(1+\cdots}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)*cos(d*x+c)*(2*(a-b)^(3/2
)*a^(3/2)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x
+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+
4*a*cos(d*x+c)+2*b)+2*(a-b)^(3/2)*a^(1/2)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b-(a-b)^(3/2)*(a+b)^(1/
2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a+ln(-(-1+cos(d*x+c))*
(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1
/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*
cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3-ln(-(-1+cos(d*x+c)
)*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^
(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a*b^2)*(-1+cos(d*x+c
))/sin(d*x+c)^2/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)/(a-b)^
(3/2)/(a+b)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(88) = 176$.
time = 5.94, size = 2420, normalized size = 22.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c)
- b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3
- a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - (a^2 + a*b)
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c
)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d
*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d), -1/4*(2*(a^2 + a*b)*sq
rt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*c
os(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 2*(a^2 - b^2)*sqrt(a)*log(-8*a^
2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos
(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^2 - a*b)*s
qrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*
cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) + 2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c)
+ b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)
*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/co
```

```
s(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x +
 c) + 1)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*b)*sqrt(-a - b)*arctan(2*sqrt
(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*co
s(d*x + c) + b)) + 2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d
), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2 - a*b)*sqrt(-a
 - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2
- 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d
*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3
*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*
d), -1/2*((a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 -
a*b)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 - b^2)*sqrt(a)*log(-
8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^3 - a*
b^2)*d), -1/2*(2*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (a^2 + a*b)*sqr
t(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*co
s(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)*sqrt(-a - b)*arctan(
2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a +
 b)*cos(d*x + c) + b)))/((a^3 - a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

3.330 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=260

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d}+$$

```
[Out] -1/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+1/4*b*arct
anh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-2*arctanh((a+b*sec(d*
x+c))^(1/2)/a^(1/2))/d/a^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/
d/(a-b)^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/4
*(a+b*sec(d*x+c))^(1/2)/(a+b)/d/(1-sec(d*x+c))+1/4*(a+b*sec(d*x+c))^(1/2)/(
a-b)/d/(1+sec(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3970, 912, 1252, 212, 205, 213}

$$\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)(\sec(c+dx)+1)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - (b*ArcTanh[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) + (b*ArcTanh[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) + ArcTanh[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]]/(Sqrt[a + b]*d) + Sqrt[a + b*Sec[c + d*x]]/(4*(a + b)*d*(1 -
 Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{b^4\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\,(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= \frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{b^4(a-x^2)} + \frac{1}{4b^3(a+b-x^2)^2} + \frac{1}{2b^4(a+b-x^2)} - \frac{1}{4b^3(-a+b+x^2)^2} - \frac{1}{2b^4(-a+\cdots}\right.\right.}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+\cdots}\right.}{d} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh\cdots}{\cdots} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{b\tan\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.19, size = 329, normalized size = 1.27

$$
\frac{\cos(c+dx)\left(\frac{a\left((a+b)^{3/2}\left(-8(-a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)-i\sqrt{a}(4a-5b)\text{ArcTan}\left(\frac{a+a\cos(c+dx)+i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\right)+i\sqrt{a}\sqrt{-a+b}(4a^2+ab-5b^2)\tanh^{-1}\left(\frac{a-a\cos(c+dx)-i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\right)\sqrt{b+a\cos(c+dx)}}{(-a+b)^{3/2}(a+b)^{3/2}(-a\cos(c+dx))^{3/2}}+\frac{(-2a^2\cot^2(c+dx)+2b^2\csc^2(c+dx))\sec(c+dx)}{a^2-b^2}\right)}{4d\sqrt{a+b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*((a*((a + b)^(3/2)*(-8*(-a + b)^(3/2)*ArcTan[Sqrt[b + a*Cos[c
 + d*x]]/Sqrt[-(a*Cos[c + d*x])]] - I*Sqrt[a]*(4*a - 5*b)*ArcTan[(a + a*Cos
[c + d*x] + I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sq
rt[-a + b])]) + I*Sqrt[a]*Sqrt[-a + b]*(4*a^2 + a*b - 5*b^2)*ArcTanh[(a - a
*Cos[c + d*x] - I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a
]*Sqrt[a + b])])*Sqrt[b + a*Cos[c + d*x]])/((-a + b)^(3/2)*(a + b)^(3/2)*(-
(a*Cos[c + d*x]))^(3/2)) + ((-2*a^2*Cot[c + d*x]^2 + 2*b^2*Csc[c + d*x]^2)*
Sec[c + d*x])/(a^2 - b^2)))/(4*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4202 vs. $2(218) = 436$.
time = 0.21, size = 4203, normalized size = 16.17

method	result	size

default	Expression too large to display	4203

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/16/d*(-1+cos(d*x+c))*(-8*(a-b)^(3/2)*a^(7/2)*(a+b)^(1/2)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)-16*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^2-4*cos(d*x+c)^2*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^4+16*cos(d*x+c)^2*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b+4*cos(d*x+c)^2*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^5-32*cos(d*x+c)*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a^2+16*(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*a*b+5*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^3*b-4*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^2*b^2-5*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a*b^3+(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^4*b+9*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^3*b^2-(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a^2*b^3-5*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*4^(1/2)*a*b^4+8*cos(d*x+c)^2*(a-b)^(3/2)*a^(5/2)*(a+b)^(1/2)*ln(4*((b+a*

```
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4
^(1/2)*b-8*cos(d*x+c)^2*(a-b)^(3/2)*a^(3/2)*(a+b)^(1/2)*ln(4*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)*b
^2-8*cos(d*x+c)^2*(a-b)^(3/2)*a^(1/2)*(a+b)^(1/2)*ln(4*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)*b^3-16*
(a-b)^(3/2)*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2
)*a^2+4*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos
(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/
2)*a^4-4*(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x
+c)^2/(a-b)^(1/2))*4^(1/2)*a^5-5*cos(d*x+c)^2*(a-b)^(3/2)*ln(-2*(2*cos(d*x+
c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos
(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^3*b+4*cos(d*x+c)^2*(a-b)^(3/2)*l
n(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a+
b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*a^2*b^2+5*cos(d*x+
c)^2*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*
x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*4^(1/2)*
a*b^3+8*(a-b)^(3/2)*a^(3/2)*(a+b)^(1/2)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*4^(1/2)*b^2-cos(d*x+c)^2*
(a+b)^(1/2)*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(216) = 432.
time = 52.93, size = 4336, normalized size = 16.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/16*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 -
 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x +
 c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3
 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a
^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2
*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b
 - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x +
c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*
a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b
- 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b
^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sq
rt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d
*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*
x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b
^4)*d), -1/16*(2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b
- 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c
) + b)) + 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2
)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos
(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2
 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x
 + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*s
qrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(c
os(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 -
(a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/
16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c
```

```
)/(2*a*cos(d*x + c) + b)) + 2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a
^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*
sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b
)*cos(d*x + c) + b)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*
a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a
*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x +
c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)
*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*
cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2
 + a*b^4)*d), 1/16*(2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a
^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
 + c) + b)) - 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
c)^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))) - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2
*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos
(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c)
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*((a^4 - a^2*b^2)*cos(d*x + c)^
2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d),
 -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(216) = 432.
time = 1.10, size = 546, normalized size = 2.10

$$\frac{\frac{(4a-5b)\sqrt{a-b}\log\left(\left|\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}\right)(a-b)-\sqrt{a-b}\,a\right|\right)}{a^2-2ab+b^2}+\frac{16\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2(4a+5b)\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}+\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}}{a-b}-\frac{2\left(\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}\right)a-(a+b)\sqrt{a-b}\right)}{\left(\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}\right)^2-a-b\right)(a+b)}}{8d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((4*a - 5*b)*sqrt(a - b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 -
sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x
+ 1/2*c)^2 + a + b))*(a - b) - sqrt(a - b)*a))/(a^2 - 2*a*b + b^2) + 16*arc
tan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^
4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) + sqrt(a
 - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 5*b)*arctan(-(sqrt(a - b)*tan(1/2*d*x
+ 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a
*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) + sq
rt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x +
1/2*c)^2 + a + b)/(a - b) - 2*((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a
*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*
c)^2 + a + b))*a - (a + b)*sqrt(a - b))/(((sqrt(a - b)*tan(1/2*d*x + 1/2*c)
^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^2 + a + b))^2 - a - b)*(a + b)))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

3.331 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=404

$$-\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}{ad}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d-2/15*(a-b)*(8*a^2-21*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d
*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/15*(-8*a^2+2*a*b+21
*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(b*(1+sec(d*x+c))/(-a+
b))^(1/2)/b^3/d-8/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+
c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.53, antiderivative size = 610, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3980, 3869, 3922, 3917, 4089, 3945, 4167, 4090}

$$-\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}-\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{4(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{4\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15b^2d}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8*a^
2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(15*b^4*d) + (4*Sqrt[a + b]*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) -
(2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(5*b*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \left(\frac{1}{\sqrt{a+b\sec(c+dx)}} - \frac{2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}} + \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right) dx$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec^4(}{\sqrt{a+b\,\mathrm{s}}}$$

$$= -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{s}}{c}}}{ad}$$

$$= \frac{4(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{b^2d}$$

$$= \frac{4(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{b^2d}$$

$$= \frac{4(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{b^2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 835 vs. 2(404) = 808.

time = 17.11, size = 835, normalized size = 2.07

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^
2)^(-1)]*(8*a^3*Tan[(c + d*x)/2] + 8*a^2*b*Tan[(c + d*x)/2] - 21*a*b^2*Tan[
(c + d*x)/2] - 21*b^3*Tan[(c + d*x)/2] - 16*a^3*Tan[(c + d*x)/2]^3 + 42*a*b
^2*Tan[(c + d*x)/2]^3 + 8*a^3*Tan[(c + d*x)/2]^5 - 8*a^2*b*Tan[(c + d*x)/2]
^5 - 21*a*b^2*Tan[(c + d*x)/2]^5 + 21*b^3*Tan[(c + d*x)/2]^5 - 30*b^3*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- 30*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*a^3 + 8*a^2*b - 21*a*b^2 - 21*b^3)*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 2*b*(4*a^2 + a*b - 18*b^2)*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
]))/(15*b^3*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(-8*a^2 + 21*b^2)*Sin[c + d
*x])/(15*b^3) - (8*a*Tan[c + d*x])/(15*b^2) + (2*Sec[c + d*x]*Tan[c + d*x])
/(5*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1778 vs. $2(365) = 730$.
time = 0.37, size = 1779, normalized size = 4.40

method	result	size
default	Expression too large to display	1779

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-4*b*a^2*cos(d*x+c)^2+b^2*a*cos(d*x+c)-4*cos(d*x+c)^4*a^2*b-21*cos(d*x
+c)^4*a*b^2-8*a^3*cos(d*x+c)^3-3*b^3-36*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+2
```

```
1*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^3+30*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3-36*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^3+21*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^3+30*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+8*cos(d*x+c)^4*a^3-21*cos(d*x+
c)^3*b^3+24*cos(d*x+c)^2*b^3+8*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b+21*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-
8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b+21*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*cos(d*x+c)^3*a^2*b+20*cos(d*x+c)
^3*a*b^2)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^4}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2), x)
```

3.332 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=310

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{b^2d}$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^
(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]

time = 0.17, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3979, 4144, 4006, 3869, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{-1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \int \frac{-1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{b^2d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{b^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.30, size = 2752, normalized size = 8.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Tan[c + d*x])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (4*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a - b)
*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
 b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (2*I)*b*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
] + Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b
 + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(b^2*Sqrt[(
-a + b)/(a + b)]*d*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[a + b*Sec[c +
 d*x]]*((2*Sec[(c + d*x)/2]^2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c
+ d*x)/2]*((-I)*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] - (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2]))/(b*Sqrt[(-a +
b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4])
 + (a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*((-I)*(a - b)*Elli
pticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (2*I)*b*EllipticP
i[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (
```

```
a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + S
qrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*
Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(b*Sqrt[(-a + b)
/(a + b)]*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4])
 - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a - b)*EllipticE[I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (2*I)*b*EllipticPi[-((a + b)/(a
- b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(-a
 + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*T
an[(c + d*x)/2])*(-(Sec[(c + d*x)/2]^4*Sin[c + d*x]) + 2*Cos[c + d*x]*Sec[(
c + d*x)/2]^4*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(b*Sqrt[(-a + b)
/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*(Cos[c + d*x]*Sec[(c + d*x)/2]^4)^(3/2))
 + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*((Sqr
t[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2)/Sqrt[2] - Sqrt[2]*a*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sin[c + d*x]*Tan[(c + d*x)/2] + (Sqrt[(-a + b
)/(a + b)]*(b + a*Cos[c + d*x])*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d
*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(Sqrt[2]*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]) - ((I/2)*(a - b)*EllipticE[I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2
]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] - (I*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/
(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a +
b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (b*Sqrt[(-a
+ b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)])/((1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b))*Sqrt[1 + ((-a +
 b)*Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a
+ b)]) + ((a - b)*Sqrt[(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2
]^2)/(a - b)])/(2*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])))/(b*Sqr
t[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x
)/2]^4]) + (((-I)*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)] - (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c
+ d*x)/2]^2)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(b*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]])))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(283) = 566$.

time = 0.23, size = 823, normalized size = 2.65

method	result
default	$-\frac{2(1+\cos(dx+c))^2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2
*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*
x+c)*cos(d*x+c)-cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*a-cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a-((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-2*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+cos(d
*x+c)^2*a-a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))/b
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.333 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{ad}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3869}

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{s}}{a}}}{ad}$$

Mathematica [A]
time = 0.22, size = 138, normalized size = 1.30

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sec(c+dx)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 0.26, size = 178, normalized size = 1.68

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,(1+\cos(dx+c))^2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{Ellipt}\right.}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d
*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(1/2), x)
```

3.334 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=361

$$\frac{\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}\,d}-\frac{\cot(c+$$

[Out] cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b)^(1/2)-cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-cot(d*x+c)/d/(a+b*sec(d*x+c))^(1/2)+b^2*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.30, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3981, 3869, 3960, 3918, 21, 3914, 3917, 4089}

$$\frac{b^2\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}-\frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - Cot[c + d*x]/(d*Sqrt[a + b*Sec[c + d*x]]) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \left(-\frac{1}{\sqrt{a+b\sec(c+dx)}}+\frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right)dx$$

$$= -\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(\ldots}{a+\ldots}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(\ldots}{a+\ldots}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(\ldots}{a+\ldots}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(\ldots}{a+\ldots}}}{ad}$$

$$= \frac{\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\ldots}}{\sqrt{a+b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.36, size = 1198, normalized size = 3.32

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(((-b + a*Cos[c + d*x])*Csc[c + d*x])/(-
a^2 + b^2) + (b*Sin[c + d*x])/(-a^2 + b^2)))/(d*Sqrt[a + b*Sec[c + d*x]]) -
 (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(a*b*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c +
d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (4*I)*a^2*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*b^2*EllipticPi[-((a + b)/
(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + (4*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - I*(2*a^2 - a*b - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*Sqrt[a + b*Se
c[c + d*x]]*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2
)]*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1407 vs. $2(330) = 660$.
time = 0.26, size = 1408, normalized size = 3.90

method	result	size
default	Expression too large to display	1408

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(2*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+sin(d
```

```
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2
*sin(d*x+c)*cos(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)+2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)
+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x
+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(
d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a^2*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*a^2-b*cos(d*x+c)^2*a+cos(d*x+c)*a*b-b
^2*cos(d*x+c))*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*co
s(d*x+c))/sin(d*x+c)^5/(a-b)/(a+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.335 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}-\frac{2a(a+b}{}$$

[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*a*(a+b*sec(d*x+c))^(3/2)/b^4/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^4/d+2*(a^2-b^2)^2/a/b^4/d/(a+b*sec(d*x+c))^(1/2)+2*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d

Rubi [A]
time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)^2)/(a*b^4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec[c + d*x]])/(b^4*d) - (2*a*(a + b*Sec[c + d*x])^(3/2))/(b^4*d) + (2*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^4d}\\ &= \frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4d}\\ &= \frac{2\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{(a^2-b^2)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a(a-x^2)}\right)\,dx, x, \sqrt{a+b\text{se}\right.}{b^4d}\\ &= \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}-\frac{2a(a+b\sec}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-2}{}\end{aligned}$$

Mathematica [A]
time = 6.43, size = 263, normalized size = 1.78

$$\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2(16a^4-20a^2b^2+5b^4)}{5a^2b^4}-\frac{2(-a^2+b^2)^2}{a^2b^3(b+a\cos(c+dx))}-\frac{6a\sec(c+dx)}{5b^3}+\frac{2\sec^2(c+dx)}{5b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}-\frac{\sqrt{a\cos(c+dx)}\,(b+a\cos(c+dx))^{3/2}\left(-\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)+\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\tan^2(c+dx)}{a^2d\,(1-\cos^2(c+dx))\,(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(16*a^4 - 20*a^2*b^2 + 5*b^4))/(
5*a^2*b^4) - (2*(-a^2 + b^2)^2)/(a^2*b^3*(b + a*Cos[c + d*x])) - (6*a*Sec[c
 + d*x])/(5*b^3) + (2*Sec[c + d*x]^2)/(5*b^2)))/(d*(a + b*Sec[c + d*x])^(3/
2)) - (Sqrt[a*Cos[c + d*x]]*(b + a*Cos[c + d*x])^(3/2)*(-Log[1 - Sqrt[b + a
```

```
*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] + Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqr
t[a*Cos[c + d*x]]]])*Tan[c + d*x]^2)/(a^2*d*(1 - Cos[c + d*x]^2)*(a + b*Sec[
c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6611 vs. $2(132) = 264$.
time = 0.79, size = 6612, normalized size = 44.68

method	result	size
default	Expression too large to display	6612

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.52, size = 194, normalized size = 1.31

$$\frac{\frac{5\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{10}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}+\frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^4}-\frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^4}+\frac{30\sqrt{a+\frac{b}{\cos(dx+c)}}\,a^2}{b^4}+\frac{10a^3}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,b^4}-\frac{20\sqrt{a+\frac{b}{\cos(dx+c)}}}{b^2}-\frac{20a}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,b^2}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) +
 sqrt(a)))/a^(3/2) + 10/(sqrt(a + b/cos(d*x + c))*a) + 2*(a + b/cos(d*x + c
))^(5/2)/b^4 - 10*(a + b/cos(d*x + c))^(3/2)*a/b^4 + 30*sqrt(a + b/cos(d*x
+ c))*a^2/b^4 + 10*a^3/(sqrt(a + b/cos(d*x + c))*b^4) - 20*sqrt(a + b/cos(d
*x + c))/b^2 - 20*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

Fricas [A]
time = 6.55, size = 467, normalized size = 3.16

$$\left[\frac{5\left(ab^4\cos(dx+c)^3+b^5\cos(dx+c)^2\right)\sqrt{a}\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)-4\left(2a^3b^2\cos(dx+c)-a^2b^3-(16a^5-20a^3b^2+5ab^4)\cos(dx+c)^3-2(4a^4b-5a^2b^3)\cos(dx+c)^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{10\left(a^3b^4d\cos(dx+c)^3+a^2b^5d\cos(dx+c)^2\right)},\ \frac{5\left(ab^4\cos(dx+c)^3+b^5\cos(dx+c)^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)-2\left(2a^3b^2\cos(dx+c)-a^2b^3-(16a^5-20a^3b^2+5ab^4)\cos(dx+c)^3-2(4a^4b-5a^2b^3)\cos(dx+c)^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{5\left(a^3b^4d\cos(dx+c)^3+a^2b^5d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cos(d*x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*cos
(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - 4*(2*a^3*b^2*cos(d
*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2*(4*a
```

```
^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2), 1/5*(5*(a*b^4*cos(d*
x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(2*a^3*b^2*
cos(d*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2
*(4*a^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(c+d\,x\right)}^5}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2), x)
```

3.336 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a+
b*sec(d*x+c))^(1/2)+2*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2))
/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]])/(b^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^2 d}\\ &= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}\\ &= -\frac{2\text{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a(a-x^2)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}\\ &= \frac{2(a^2-b^2)}{ab^2 d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2 d}+\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{\ }\right.}{ad}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2 d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec}}{b^2 d}\end{aligned}$$

Mathematica [A]
time = 0.99, size = 167, normalized size = 1.90

$$\frac{4a^2-2b^2-\frac{b^2\sqrt{b+a\cos(c+dx)}\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)}{\sqrt{a\cos(c+dx)}}+\frac{b^2\sqrt{b+a\cos(c+dx)}\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)}{\sqrt{a\cos(c+dx)}}+2ab\sec(c+dx)}{ab^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a^2 - 2*b^2 - (b^2*Sqrt[b + a*Cos[c + d*x]]*Log[1 - Sqrt[b + a*Cos[c + d
*x]]/Sqrt[a*Cos[c + d*x]]])/Sqrt[a*Cos[c + d*x]] + (b^2*Sqrt[b + a*Cos[c +
d*x]]*Log[1 + Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]])/Sqrt[a*Cos[c
+ d*x]] + 2*a*b*Sec[c + d*x])/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2829 vs. $2(78) = 156$.

time = 0.25, size = 2830, normalized size = 32.16

method	result	size
default	Expression too large to display	2830

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^3*(4*cos(d*x+c)^3*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^4+4*cos(d*x+c)^2*(a-b)^(3/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^4+12*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^2*(a-b)^(3/2)*a^3-cos(d*x+c)^3*ln(-2*(-
1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d
*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a
-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^5*b+co
s(d*x+c)^3*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(
d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a
-b)^(1/2))*a^4*b^2+ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c
)^2/(a-b)^(1/2))*cos(d*x+c)^3*a^5*b-ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c)
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*
x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^3*a^4*b^2-2*ln(-2*(-1+cos(d*x+
c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b
)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)^2*a^4*b
^2+2*cos(d*x+c)^2*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+
c)^2/(a-b)^(1/2))*a^3*b^3+2*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d
*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*c
os(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4*b^2-2*cos(d*x+c)^2*ln(-(-1+cos(d
*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(
a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/
2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^3-4*((b+a
*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a*b^2-ln(-2*(-1
+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*
x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-
b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c
```

```
)*a^3*b^3+cos(d*x+c)*ln(-2*(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d
*x+c)^2/(a-b)^(1/2))*a^2*b^4+ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)
/sin(d*x+c)^2/(a-b)^(1/2))*cos(d*x+c)*a^3*b^3-cos(d*x+c)*ln(-(-1+cos(d*x+c)
)*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^
(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^4+4*((b+a*cos(
d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(3/2)*a^2*b^2+4*cos(d*x+c)
^3*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a^3+12*
cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)
*a^3+2*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)
*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a
*cos(d*x+c)+2*b)*cos(d*x+c)*(a-b)^(3/2)*a^(1/2)*b^4+8*cos(d*x+c)*(a-b)^(3/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^3*b-4*cos(d*x+c)^3
*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a*b^2-12*
cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)
*a*b^2+4*cos(d*x+c)*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)*a^2*b^2+2*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))
^2)^(1/2)+4*a*cos(d*x+c)+2*b)*cos(d*x+c)^3*(a-b)^(3/2)*a^(5/2)*b^2+4*ln(4*(
(b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^
(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2
*b)*cos(d*x+c)^2*(a-b)^(3/2)*a^(3/2)*b^3-12*cos(d*x+c)^2*((b+a*cos(d*x+c))*
cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a*b^2+8*cos(d*x+c)^2*(a-b)^(
3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^3*b+4*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(a-b)^(3/2)*a^3)*cos(d*x+c)*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)*4^(1/2)/(b+a*cos(d*x+c))/((b+a*cos(d*x+c))
*cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)/sin(d*x+c)^6/b^2/(a-b)^(3/2)/a^2
```

Maxima [A]

time = 0.50, size = 110, normalized size = 1.25

$$-\frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}-\frac{2\sqrt{a+\frac{b}{\cos(dx+c)}}}{b^2}-\frac{2a}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt
(a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a) - 2*sqrt(a + b/cos(d*x + c))
/b^2 - 2*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

Fricas [A]
time = 4.58, size = 317, normalized size = 3.60

$$\left[\frac{(ab^2\cos(dx+c)+b^3)\sqrt{a}\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right)+4\left(a^2b+(2a^3-ab^2)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{2\left(a^3b^2d\cos(dx+c)+a^2b^3d\right)}, -\frac{(ab^2\cos(dx+c)+b^3)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)-2\left(a^2b+(2a^3-ab^2)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{a^3b^2d\cos(dx+c)+a^2b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cos(d*x + c) + b^3)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + 4*(a^2*b + (2*a^3 - a*b^2)*cos(d*x + c
))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a^3*b^2*d*cos(d*x + c) + a^2*b
^3*d), -((a*b^2*cos(d*x + c) + b^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2*b
 + (2*a^3 - a*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
a^3*b^2*d*cos(d*x + c) + a^2*b^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(78) = 156.
time = 1.22, size = 258, normalized size = 2.93

$$-\frac{2\left(\frac{\frac{\left(2a^3\mathrm{sgn}(\cos(dx+c))-a^2b\mathrm{sgn}(\cos(dx+c))-ab^2\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2b^2}-\frac{2a^3\mathrm{sgn}(\cos(dx+c))+a^2b\mathrm{sgn}(\cos(dx+c))-ab^2\mathrm{sgn}(\cos(dx+c))}{a^2b^2}}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}}+\frac{\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2*(((2*a^3*sgn(cos(d*x + c)) - a^2*b*sgn(cos(d*x + c)) - a*b^2*sgn(cos(d*x
 + c)))*tan(1/2*d*x + 1/2*c)^2/(a^2*b^2) - (2*a^3*sgn(cos(d*x + c)) + a^2*b
*sgn(cos(d*x + c)) - a*b^2*sgn(cos(d*x + c)))/(a^2*b^2))/sqrt(a*tan(1/2*d*x
 + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b
) + arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) +
 sqrt(a - b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

3.337 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+b*sec(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 53, 65, 213}

$$\frac{2}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{2}{ad\sqrt{a+b\sec(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{ad}\\ &= \frac{2}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2}{ad\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(54) = 108.
time = 0.40, size = 128, normalized size = 2.37

$$\frac{\left(2a\cos(c+dx)+\sqrt{a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}\left(\log\left(1-\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)-\log\left(1+\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{a\cos(c+dx)}}\right)\right)\right)\sec(c+dx)}{a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*a*Cos[c + d*x] + Sqrt[a*Cos[c + d*x]]*Sqrt[b + a*Cos[c + d*x]]*(Log[1 -
 Sqrt[b + a*Cos[c + d*x]]/Sqrt[a*Cos[c + d*x]]] - Log[1 + Sqrt[b + a*Cos[c
+ d*x]]/Sqrt[a*Cos[c + d*x]]]))*Sec[c + d*x])/(a^2*d*Sqrt[a + b*Sec[c + d*x
]])
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec\left(dx+c\right)}}}{d}$	45
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec\left(dx+c\right)}}}{d}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^(3/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+b*sec(d*x+c)
)^(1/2))
```

Maxima [A]
time = 0.47, size = 70, normalized size = 1.30

$$\frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(dx+c\right)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos\left(dx+c\right)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{\cos\left(dx+c\right)}}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(
a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(46) = 92.
time = 3.82, size = 260, normalized size = 4.81

$$\left[\frac{4\,a\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)+b\right)\sqrt{a}\log\left(-8\,a^{2}\cos\left(dx+c\right)^{2}-8\,ab\cos\left(dx+c\right)-b^{2}+4\,\left(2\,a\cos\left(dx+c\right)^{2}+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\right)}{2\,\left(a^{3}d\cos\left(dx+c\right)+a^{2}bd\right)},\ \frac{\left(a\cos\left(dx+c\right)+b\right)\sqrt{-a}\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{2\,a\cos\left(dx+c\right)+b}\right)+2\,a\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{a^{3}d\cos\left(dx+c\right)+a^{2}bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a*cos(d*x
 + c) + b)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4
*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))))/(a^3*d*cos(d*x + c) + a^2*b*d), ((a*cos(d*x + c) + b)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2
*a*cos(d*x + c) + b)) + 2*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c))/(a^3*d*cos(d*x + c) + a^2*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [B]
time = 1.95, size = 50, normalized size = 0.93

$$\frac{2}{a\,d\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] 2/(a*d*(a + b/cos(c + d*x))^(1/2)) - (2*atanh((a + b/cos(c + d*x))^(1/2)/a^
(1/2)))/(a^(3/2)*d)
```

3.338 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}+\frac{}{a\,(a^2}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 912, 1301, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)}\,dx,x,b\sec(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx,x,\sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(\frac{1}{a(a^2-b^2)x^2}-\frac{1}{ab^2(a-x^2)}+\frac{1}{2(a-b)b^2(a-b-x^2)}+\frac{1}{2b^2(a+b)(a+b-x^2)}\right)\,c\right.}{d}$$

$$= \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\sqrt{a+b\sec(c+dx)}\right.}{ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d}-\frac{\tanh}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.58, size = 322, normalized size = 2.27

$$-\frac{ia^{3/2}(-a+b)^{3/2}\tanh^{-1}\left(\frac{a-a\cos(c+dx)-i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\sqrt{b+a\cos(c+dx)}+\sqrt{a+b}\left(2b^2\sqrt{-a+b}\sqrt{-a\cos(c+dx)}-2\sqrt{-a+b}\,(a^2-b^2)\,\text{ArcTan}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)\sqrt{b+a\cos(c+dx)}+ia^{3/2}(a+b)\text{ArcTan}\left(\frac{a+a\cos(c+dx)+i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\sqrt{b+a\cos(c+dx)}\right)}{a(-a+b)^{3/2}(a+b)^{3/2}d\sqrt{-a\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((I*a^(3/2)*(-a + b)^(3/2)*ArcTanh[(a - a*Cos[c + d*x] - I*Sqrt[-(a*Cos[c
+ d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[a + b])]*Sqrt[b + a*Cos[c
+ d*x]] + Sqrt[a + b]*(2*b^2*Sqrt[-a + b]*Sqrt[-(a*Cos[c + d*x])] - 2*Sqrt[
-a + b]*(a^2 - b^2)*ArcTan[Sqrt[b + a*Cos[c + d*x]]/Sqrt[-(a*Cos[c + d*x])]
]*Sqrt[b + a*Cos[c + d*x]] + I*a^(3/2)*(a + b)*ArcTan[(a + a*Cos[c + d*x] +
```

```
I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])
]*Sqrt[b + a*Cos[c + d*x]]))/(a*(-a + b)^(3/2)*(a + b)^(3/2)*d*Sqrt[-(a*Cos
[c + d*x])]*Sqrt[a + b*Sec[c + d*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2765 vs. $2(122) = 244$.
time = 0.18, size = 2766, normalized size = 19.48

method	result	size
default	Expression too large to display	2766

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(2*cos(d*x+c)*(a-b)^(3/2)*a^(9/2)*ln(4*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)+2*cos(d*
x+c)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a
^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*c
os(d*x+c)+2*b)*(a-b)^(3/2)*a^(7/2)*b-2*cos(d*x+c)*(a-b)^(3/2)*a^(5/2)*ln(4*
((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+
2*b)*b^2+2*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*
x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
+4*a*cos(d*x+c)+2*b)*(a-b)^(3/2)*a^(7/2)*b-2*cos(d*x+c)*ln(4*((b+a*cos(d*x+
c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*(a-b)^(3/
2)*a^(3/2)*b^3+2*(a-b)^(3/2)*a^(5/2)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^2-cos(d*x+c)*(a-b)^(3/2)*(
a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a^4+cos(d*x+c)
*(a-b)^(3/2)*(a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*
a^3*b-2*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c
)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*
a*cos(d*x+c)+2*b)*(a-b)^(3/2)*a^(3/2)*b^3+4*cos(d*x+c)*(a-b)^(3/2)*((b+a*co
s(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*a^2*b^2+4*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(3/2)*a*b^3-(a-b)^(3/2)*(
a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a^3*b+(a-b)^(3
/2)*(a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x
```

```
+c))^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+b*cos(d*x+c)+2*(a+b)^(1/2)*((b+a*c
os(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+b)/(-1+cos(d*x+c)))*a^2*b^2-2
*(a-b)^(3/2)*a^(1/2)*ln(4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)*cos(d*x+c)*a^(1/2)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)+4*a*cos(d*x+c)+2*b)*b^4+cos(d*x+c)*ln(-(-1+cos(d*x+c))*(2*((b+a*
cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b
+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c
)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^6+cos(d*x+c)*ln(-(-1+cos(d*x+
c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b
)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-
2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^5*b-cos(d*x+c)*l
n(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*
cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^4
*b^2-ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^
(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^
2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2
))*cos(d*x+c)*a^3*b^3+4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)*(a-b)^(3/2)*a^2*b^2+4*((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)*(a-b)^(3/2)*a*b^3+ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x+c)/
(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+
c)^2/(a-b)^(1/2))*a^5*b+ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*cos(d*x
+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(
d*x+c)^2/(a-b)^(1/2))*a^4*b^2-ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c))*cos(
d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x+c))*c
os(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*x+c)-b
)/sin(d*x+c)^2/(a-b)^(1/2))*a^3*b^3-ln(-(-1+cos(d*x+c))*(2*((b+a*cos(d*x+c)
)*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*(a-b)^(1/2)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+b*cos(d*
x+c)-b)/sin(d*x+c)^2/(a-b)^(1/2))*a^2*b^4)*cos(d*x+c)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)*4^(1/2)/((b+a*cos(d*x+c))*cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
/(b+a*cos(d*x+c))/sin(d*x+c)^2/(a-b)^(5/2)/a^2/(a+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(122) = 244$.

time = 46.02, size = 3924, normalized size = 27.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) + (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x
 + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a
^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 +
 a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a
^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - (a^4*b - 2*a^3*
b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((
8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a
*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2
*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4
*(2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*c
os(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - 2*(a^4*b - 2*a^2*b^3 + b^5 +
(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 -
 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b - 2*a^3*b^2 + a^2*b^3 +
(a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b +
b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(
d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b
^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a
```

```
^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + b)) + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d
*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^4*b - 2*a^3*b^2 +
 a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2
 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(
d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b +
3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*
b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/4*(2*(a
^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-
a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*
x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 -
2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 +
2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*c
os(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3
+ b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) - 2*(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c)
)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

3.339 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{(4a-7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d}+\frac{(4a+7b)\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{4(a+b)^{5/2}}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*(4*a-7*b)*arctanh(
(a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*(4*a+7*b)*arctanh((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d+2*b^4/a/(a^2-b^2)^2/d/(a+b*s
ec(d*x+c))^(1/2)+1/4*(a+b*sec(d*x+c))^(1/2)/(a+b)^2/d/(1-sec(d*x+c))+1/4*(a
+b*sec(d*x+c))^(1/2)/(a-b)^2/d/(1+sec(d*x+c))
```

Rubi [A]
time = 0.29, antiderivative size = 316, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3970, 912, 1349, 212, 205}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)^2(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)^2(\sec(c+dx)+1)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{5/2}}+\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2d(a-b)^{5/2}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) - (b*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) + (b*ArcTanh
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) + ((2*a + 3*b)*
ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) + (2*b^4
)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + Sqrt[a + b*Sec[c + d*x]]/(
4*(a + b)^2*d*(1 - Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)^2*d
*(1 + Sec[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4(a-x^2)} - \frac{1}{4(a-b)b^3(a-b-x^2)^2} + \frac{2a-3b}{4(a-b)^2b^4(a-b-\right.\right.}{d}$$

$$= \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx}\right.}{ad}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a-b}}\right.}{2(a-b)^{5/2}d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a-b}}\right.}{2(a-b)^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.86, size = 381, normalized size = 1.61

$$\frac{\left(\frac{\left(\sqrt{a}(a+b)^{5/2}\left(8(-a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b+a\cos(c+dx)}}{\sqrt{-a\cos(c+dx)}}\right)-ia^{3/2}(4a-7b)\text{ArcTan}\left(\frac{a+a\cos(c+dx)+i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\right)+ia^2(-a+b)^{5/2}(4a+7b)\tanh^{-1}\left(\frac{a-a\cos(c+dx)-i\sqrt{-a\cos(c+dx)}\sqrt{b+a\cos(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\right)(b+a\cos(c+dx))^{3/2}}{(-a+b)^{5/2}(a+b)^{5/2}\sqrt{-a\cos(c+dx)}} - \frac{\sqrt{a}(b+a\cos(c+dx))(a^4-3a^2b^2-4b^4-2ab(a^2-b^2)\cos(c+dx)+(a^4+a^2b^2+4b^4)\cos(2(c+dx)))\csc^2(c+dx)}{(a^2-b^2)^2}\right)\sec(c+dx)}{4a^{3/2}d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((((Sqrt[a]*(a + b)^(5/2)*(8*(-a + b)^(5/2)*ArcTan[Sqrt[b + a*Cos[c + d*x]]
/Sqrt[-(a*Cos[c + d*x])]] - I*a^(3/2)*(4*a - 7*b)*ArcTan[(a + a*Cos[c + d*x
] + I*Sqrt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[-a +
b])]) + I*a^2*(-a + b)^(5/2)*(4*a + 7*b)*ArcTanh[(a - a*Cos[c + d*x] - I*Sq
rt[-(a*Cos[c + d*x])]*Sqrt[b + a*Cos[c + d*x]])/(Sqrt[a]*Sqrt[a + b])])*(b
+ a*Cos[c + d*x])^(3/2))/((-a + b)^(5/2)*(a + b)^(5/2)*Sqrt[-(a*Cos[c + d*x
])]) - (Sqrt[a]*(b + a*Cos[c + d*x])*(a^4 - 3*a^2*b^2 - 4*b^4 - 2*a*b*(a^2
- b^2)*Cos[c + d*x] + (a^4 + a^2*b^2 + 4*b^4)*Cos[2*(c + d*x)])*Csc[c + d*x
]^2)/(a^2 - b^2)^2)*Sec[c + d*x])/(4*a^(3/2)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. $2(204) = 408$.
time = 0.26, size = 10977, normalized size = 46.51

method	result	size
default	Expression too large to display	10977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(202) = 404.
time = 116.62, size = 8098, normalized size = 34.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*cos(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x
+ c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(a)*log(-8
*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (4*a^6*b +
 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*
b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4
*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^
2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b
+ b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))
*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*co
s(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^6*b - 5*a^5*b^2 -
 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4
*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^
3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b
^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)
```

```
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/
(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos
(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a
^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((
a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^
3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 -
a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d), -1/
16*(16*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6)*cos(d*x + c)^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c
)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(-a)*arctan(2
*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
 + c) + b)) - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - (
4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a
^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*
a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a -
b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
+ (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^
6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c)^3 - (4*a^6*b - 5*a^5
*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cos(d*x + c)^2 + (4*a^7 - 5*a^6*
b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x + c))*sqrt(a + b)*log(-((8*
a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*c
os(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b
 + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^7 +
3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x
 + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c)))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x +
c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 - (a^9 -
3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d), -1/16*(2*(4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4
 - 7*a^2*b^5 - (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d
*x + c)^3 - (4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cos(
d*x + c)^2 + (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cos(d*x
 + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*
x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + 8*(a^6*b - 3*a^4*b^3 +
 3*a^2*b^5 - b^7 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (
a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*
a^3*b^4 - a*b^6)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^
3*b^4 - 7*a^2*b^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*
cos(d*x + c)^3 - (4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)
*cos(d*x + c)^2 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*co
s(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 +
```

```
4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 -
2*cos(d*x + c) + 1)) - 8*((a^7 + 3*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (a^6
*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 2*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*
cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

3.340 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=530

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{a^2d}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a^2/d+2/3*(8*a^4-11*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^4/d/(a+b)^(1/2)+2/3*(2*a+b)*(4
*a^2+a*b-3*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b^3/d/(a+b)^(1/2)-4*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/
2)+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*a^2*sec(d*x+c)*t
an(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sec(d*x
+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A]
time = 0.89, antiderivative size = 907, normalized size of antiderivative = 1.71, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3980, 3870, 4143, 4006, 3869, 3917, 4089, 3921, 4090, 3930, 4167}

$$-\frac{2\sec(c+dx)\tan(c+dx)a^2}{b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}-\frac{4\tan(c+dx)a}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}+\frac{2(8a^2-5b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,a}{3b^4\sqrt{a+b}\,d}-\frac{4\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,a}{b^2\sqrt{a+b}\,d}+\frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2(a^2-b^2)d}+\frac{2(2a+b)(4a+b)\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{3b^3\sqrt{a+b}\,d}-\frac{4\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{b\sqrt{a+b}\,d}+\frac{2b^2\tan(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}\,a}+\frac{2\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{\sqrt{a+b}\,da}-\frac{2\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{\sqrt{a+b}\,da}-\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{da^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d)
 + (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) - (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*(2*a + b)*(
4*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
```

```
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))]])/(3*b^3*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))]])/(a^2*d) - (4*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) -
 (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
```

```
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \left(\frac{1}{(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}} + \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right)$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx\right) + \int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx + \int \frac{\text{sec}}{(a+b\text{s}}$$

$$= -\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2}{b\,(a^2-}$$

$$= -\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2}{b\,(a^2-}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{a\sqrt{a+b}\,d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{a\sqrt{a+b}\,d}$$

Mathematica [**A**]
time = 16.82, size = 859, normalized size = 1.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]
^2)^(-1)]*(8*a^3*Tan[(c + d*x)/2] + 8*a^2*b*Tan[(c + d*x)/2] - 3*a*b^2*Tan[
(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2] - 16*a^3*Tan[(c + d*x)/2]^3 + 6*a*b^2
```

```
*Tan[(c + d*x)/2]^3 + 8*a^3*Tan[(c + d*x)/2]^5 - 8*a^2*b*Tan[(c + d*x)/2]^5
 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Tan[(c + d*x)/2]^5 + 6*b^3*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b
^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/
2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] + (8*a^3 + 8*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 2*a*b*(4*a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a*b^3*d*(a +
 b*Sec[c + d*x])^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + ((b + a*
Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-8*a^2 + 3*b^2)*Sin[c + d*x])/(3*a*b^3)
 - (2*(-(a^2*Sin[c + d*x]) + b^2*Sin[c + d*x]))/(a*b^2*(b + a*Cos[c + d*x])
) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(489) = 978$.
time = 0.32, size = 1545, normalized size = 2.92

method	result	size
default	Expression too large to display	1545

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*4^(1/2)*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^3+8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-3
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3+6*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3-8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b^2+8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
```

```
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*a^3+8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-3*sin(d*x+c)*cos(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-3*sin(d*x+c)*cos(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^3+6*s
in(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*b^3-8*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-2*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-8*a^3*cos(d*x+c)^3+4*c
os(d*x+c)^3*a^2*b+3*cos(d*x+c)^3*a*b^2-3*cos(d*x+c)^3*b^3+8*cos(d*x+c)^2*a^
3-8*b*a^2*cos(d*x+c)^2-2*cos(d*x+c)^2*a*b^2+3*cos(d*x+c)^2*b^3+4*cos(d*x+c)
*a^2*b-b^2*a)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+
c)/cos(d*x+c)/b^3/a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^4}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2), x)
```

3.341 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c\cdots}{a-b}}}{ab^2d}$$

[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*tan(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.27, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4146, 4144, 4006, 3869, 3917, 4089}

$$\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ab^2d}+\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd}+\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]

[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*Tan[c + d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]])

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(a^2-b^2)+\frac{1}{2}(a^2-b^2)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a(a^2-b^2)}$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - \frac{2\int \frac{\frac{1}{2}(a^2-b^2)-\frac{1}{2}(a^2-b^2)\sec}{\sqrt{a+b\sec(c+a}}}{a(a^2-b^2)}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s}{}}}{ab^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s}{}}}{ab^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.72, size = 5162, normalized size = 15.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(315) = 630.
time = 0.21, size = 633, normalized size = 1.84

method	result
default	$-\frac{\sqrt{4}\left(\cos(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin(dx+c)\,\mathrm{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a+\cos(dx+c)\sqrt{\frac{}{(1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*4^(1/2)*(cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
```

```
)/(1+cos(d*x+c)))^(1/2)*a+cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b+((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a+((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-2*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b-cos(d*x+c)
^2*a+cos(d*x+c)^2*b+a*cos(d*x+c)-b*cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c)
)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/b/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.342 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}-\frac{2\,\text{co}}{}$$

```
[Out] 2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b
)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int}{}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a\sqrt{a+b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.15, size = 1249, normalized size = 3.60

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a -
```

```
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)
*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(318) = 636$.
time = 0.22, size = 1209, normalized size = 3.48

method	result	size
default	Expression too large to display	1209

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(a^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2*sin(d*x+c)*cos(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)*cos(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)*cos(d*x+c)+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)
-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x
+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(
d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
```

```
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+b*cos(d*x+c)^2*a-cos(d*x+c)^2*b^2-cos(d*x+c)*a*b+b
^2*cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)/a/(a+b)/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(1/(a + b/cos(c + d*x))^(3/2), x)
```

3.343 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{a^2d}$$

```
[Out] -cot(d*x+c)/d/(a+b*sec(d*x+c))^(3/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x
+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*(a^2+b^2)*cot(d*x
+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(
-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(
3/2)/d-(a^2-a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*
x+c))^(3/2)+2*b^2*(a^2+b^2)*tan(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/
2)
```

Rubi [A]

time = 0.65, antiderivative size = 664, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3981, 3870, 4143, 4006, 3869, 3917, 4089, 3960, 3918, 4088, 4090}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a +
 b]*d) - ((3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) -
Cot[c + d*x]/(d*(a + b*Sec[c + d*x])^(3/2)) + (b^2*Tan[c + d*x])/((a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*b^2*Tan[c + d*x])/((a^2 - b^2)^2*d*
```

```
Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
```

```
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \left(-\frac{1}{(a+b\sec(c+dx))^{3/2}} + \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right)\,dx$$

$$= -\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx + \int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}} - \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{1}{2}(3b)\int \frac{}{(a+}$$

$$= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}} + \frac{b^2\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2b^2\tan}{a\,(a^2-b^2)\,d\sqrt{}}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= \frac{4a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{(a-b)(a+b)^{3/2}d}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 13.68, size = 663, normalized size = 1.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(((2*a*b - a^2*Cos[c + d*x] - b^2*Co
s[c + d*x])*Csc[c + d*x])/(-a^2 + b^2)^2 - (2*b*(a^2 + b^2)*Sin[c + d*x])/(
a*(a^2 - b^2)^2) + (2*b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x
]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*Cos[(c + d*x)/2]^2*(b + a*Cos[c +
 d*x])*Sec[c + d*x]^2*((-2*I)*b*(-a^3 + a^2*b - a*b^2 + b^3)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] + I*(2*a^4 - a^3*b - 2*a^2*b^2 - 3*a*b^3 + 4*b^4)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
```

```
 - b)] - (4*I)*(a^2 - b^2)^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b
)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] -
b*Sqrt[(-a + b)/(a + b)]*(a^2 + b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(416) = 832$.
time = 0.21, size = 2239, normalized size = 4.99

method	result	size
default	Expression too large to display	2239

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-2*cos(d*x+c)^2*b^4+4*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+4*cos(d*x+c)*sin(d*x+c)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4-2*cos(d*
x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*a^4+2*a^3*b*cos(d*x+c)^2+2*a*b^3*cos(d*x+c)^2+2*a^2*b^2*cos(d*x+c)-8*co
s(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*a^2*b^2+cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+2*b^4*cos(d*x+c)+4*sin(d*x+c)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+4*sin(d*x+c)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4-2*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4
-2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*b^4-cos(d*x+c)*a^3*b-3*cos(d*x+c)*a*b^3-a^4*cos(d*x+c)^2-a^2*b^2*cos(d*x+
c)^2-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*b^4-8*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2+sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
```

```
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+6*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2+3*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3-2*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-2*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-2*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b
^3+6*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3-2*cos(d*x+c)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-2*cos(d*x+c)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*
b^2-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*a*b^3)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/sin(d*x+c)/a/(a-b)^2/(a+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.344 $\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=245

$$\frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{a^3\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\right.}{}$$

```
[Out] 3*a*b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^3*hypergeom([1, 1/2+1/2*n],[3/2+1/
2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+3*a^2*b*(cos(f*x+e)^2)^(
1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e
)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+b^3*(cos(f*x+e)^2)^(2+1/2*n)*hypergeom([2+
1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)^3*(d*tan(f*x+e))^(1+
n)/d/f/(1+n)
```

Rubi [A]
time = 0.19, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3971, 3557, 371, 2697, 2687, 32}

$$\frac{a^3(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)}+\frac{3ab^2(d\tan(e+fx))^{n+1}}{df(n+1)}+\frac{b^3\sec^3(e+fx)\cos^2(e+fx)^{\frac{n+4}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (3*a*b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^3*Hypergeometric2F1[1
, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1
+ n)) + (3*a^2*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2,
(2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n)
)/(d*f*(1 + n)) + (b^3*(Cos[e + f*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 +
n)/2, (4 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(d*Tan[e + f*x])
^(1 + n))/(d*f*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int \left(a^3(d\tan(e+fx))^n + 3a^2b\sec(e+fx)(d\tan(e+fx))^n + 3\right.$$

$$= a^3\int (d\tan(e+fx))^n\,dx + \left(3a^2b\right)\int \sec(e+fx)(d\tan(e+fx$$

$$= \frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+}{df(1+n)}$$

$$= \frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^3\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)}{df(1+n)}$$

Mathematica [A]
time = 3.66, size = 238, normalized size = 0.97

$$\frac{d(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{-n/2}\left(9a^2b(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\sec^2(e+fx)\right)\sec(e+fx)\sqrt{-\tan^2(e+fx)}+b^3(1+n)\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{5}{2};\sec^2(e+fx)\right)\sec^3(e+fx)\sqrt{-\tan^2(e+fx)}-3a^3\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{\frac{2+n}{2}}+9ab^2\left(\sqrt{-\tan^2(e+fx)}-\left(-\tan^2(e+fx)\right)^{\frac{2+n}{2}}\right)\right)}{3f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(9*a^2*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n
)/2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] + b^3*(1 + n)*
Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[e + f*x]^2]*Sec[e + f*x]^3*Sqrt[
-Tan[e + f*x]^2] - 3*a^3*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e
+ f*x]^2]*(-Tan[e + f*x]^2)^((2 + n)/2) + 9*a*b^2*(Sqrt[-Tan[e + f*x]^2] -
(-Tan[e + f*x]^2)^((2 + n)/2))))/(3*f*(1 + n)*(-Tan[e + f*x]^2)^(n/2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(fx+e\right)\right)^{3}\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{n}\left(a+b\sec\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3*(d*tan(f*x+e))**n,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
```

```
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

3.345 $\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=160

$$\frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{a^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2}\right.}{}$$

```
[Out] b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n]
,-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a*b*(cos(f*x+e)^2)^(1+1/2*
n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*t
an(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A]
time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3971, 3557, 371, 2697, 2687, 32}

$$\frac{a^2(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)}+\frac{b^2(d\tan(e+fx))^{n+1}}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^2*Hypergeometric2F1[1, (1
 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)
) + (2*a*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n
)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n))/(d*f
*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int \left(a^2(d\tan(e+fx))^n + 2ab\sec(e+fx)(d\tan(e+fx))^n + b^2\right.$$

$$= a^2\int (d\tan(e+fx))^n\,dx + (2ab)\int \sec(e+fx)(d\tan(e+fx)$$

$$= \frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+f}{df(1+n)}$$

$$= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d}{df(1+n)}$$

Mathematica [A]
time = 1.31, size = 178, normalized size = 1.11

$$\frac{d(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{-n/2}\left(2ab(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\sec^2(e+fx)\right)\sec(e+fx)\sqrt{-\tan^2(e+fx)}-a^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{\frac{2+n}{2}}+b^2\left(\sqrt{-\tan^2(e+fx)}-\left(-\tan^2(e+fx)\right)^{\frac{2+n}{2}}\right)\right)}{f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(2*a*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] - a^2*Hypergeome
tric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(-Tan[e + f*x]^2)^((2 + n
```

```
)/2) + b^2*(Sqrt[-Tan[e + f*x]^2] - (-Tan[e + f*x]^2)^((2 + n)/2))))/(f*(1
+ n)*(-Tan[e + f*x]^2)^(n/2))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (a+b\sec(fx+e))^2 (d\tan(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
[Out] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*tan(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^n (a+b\sec(e+fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

3.346 $\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=129

$$\frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\mathrm{se}}{df(1+n)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/
d/f/(1+n)+b*(cos(f*x+e)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/
2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3969, 3557, 371, 2697}

$$\frac{a(d\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{df(n+1)}+\frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/(d*f*(1 + n)) + (b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometri
c2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e
 + f*x])^(1 + n))/(d*f*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = a\int (d\tan(e+fx))^n\,dx + b\int \sec(e+fx)(d\tan(e+fx))^n\,dx$$
$$= \frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)\sec(e+fx)(}{df(1+n)}$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b\cos^2(}{}$$

Mathematica [A]
time = 0.72, size = 106, normalized size = 0.82

$$\frac{(d\tan(e+fx))^n\left(\frac{a\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{1+n}+b\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\sec^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] ((d*Tan[e + f*x])^n*((a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x])/(1 + n) + b*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 -
 n)/2, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^((1 - n)/2)))/f
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (a+b\sec(fx+e))(d\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan{(e+fx)})^n\,(a+b\sec{(e+fx)})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)),x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

3.347 $\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=266

$$\frac{dF_1\left(1-n;\frac{1-n}{2},\frac{1-n}{2};2-n;\frac{a+b}{a+b\sec(e+fx)},\frac{a-b}{a+b\sec(e+fx)}\right)\left(-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}\left(\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}(d\tan(e+fx}{af(1-n)}$$

```
[Out] d*AppellF1(1-n,1/2-1/2*n,1/2-1/2*n,2-n,(a-b)/(a+b*sec(f*x+e)),(a+b)/(a+b*se
c(f*x+e)))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(b*(1+sec(f*x+e
))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(d*tan(f*x+e))^(-1+n)/a/f/(1-n)+d*hypergeo
m([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(-1+n)*tan(f*x+e
)^2/a/f/(1+n)
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] Defer[Int][(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]), x]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 786 vs. 2(266) = 532.
time = 4.92, size = 786, normalized size = 2.95

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (2*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2,
```

```
((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]*(d*Tan[e + f*x])^n
)/(f*(a + b*Sec[e + f*x])*(((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n
)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*
x)/2]^2 - 16*n*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Cos[(e + f*x)/2]*Csc[e
+ f*x]^3*Sec[e + f*x]*Sin[(e + f*x)/2]^5 + 2*n*((a + b)*AppellF1[(1 + n)/2,
 n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1
+ n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)])*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2] - (2*(1 + n)*((a - b)*b
*AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e +
 f*x)/2]^2)/(a + b)] + (a + b)^2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]) + b*(a + b)*n*AppellF1[(3
+ n)/2, 1 + n, 1, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^
2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2)/((a + b)*(3 + n))))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(d \tan (fx+e))^n}{a+b \sec (fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
[Out] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan{(e+fx)})^n}{a+b\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*tan(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(e+f\,x)}\,(d\tan(e+f\,x))^n}{b+a\,\cos{(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n/(a + b/cos(e + f*x)),x)
[Out] int((cos(e + f*x)*(d*tan(e + f*x))^n)/(b + a*cos(e + f*x)), x)
```

3.348 $\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Mathematica [A]
time = 14.79, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\tan\left(c+dx\right)\right)^{m}\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e\,\tan(c+d\,x)\right)}^m\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

3.349 $\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx=\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec\left(dx+c\right)}\,\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\tan (c+dx))^m \sqrt{a+b\sec (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral((e*tan(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e\tan(c+d\,x))^m \sqrt{a+\frac{b}{\cos (c+d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

3.350 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 5.03, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*tan(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

3.351 $\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 6.03, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(e\tan(dx+c))^m}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \tan (c + dx))^m}{(a + b \sec (c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((e*tan(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\tan(c+dx))^m}{\left(a+\frac{b}{\cos(c+dx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

3.352 $\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}((a+b\sec(c+dx))^n(e\tan(c+dx))^m,x)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Mathematica [A]
time = 3.52, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\tan{(c+dx)})^m\,(a+b\sec{(c+dx)})^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
```

```
[Out] Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e\,\mathrm{tan}\left(c+d\,x\right)\right)}^m\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

3.353 $\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{a(a^2-2b^2)\,(a+b\sec(c+dx))^{1+n}}{b^4d(1+n)}-\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}+\frac{(3a^2-2b$$

```
[Out] -a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1+n)/b^4/d/(1+n)-hypergeom([1, 1+n],[2+n],
1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(3*a^2-2*b^2)*(a+b*sec(d
*x+c))^(2+n)/b^4/d/(2+n)-3*a*(a+b*sec(d*x+c))^(3+n)/b^4/d/(3+n)+(a+b*sec(d*
x+c))^(4+n)/b^4/d/(4+n)
```

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 966, 1634, 67}

$$-\frac{a(a^2-2b^2)\,(a+b\sec(c+dx))^{n+1}}{b^4d(n+1)}+\frac{(3a^2-2b^2)\,(a+b\sec(c+dx))^{n+2}}{b^4d(n+2)}-\frac{3a(a+b\sec(c+dx))^{n+3}}{b^4d(n+3)}+\frac{(a+b\sec(c+dx))^{n+4}}{b^4d(n+4)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(1 + n))/(b^4*d*(1 + n))) - (Hyperg
eometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(
1 + n))/(a*d*(1 + n)) + ((3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(2 + n))/(b^4
*d*(2 + n)) - (3*a*(a + b*Sec[c + d*x])^(3 + n))/(b^4*d*(3 + n)) + (a + b*S
ec[c + d*x])^(4 + n)/(b^4*d*(4 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^2-x^2\right)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)} + \frac{\text{Subst}\left(\int \frac{(a+x)^n\left(b^4(4+n)-a^3(4+n)x-\left(3a^2+2b^2\right)(4\cdots}{x}\cdots\right.}{b^4 d(4+\cdots}$$

$$= \frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)} + \frac{\text{Subst}\left(\int \left(-a\left(a^2-2b^2\right)(4+n)(a+x)^n + \cdots\right.\right.}{\cdots}$$

$$= -\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{1+n}}{b^4 d(1+n)} + \frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx\cdots}{b^4 d(2+n)}$$

$$= -\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{1+n}}{b^4 d(1+n)} - \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec\cdots}{\cdots}\right.}{ad(1\cdots}$$

Mathematica [A]
time = 3.26, size = 298, normalized size = 1.68

$$-\frac{\left(n(b+a\cos(c+dx))\left(-6a^2b+12b^3-6a^2bn+16b^3n+4b^3n^2+3a\left(3a^2+b^2\left(-8-n+n^2\right)\right)\cos(c+dx)+2b(1+n)\left(-3a^2+b^2\left(12+7n+n^2\right)\right)\cos(2(c+dx))+3a^3\cos(3(c+dx))-12ab^2\cos(3(c+dx))-7ab^2n\cos(3(c+dx))-ab^2n^2\cos(3(c+dx))\right)-2b^4\left(24+50n+35n^2+10n^3+n^4\right)\cos^4(c+dx)\,{}_2F_1\left(1,-n;1-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)\right)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^n}{2b^4dn(1+n)(2+n)(3+n)(4+n)\left(-1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -1/2*((n*(b + a*Cos[c + d*x])*(-6*a^2*b + 12*b^3 - 6*a^2*b*n + 16*b^3*n + 4
*b^3*n^2 + 3*a*(3*a^2 + b^2*(-8 - n + n^2))*Cos[c + d*x] + 2*b*(1 + n)*(-3*
a^2 + b^2*(12 + 7*n + n^2))*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)] - 12*
a*b^2*Cos[3*(c + d*x)] - 7*a*b^2*n*Cos[3*(c + d*x)] - a*b^2*n^2*Cos[3*(c +
```

```
d*x)]) - 2*b^4*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*Cos[c + d*x]^4*Hypergeom
etric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x
)/2]^8*(a + b*Sec[c + d*x])^n)/(b^4*d*n*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(-1
 + Tan[(c + d*x)/2]^2)^4)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(dx + c\right)\right)^{n} \left(\tan^{5}\left(dx + c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(c + dx\right)\right)^{n} \tan^{5}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^5\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

3.354 $\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx$

Optimal. Leaf size=102

$$-\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)}+\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}+\frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)}$$

```
[Out] -a*(a+b*sec(d*x+c))^(1+n)/b^2/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+
c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(a+b*sec(d*x+c))^(2+n)/b^2/d/(2+n)
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 966, 81, 67}

$$-\frac{a(a+b\sec(c+dx))^{n+1}}{b^2d(n+1)}+\frac{(a+b\sec(c+dx))^{n+2}}{b^2d(n+2)}+\frac{(a+b\sec(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] -((a*(a + b*Sec[c + d*x])^(1 + n))/(b^2*d*(1 + n))) + (Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(
1 + n)) + (a + b*Sec[c + d*x])^(2 + n)/(b^2*d*(2 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
```

```
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(a+x)^n(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}$$

$$= \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n(b^2(2+n)+a(2+n)x)}{x}\,dx, x, b\sec\right.}{b^2d(2+n)}$$

$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x}{x}\right.}{}$$

$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a\ }{ad(1+n)}$$

Mathematica [**A**]
time = 1.43, size = 118, normalized size = 1.16

$$\frac{\left(n(b+bn-a\cos(c+dx))(b+a\cos(c+dx))-b^2(2+3n+n^2)\cos^2(c+dx)\,{}_2F_1\left(1,-n;1-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)\right)\sec^2(c+dx)(a+b\sec(c+dx))^n}{b^2dn(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((n*(b + b*n - a*Cos[c + d*x])*(b + a*Cos[c + d*x]) - b^2*(2 + 3*n + n^2)*C
os[c + d*x]^2*Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c
 + d*x])])*Sec[c + d*x]^2*(a + b*Sec[c + d*x])^n)/(b^2*d*n*(1 + n)*(2 + n))
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^n\left(\tan^3\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^n \tan^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.355 $\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx$

Optimal. Leaf size=48

$$-\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n
)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3970, 67}

$$-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a*d*(1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx,x,b\sec(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 49, normalized size = 1.02

$$\frac{{}_2F_1\left(1,-n;1-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)(a+b\sec(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])]*(a
+ b*Sec[c + d*x])^n)/(d*n)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{n}\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(c+d\,x)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

3.356 $\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=162

$$-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+d$$

```
[Out] -1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n
)/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*s
ec(d*x+c))^(1+n)/(a+b)/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(
a+b*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3970, 975, 67, 845, 70}

$$-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a +
 b*Sec[c + d*x])^(1 + n))/((a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a
+ b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/
a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
```

```
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx &= -\frac{b^2\text{Subst}\left(\int \frac{(a+x)^n}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{b^2\text{Subst}\left(\int \left(\frac{(a+x)^n}{b^2x}-\frac{x(a+x)^n}{b^2(-b^2+x^2)}\right)dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}+\frac{\text{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\,\text{se}\right.}{d} \\
&= \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}+\frac{\text{Subst}}{} \\
&= \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}-\frac{\text{Subst}}{} \\
&= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}-\frac{{}_2F_1\left(1\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 1.57, size = 163, normalized size = 1.01

$$
\frac{\left(-2\,{}_2F_1\left(1,-n;1-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)+{}_2F_1\left(1,-n;1-n;\frac{(a+b)\cos(c+dx)}{b+a\cos(c+dx)}\right)+2^n\,{}_2F_1\left(-n,-n;1-n;\frac{(-a+b)\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2b}\right)\left(\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{b}\right)^{-n}\right)(a+b\sec(c+dx))^n}{2dn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((-2*Hypergeometric2F1[1, -n, 1 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])]
 + Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*
x])] + (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c
+ d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a +
b*Sec[c + d*x])^n)/(2*d*n)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \cot \left(d x+c\right) \left(a+b \sec \left(d x+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec{\left(c + d x \right)}\right)^{n} \cot{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

3.357 $\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=279

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}+\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx)}{2(a+b)d(1+n)}$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)+1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)-hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a
+b*sec(d*x+c))^(1+n)/a/d/(1+n)-1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*x+
c))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+n],
[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

Rubi [A]
time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 975, 70, 67, 845}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}-\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)^2}+\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*
(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n)) - (b*Hypergeometric2F1[2, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a
- b)^2*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*
x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \frac{b^4\text{Subst}\left(\int \frac{(a+x)^n}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{(a+x)^n}{4b^3(b-x)^2}+\frac{(a+x)^n}{b^4x}-\frac{(a+x)^n}{4b^3(b+x)^2}-\frac{x(a+x)^n}{b^4(-b^2+x^2)}\right)\,dx, x, b\sec$$

$$= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec\right.}{d}$$

$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,{}_2F}{}$$

$$= -\frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,{}_2F}{}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} + \frac{{}_2F_1\left(1,\right.}{}$$

Mathematica [A]
time = 7.09, size = 256, normalized size = 0.92

$$\frac{\cot^2(c+dx)(a+b\sec(c+dx))^n\left(a+b\sqrt{\sec^2(c+dx)}\right)\left(a(a+b)^2(2a+b(-2+n))\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\sqrt{\sec^2(c+dx)}}{a-b}\right)\tan^2(c+dx)+(a-b)\left(a(a-b)(2a-b(-2+n))\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\sqrt{\sec^2(c+dx)}}{a+b}\right)\tan^2(c+dx)-2(a+b)\left(a(1+n)\left(a-b\sqrt{\sec^2(c+dx)}\right)+2(a^2-b^2)\,{}_2F_1\left(1,1+n;2+n;1+\frac{b\sqrt{\sec^2(c+dx)}}{a}\right)\tan^2(c+dx)\right)\right)\right)}{4a(a-b)^2(a+b)^2d(1+n)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n*(a + b*Sqrt[Sec[c + d*x]^2])*(a*(a +
 b)^2*(2*a + b*(-2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sqrt[Sec
[c + d*x]^2])/(a - b)]*Tan[c + d*x]^2 + (a - b)*(a*(a - b)*(2*a - b*(-2 + n
))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sqrt[Sec[c + d*x]^2])/(a + b)]
*Tan[c + d*x]^2 - 2*(a + b)*(a*(1 + n)*(a - b*Sqrt[Sec[c + d*x]^2]) + 2*(a^
2 - b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sqrt[Sec[c + d*x]^2])/a]
*Tan[c + d*x]^2))))/(4*a*(a - b)^2*(a + b)^2*d*(1 + n))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\cot^3(dx+c)\right)\left(a+b\sec(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \cot^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.358 $\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left((a+b\sec(c+dx))^n\tan^4(c+dx),x\right)$$

[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]

Rubi steps

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx$$

Mathematica [A]
time = 6.97, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]

[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\tan^4(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^n \tan^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c + d\,x)^4 \left(a + \frac{b}{\cos (c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.359 $\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal. Leaf size=237

$$\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(c}{bd\sqrt{1+\sec(c+dx)}}$$

```
[Out] (a+b)*AppellF1(1/2,-1-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+b))^n)/(1+s
ec(d*x+c))^(1/2)-a*AppellF1(1/2,-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+
b))^n)/(1+sec(d*x+c))^(1/2)-Unintegrable((a+b*sec(d*x+c))^n,x)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^n) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
+ b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^n) - Defer[Int][(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \left(-1+\sec^2(c+dx)\right)\,dx$$

$$= \frac{\int (-b-a\sec(c+dx))(a+b\sec(c+dx))^n\,dx}{b} + \frac{\int \sec(c+dx)(a+}{}$$

$$= -\frac{a\int \sec(c+dx)(a+b\sec(c+dx))^n\,dx}{b} - \frac{\tan(c+dx)\text{Subst}\Big(\int \frac{}{\sqrt{}}}{bd\sqrt{1-\sec(c}}$$

$$= \frac{(a\tan(c+dx))\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{(}{}$$

$$= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a}{bd\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a}{bd\sqrt{1+\sec(c+dx)}}$$

Mathematica [A]
time = 3.68, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^n \left(\tan^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec{(c+dx)})^n \tan^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^2\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.360 $\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\cot^2(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^2\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.361 $\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\cot^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [A]
time = 5.82, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^4\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.362 $\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left((a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [A]
time = 5.35, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\tan^{\frac{3}{2}}(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

3.363 $\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sec(c+dx))^n\sqrt{\tan(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\, dx = \int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\, dx$$

Mathematica [A]
time = 6.78, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\left(\sqrt{\tan}\left(dx+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec{(c + dx)})^n \sqrt{\tan{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(tan(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\tan\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

3.364 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Mathematica [A]
time = 6.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^n}{\sqrt{\tan(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(tan(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.365 $\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [A]
time = 7.55, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec{(c + dx)})^n}{\tan^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n/tan(c + d*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```